Chapter 1

Reassessing Dynamic Efficiency

Abstract

In a seminal paper, Abel et al. (1989) argue that the United States and 6 other major advanced economies are dynamically efficient. Updating data on mixed income and land rents, I find in contrast that the criterion for dynamic efficiency is not verified for any advanced economy ; and that Japan and South Korea have unambiguously over-accumulated capital. This world "savings glut" can potentially explain otherwise hard-to-understand macroeconomic stylized facts (low interest rates, cash holding by firms, financial bubbles). Subject to some caveats, an increase of public debt, or a generalization of pay-as-you-go systems could therefore be Pareto-improving.

Keywords: Dynamic efficiency, saving and investment, intergenerational income distribution.

JEL classification: E21, E22, E24

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Introduction

Dynamic efficiency is an important macroeconomic issue. The presumption that increasing investment is always good for the economy relies on the fact that more investment leads to more output in the long run, and that more output helps achieve higher consumption. Government debt is similarly often criticized as having crowding-out effect on capital accumulation, because decreasing investment is assumed to be detrimental to economic growth. But this need not be the case. A competitive equilibrium with optimizing agents, market clearing, price taking and rational expectations can fail to be Pareto-optimal when interest rates are low in the competitive equilibrium, even without assuming any type of inefficiency (externalities, information asymmetries, or price stickiness): all that is needed is that the economy is expected to run forever, which is ruled out in the canonical Arrow-Debreu model.[1] Samuelson (1958) and Diamond (1965) showed that the government can in this case make every agent better off by borrowing.[2] [3]

For example, in a Diamond (1965) overlapping-generations capital accumulation model without uncertainty, the capital stock can be too large in the competitive steady-state: in that case, the interest rate r is lower than the rate of growth of the economy g. The economy is *dynamically inefficient*. To the contrary, the economy is dynamically efficient if the interest rate is high: $r > g$. When $r = g$, the economy is said to be at the Golden Rule level of capital accumulation.[4]. This result is not as counterintuitive as it might seem at first glance. Sure, more investment always leads to higher output. But if agents care about consumption, and not output, a social planner might realize that maintaining the capital/output ratio requires more in investment each year (gK) than the economy actually produces in capital income (rK). Intuitively, a capital sector that is on net producing more output than it is using for new investment is contributing to consumption, whereas one that is using more in resources than it is producing is a sink for scarce resources.

The idea that the economy has accumulated enough capital may seem counterintuitive, to say the least, to many. With an ageing population, shouldn't the old generation care more about the less numerous young generation and help reduce public debt, rather than the other way around? But the idea behind dynamic inefficiency (and in the OLG model) is that capital might actually do a poor job at transferring resources ; it might well be more efficient to have the young work for the old in each period (a pay-as-you-go system), rather than having every individual save for retirement through its own retirement account, because capital is not so efficient at producing fruits needed at retirement. More importantly, this analysis is not only normative but also positive. For if the economy is in a dynamically inefficient state then rational bubbles can appear[5], and transfer resources from young to old agents though

[1]Technically, the failure of the first welfare theorem relies on the double infinity of agents (new agents are born in each period $t = 1, 2, ..., \infty$) and goods (if the consumption good is perishable, then there is at least one good in each period $t = 1, 2, ..., \infty$), as explained in Shell (1971). In order to prove the first welfare theorem, one needs that the sum of endowment values are summable at the equilibrium price vector, and hence that interest rates are not too low. The first welfare theorem extends to cases in which there are an infinite number of agents or of goods, but not when there are both an infinite number of agents and goods. What is interesting is that the usual suspects for inefficiencies are here absent : there are no externalities, no information asymmetries, and all markets are assumed to be clearing (no price stickiness in particular). As Geanakoplos (2008) remarks: "On the whole, it seems at least as realistic to suppose that everyone believes the world is immortal as to suppose that everyone believes in a definite date by which it will end. (In fact, it is enough that people believe, for every T, that there is positive probability the world lasts past T.)"

[2]The government can just as well run other types of Ponzi-schemes: social security, money, etc.

[3]Of course, unbacked public debt is akin to a rational bubble, and its uncertain refinancing can introduce additional uncertainty in the economy. But the government can introduce a broad spectrum of measures to ensure coordination on this rational bubble rather than on others, ranging from accepting Treasuries in open-market operations to outright financial repression. About rational bubbles, see Tirole (1985) and Santos and Woodford (1997). Hellwig and Lorenzoni (2009) deals with rational bubbles in international debt: it shows that it can only be sustained is the economy is dynamically inefficient, so that the gain from defaulting is not too high.

[4]On the Golden-Rule level of capital, see Ramsey (1928), Phelps (1961), Phelps (1965), Cass (1965), Diamond (1965)

[5]This is because bubbles can in that case grow at a rate higher than the rate of interest, even with a constant fraction of optimists in the population: their wealth grows at rate g, while the interest rate is $r < g$. Dynamic inefficiency thus provides a strong rationale for buying overvalued assets and chasing capital gains rather than dividends.

in a more unpredictable way.[6] Still on the positive front, dynamically inefficient economies have many of the properties of Keynesian economies as discussed by Geanakoplos (2008): in particular, they are isomorphic to Arrow-Debreu economies where markets do not clear at infinity. They therefore leave room for "animal spirits" to determine prices and drive business cycles.

But the real world provides the empiricist with multiple interest rates r to choose from. Should he use the safe interest rates on government bonds, in which case he would conclude that the economy is strongly inefficient?[7] Or should he use returns on equities, which almost always exceed the rate of growth?[8] Abel et al. (1989) extend the overlapping-generations capital accumulation model to account for more general production functions, in particular stochastic, with risk-varying interest rates. They conclude that the following sufficient criterion should be looked at: if capital income always exceeds investment, then the economy is in a dynamically efficient state. If investment to the contrary always exceeds capital income, then the economy is dynamically inefficient.[9] Empirically, Abel et al. (1989) find that this sufficient criterion for dynamic efficiency is satisfied by a wide margin for the United States $(1929-1985)$ and 6 other advanced economies $(1960-1985)$.

In contrast, I find that sufficient conditions for dynamic efficiency are verified for none of the advanced economies. To the contrary, Japan and South Korea verify the criterion for dynamic inefficiency.[10] And so do most advanced economies, including the United States, across a reasonable range of parameter estimates - in particular if one assumes that average Tobin's q is in the range of 1.5, an underestimation according to most analyses into this issue. For Australia and Canada, dynamic inefficiency is confirmed if Tobin's q is just a bit higher than 1. To the least, even taking a very conservative value of 1 for Tobin's q, that is assuming that monopoly rents and decreasing returns are nowhere present, dynamic inefficiency cannot be rejected using Abel et al. (1989)'s criterion. Moreover, taking the dynamic inefficiency of Japan as given (see Figure 1, which shows that investment has always been higher than capital income), an arbitrage argument suggests that other economies with which Japan has opened its capital account are dynamically inefficient too. Wouldn't capital otherwise flow out of Japan even more than it does?

Dynamic inefficiency could help explain a number of macroeconomic and financial stylized facts. A microeconomic counterpart to macroeconomic dynamic inefficiency is the claim in Fama and French (2002) and Campbell (2003), that firms seem to sink resources, since they

[6]For example, when the young buy overvalued houses from the old, it is effectively a transfer of wealth from the young to the old. Note that the bailing out of the financial sector also amounts to such a transfer that the private sector had been achieving on its own until the housing bubble collapsed.

[7]Interest rates on 10-year US Treasury Inflation Protected Securities (TIPS) are now negative.

[8]However, calculating returns on equity from stock market returns is a contaminated test of dynamic efficiency because dynamic inefficiency creates the potential for rational bubbles, which increase the rate of return to capital through capital gains. In particular, Fama and French (2002) note that the income return on book equity is much lower than the average stock return. More on this below.

[9]This criterion is actually not so different from the $g > r$ criterion. In steady-state, investment required to maintain the capital-over-output ratio is gK, while income coming from capital is $r_a K$ on average, with r_a being the average r_a obtained from investment. Hence, condition $gK > r_a K$ in all periods implies $g > r_a$. What Abel et al. (1989) therefore teach us is that one should look at the *average* interest rate in the economy, rather that at the *safe* interest rate.

[10]While one could argue that South Korea still is in a capital accumulation phase, it is certainly not the case for Japan.

get lower returns on dividends than is implied by their own measured cost of capital[11] (note that dynamic inefficiency, strictly speaking, arises when returns from dividends are lower than the rate of growth, which is usually more restrictive than the previous condition). It can explain financial instability, as with dynamic inefficiency, asset prices are no longer pinned down.[12] Linked to the possibility of rational bubbles, equity prices can command an equity premium over finitely-lived assets, if an inception of rational bubbles has increased their rate of return by higher-than-r returns from capital gains. Finally, among other examples (see Section 3), dynamic inefficiency can help shed a new light on the Japanese "lost decade", during which investment was substantially scaled down but consumption did not go down so much.[13]

The main reason for the difference between my results and Abel et al. (1989)'s results is new data from a recent release of a harmonized system of national accounts by the OECD, in particular with a different treatment of mixed income.[14] I also use more detailed data concerning land rents, which differs quite substantially from the one used in Abel et al. (1989). By means of an example, Abel et al. (1989) estimate land rents in Japan to be roughly equal to 5% of GDP, while they were according to both OECD and Goldsmith (1985)'s estimates, rather in the 17% of GDP range.

Related literature. To the best of my knowledge, no paper has reassessed dynamic efficiency since Abel et al. (1989) so far. The breadth of the literature which this paper speaks to is potentially very large, given the importance of dynamic efficiency for intergenerational transfers. From a methodological standpoint, there has been a renewed interest in national accounts data recently, in particular since the release of harmonized national accounts by the OECD. This has led to reassessing many common wisdoms. Gollin (2002) shows that labor income shares are not so variable across countries than economists once thought, and this paper tries to share the care he gives to the treatment of capital income. Caselli and Feyrer (2007) revisit Lucas' puzzle, showing that marginal product of capital is equalized across countries, once one accounts in particular for the effect of land and other non-reproducible resources. Piketty and Zucman (2012) use new balance sheet data to investigate the long run evolutions of wealth-income ratios over the courses of the nineteenth and twentieth centuries. A big advantage from using flows instead of stocks to study capital overaccumulation is that I do not have to make assumptions about the relative price of capital and consumption goods.[15]

[11] Fama and French (2002): "Most important, the average stock return for 1951 to 2000 is much greater than the average income return on book equity. Taken at face value, this says that investment during the period is on average unprofitable: its expected return is less than the cost of capital. In contrast, the lower estimates of the expected stock return from the dividend and earnings growth models are less than the income return on investment, so the message is that investment is on average profitable." ; and Campbell (2003): "if one uses average returns as an estimate of the true cost of capital, one is forced to the implausible conclusion that corporations destroyed stockholder value by retaining and reinvesting earnings rather than paying them out."

[12] The recursive equation $p_t = p_{t+1}/(1+r) + d_t$ admits an infinity of solutions depending on expectations at infinity. With dynamic efficiency, uniqueness is guaranteed through a transversality condition. Dynamic efficiency therefore comes naturally from optimization of an infinitely-lived agent, and dynamic inefficiency arises in overlapping-generations economies.

[13] Note that in an overlapping-generations model, GDP is no good measure for welfare, as more investment can lead to lower consumption always, and yet higher GDP.

[14] National Income and Product Accounts (NIPA) used by Abel et al. (1989) do not account for income of unincorporated enterprises, which include labor income.

[15] Similarly, it is tempting to back out an average return on capital from observed wealth income ratios as

From a substantive standpoint, this paper provides some support for Bernanke (2005)'s view that there indeed is a "savings glut" at the world level. It relates to a broad literature on the lack of safe assets, for example Caballero and Krishnamurthy (2006), Caballero et al. (2008) or Krishnamurthy and Vissing-Jorgensen (2012). The claim of this paper that the world does not so much lack safe assets, but lacks assets per say.[16]

The rest of the paper proceeds as follows. In section 1, I review very briefly Abel et al. (1989)'s sufficient conditions for dynamic efficiency. In Section 2.1, I reassess dynamic efficiency for the United States, using the same primary dataset as Abel et al. (1989) did use - that is, the National Income and Product Accounts - in order to highlight what differs in my calculations and leads me to a different conclusion. I then review dynamic efficiency in other advanced economies (15 additional countries) in Section 2.2. These two sections will lead to the conclusion that Japan and South Korea are dynamically inefficient, while dynamic inefficiency cannot be rejected for any advanced economy. Section 3 tries to go further at the cost of more assumptions; I let the reader make his mind about dynamic inefficiency of the other 12 advanced economies. Finally, I review in section 3 some stylized facts consistent with dynamic inefficiency, and I discuss some potential caveats to this study.

1 Sufficient conditions for dynamic efficiency

I here briefly review Abel et al. (1989)'s result that one should compare aggregate capital income and investment to assess dynamic efficiency. The setup generalizes Diamond (1965) to an economy with uncertainty, and a very general production technology.

There are overlapping generations of agents living for 2 periods: they are young then old. Time is discrete $t = 0, 1, 2, ...$ Agents have a Von-Neumann Morgentern utility function. There are L_t agents in each cohort. Young supply 1 unit of labor inelastically, get wage w_t, buy shares s_t of market portfolio V_t, and sell shares to young when old. Utility is additively separable across time for simplicity. Individual born in t solves :

$$\max_{s_t}\{u(c_t^y) + \mathbb{E}_t v(c_{t+1}^o)\}$$
$$\text{s.t.} \quad c_t^y = w_t - V_t s_t$$
$$\text{s.t.} \quad c_{t+1}^o = (D_{t+1} + V_{t+1}) s_t$$

measured for example by Piketty and Zucman (2012), and capital income: it would be Capital Income as a % of GDP divided by Wealth Income ratios as a % of GDP. This approach is however misleading, as not all future capital income is capitalized in today's capital values, like revenues from future ideas and companies. For example, assume that assets are trees decaying at a rate δ, giving a first dividend equal to R, and that new ideas come up each period to replace those decaying investments, as in Tirole (1985). Then the method outlined above would lead one to overstate return on capital by an amount δ, since the capitalized value of assets would be $R/(r + \delta)$ and capital income would be R in each period.

[16] However, the two theories have a lot in common. With dynamic inefficiency, there is a shortage of assets and so rational bubbles can form naturally as discussed above. These bubbles increase the supply of assets but as they are subject to a coordination problem, this supply is not "safe". Agents therefore would ideally need assets that they perceive as safe, even though they are valued above their fundamental values. In Krishnamurthy and Vissing-Jorgensen (2012), US Treasuries play this role.

The standard first-order condition for this maximization problem is:

$$\mathbb{E}_t \left[\frac{v'(c^o_{t+1})}{u'(c^y_t)} \frac{V_{t+1} + D_{t+1}}{V_t} \right] = 1.$$

Total consumption at date t is given by:

$$C_t = L_t c^y_t + L_{t-1} c^o_t.$$

Asset market clearing implies that $L_t s_t = 1$.

Defining profit: $\pi_t = Y_t - \frac{\partial F}{\partial L_t} L_t$, and investment I_t as being consumption's complement in output, Abel et al. (1989) prove the following proposition:

Proposition 1 (Abel et al. (1989))**.** *A sufficient condition for dynamic inefficiency is* $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - I_t \leq -\epsilon V_t$. *A sufficient condition for dynamic efficiency is:* $\exists \epsilon > 0, \forall t, \pi_t - I_t \geq \epsilon V_t$.

The intuition is pretty straightforward: the economy is dynamically inefficient if it is always investing more than it is getting from capital income, or sinking resources each period (at the steady state growth path). To the contrary, it is efficient if it is always investing less than it is getting out. The next section looks at this criterion again for the US economy first, and at other economies next; because the data sources are not the same for the two (the US has its own national accounting system, the NIPA, which Abel et al. (1989) use).

2 Reassessing dynamic efficiency

2.1 In the United States

In this section, I investigate dynamic efficiency in the United States assuming that Tobin's q is equal to 1. Since there are monopoly rents and decreasing returns to scale in the real world, this leads me to be too sanguine about dynamic efficiency (see Section 3.1 for different assumptions about Tobin q); yet the conclusion here will be that even with $q = 1$, dynamic inefficiency cannot be rejected in the United States. In order to pinpoint how my assessment differs from the seminal Abel et al. (1989), I follow them in using the National Income of Product Accounts , even though these are not the harmonized national accounts from OECD I use later.

I use the NIPA data maintained by the Bureau of Economic Analysis (BEA) to compare Gross Private Domestic Investment on the one hand, and Gross Capital Income on the other hand. While investment is available as a series, Gross Capital Income has to be calculated. Again, I follow their methodology in adding profit (including taxes on profit), rental income, interest income, capital income of proprietors, and private Capital Consumption Allowances (which are the difference of total and government Capital Consumption Allowances). On Figure 2 I compare data obtained from their Tables with data I calculate from today's series of the NIPA. I do not systematically over or understate capital income in any way, and the fit is good.

As these results are often remembered, capital income is about 25% of GDP, while investment is 15% of GDP. Therefore, dynamic efficiency seems to be satisfied by a wide margin for the United States. Yet there are 2 ways in which these calculations are being too sanguine about dynamic efficiency of the US economy. The first is that entrepreneurial income is not properly accounted for, as unincorporated enterprises are not taken into account in the NIPA, unlike in OECD mixed income. Second, because land rents are a bit higher than 5% of GDP as revealed by land values. I make both adjustments in turn:

- **Entrepreneurial income.** Quoting Mead et al. (2004), "Some aggregates exist in one system but not in the other. For example, NIPA corporate profits and personal income do not have precise counterparts in the SNA, and the SNA concept of "mixed income" - that is, the residual business income of unincorporated corporations that is attributable to labor and to capital has not been implemented in the NIPAs, pending a review of the sectoring of unincorporated businesses." NIPA only accounts for the income of proprietors, while OECD notion of mixed income includes also that of unincorporated enterprises. In other words, NIPA misses part of mixed income. How should I attribute mixed income between capital and labor income? This is extensively discussed in Gollin (2002), who goes as far as attributing the entirety of mixed income to labor. For robustness, I do not take such an extreme stand here: I only impute 2/3 of mixed income to labor. One should however bear in mind that this estimate is very conservative in many ways. First, this imputation is traditionally used since the work of Christensen (1971). At the time, proprietors' income was mostly that of farmers working in agriculture, where wages were relatively low; today, unincorporated enterprises use more skilled labor. in particular, doctors, lawyers operate in such structures. Furthermore, in most countries there exists a tax incentive to create an incorporated enterprise if the business is capital intensive; this is because unincorporated enterprise (e.g. LLC in the US) tax capital as personal income.[17] And finally, the share of capital in aggregate output is closer to 3/4 than 2/3. Figure 5 should really be seen as a higher bound on capital income excluding mixed income coming from labor.

- **Land rents, non-reproducible assets.** Land is a fixed non-reproducible factor, its return should therefore not be included to assess whether investment is on average productive or not.[18] Abel et al. (1989) use data from Rhee (1991).[19] They settle for 5% of GDP for the US, because it is consistent with an estimate of the aggregate value of land at 2/3 of GDP and a return of 8%.[20] Data on land rents is not directly available, as it is tied to residential structures, and both often trade as a bundle. I use two sources for

[17] More generally, there is an incentive to treat labor income as capital income for tax reasons. Capital income is thus always more likely to be overestimated than the contrary.

[18] In agricultural societies, one could argue that land needed to be somewhat maintained in order to remain productive. Today, land rents overwhelmingly consist of urban land, which has value because of economic geography considerations. For example, in the Alonso-Muth-Mills monocentric city model, higher land rents in the core are the exact counterpart of lower transportation cost.

[19] Rhee (1991) is using Denison (1962), itself updating Goldsmith(1959).

[20] Notwithstanding the very low assumed aggregate value of land in total assets, calculating land rents using returns is precisely what assessing dynamic efficiency is about, which is a bit circular. Instead, I will impute factor returns using their proportion in total assets.

calculating land rents in the United States: Goldsmith (1985) estimates a replacement cost of structure and attributes the remainder to land., which leads him to impute 25% of total assets to land (this is consistent with Davis and Heathcote (2007)).[21] Imputing land rents in this way leads to the results in Figure 6. Note that land is not the only non-reproducible asset: natural resources are another one. For the sake of conservativeness, I will however not make an attempt at substracting them from capital income.

2.2 In other countries

As in section 2.1, I hereby assume that Tobin's q is equal to 1, which goes against the conclusions of this paper. I compare capital income and investment for 15 additional countries (those for which OECD releases Gross Investment and Capital Income series): Australia, Belgium, Canada, Denmark, France, Germany, Hungary, Italy, Japan, Norway, Russia, South Korea, Sweden, Switzerland, United Kingdom.[22]

Reproducing. Abel et al. (1989) investigate dynamic efficiency in England, France, Germany, Italy, Canada, Japan. Their results are in Table 3 to the paper, which are plotted on Figure 9 as the difference between gross profit and gross investment. But they do not account for mixed income and land, which together account for about 9% of GDP. Note that with the Abel et al. (1989) data, Japan and Germany could already not be confirmed as being dynamically efficient, since investment is not lower than capital income over the whole period.

Updating. Land rents and mixed income do vary a lot across countries. OECD provides estimates of both for many countries. In contrast, Abel et al. (1989) used 5% of GDP for every country for lack of better data. When not available, I complement land price data with Goldsmith (1985) estimates from Comparative National Balance Sheets. Since Goldsmith (1985) does not provide data on land since 1978, I assume that land shares were constant ever since, and take the lowest of the 1973 or 1978 to be more conservative. These details are discussed more precisely in Appendix 3. Moreover, I perform a number of robustness checks in Section 3.3.

Results. I present in the main text the most inefficient economies according to OECD data: Japan and South Korea. I will not put too much of an emphasis on South Korea as it has developed only recently. The criterion for dynamic efficiency applies only at the (stochastic) steady-state, after the period of capital accumulation. Since I do not always have much data on mixed income, I present both capital income correcting for the value of mixed income and capital income containing mixed income. More precisely, capital income containing mixed income is an over-estimation of capital income coming from investment, but an under-estimation of capital income containing mixed income. This is because I calculate the value of land rents through the proportion of land in non-financial assets; so that I overestimate land rents when using capital income containing labor income from mixed income. More

[21] See Appendix 3 for a discussion on land data.

[22] I drop Mexico from the sample, because it has relative income to the US (in purchasing-power parity) significantly lower then the rest of my papers: about 30% on average. Dynamic efficiency is a steady-state concept, and such an emerging economy has not reached its steady-state of capital accumulation.

precisely, I use the following inequality:

$$\underbrace{\text{CapIncNolandNomixed}}_{\textbf{Capital Income}} = (\text{CapInc} - 2/3\text{mixed}) * \left(1 - \frac{\text{land}}{\text{assets}}\right) \leq \underbrace{\text{CapInc} * \left(1 - \frac{\text{land}}{\text{assets}}\right)}_{\textbf{Capital Income w/ mixed L}}$$

"Capital income with mixed Labor" is therefore a slight abuse of language in the graphs, only a fraction $1 - \text{land/assets}$ of mixed labor is actually included.

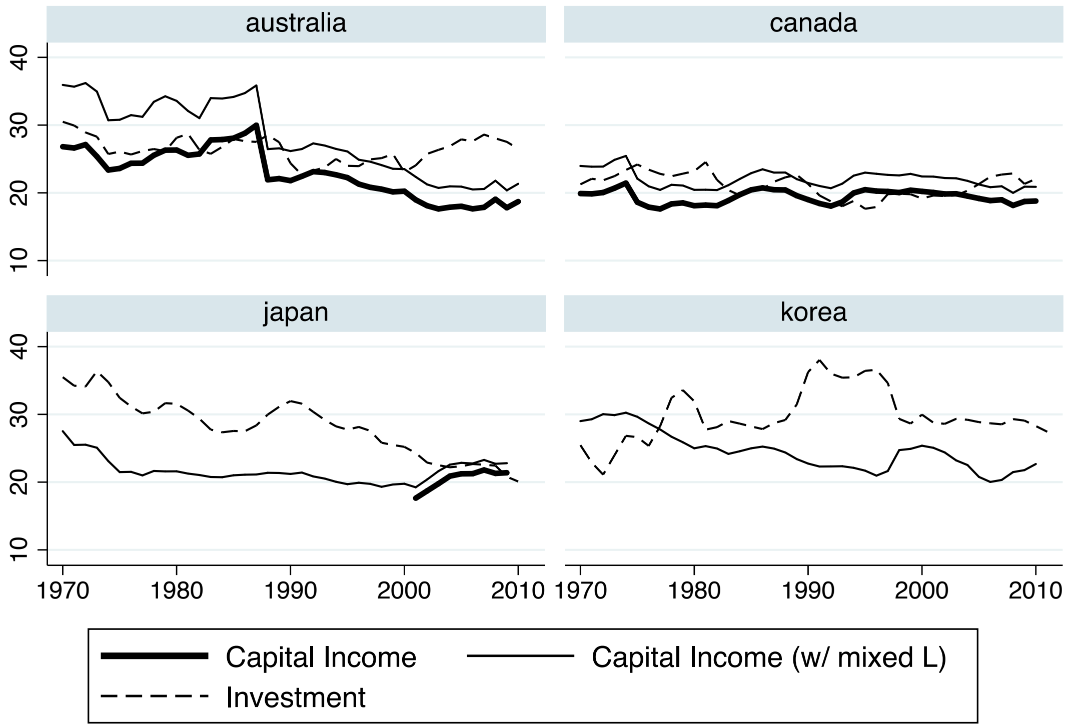
Description of the results. As can be seen on Figure 1, OECD does not provide data on mixed income for Japan before 2001. But the continuous thin line is always a higher bound on the thick line (capital income). Data from the OECD therefore suggests that Japan is inefficient. This contrasts starkly with Abel et al. (1989)'s results. The reason is that Japan has much higher land rents than the United States, because land is far more scarce. Therefore, by assuming that the United States have 5% of GDP in land revenues, Abel et al. (1989) strongly underestimate Japanese land rents. Data for mixed income in South Korea unfortunately is not available, but capital income including mixed income has been lower than investment since 1980, suggesting strong inefficiency. However, as discussed earlier, South Korea might already be in a stage of capital accumulation. Finally, Australia and Canada have low capital income compared to investment (excluding a few years in the eighties for Australia, and the nineties for Canada). This is surprising as Australia and Canada are not usual suspects for capital over-accumulation. Finally, Figures 13, 14 and 15 show that dynamic inefficiency cannot be rejected for any country (France, Germany, Hungary, Italy, Russia, Norway, Sweden, Switzerland, Beigium, United Kingdom).

3 Further calculations

From the upper left hand graph of Figure 1, one could boldly conclude that every advanced economy having an open financial account with Japan is dynamically inefficient. The reasoning is the following. In the absence of financial frictions, any dynamically efficient country could borrow from Japan and invest at a higher rate than Japan.[23] Therefore, a simple arbitrage argument would have all countries be inefficient if one of them is. But why is it then that investment is not always higher than capital income in other advanced economies as well? This section is about discussing other factors influencing the calculation of capital income, without necessarily being able to take a quantitative stance on these factors. In subsection 3.1, I argue that Tobin's average q is significantly higher than 1, which has led me to be too sanguine about dynamic efficiency. In subsection 3.2, I review other reasons which might have led me to be too sanguine about dynamic efficiency. In subsection 3.3 I do the opposite and examine the robustness of my calculations to other assumptions, notably about government investment.

[23] In practice, Japan has a current account surplus but it still is inefficient, perhaps less so today.

Figure 1: ASSESSING DYNAMIC EFFICIENCY IN AUSTRALIA, CANADA, JAPAN, SOUTH KOREA



Source: OECD, Kuznets (1985), and author's calculations

Notes: The dotted line represents Gross Capital Formation as a % of GDP. The thick continuous line is Capital Income as a % of GDP (including Capital Consumption Allowances), excluding land rents and mixed income coming from labor. Those two are to be compared to assess dynamic efficiency. Because data on mixed income is not always available, I also plot as a thin continuous line Capital Income excluding land rents but including labor income in mixed income (thus overstating "economic" Capital Income). "Korea" is South Korea. Data comes from OECD.

3.1 Taking into account Tobin's q

Extension of the Abel et al. (1989) model to decreasing returns

This model differs with Abel et al. (1989) in the production function $Y_t = F(I_{t-n}^{t-1}, L_t, \theta_t)$,, which has decreasing returns. $(I_{t-n}^{t-1} = (I_{t-1}, ..., I_{t-n}))$ Defining profit: $\pi_t = Y_t - \frac{\partial F}{\partial L_t} L_t$, and pure profit:

$$\pi_t^p = Y_t - \frac{\partial F}{\partial L_t} L_t - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}.$$

Proposition 2. ***(Decreasing returns.)*** *With decreasing returns to scale, a sufficient condition for dynamic inefficiency is* $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^p - I_t \leq -\epsilon V_t$. *Moreover,* $\exists \epsilon > 0, \forall t, \pi - I_t \geq \epsilon V_t$ *is not sufficient for dynamic efficiency. A sufficient condition for dynamic efficiency is:* $\exists \epsilon > 0, \forall t, \pi_t - \pi_t^p - I_t \geq \epsilon V_t$.

Proof. See Appendix 1.1. □

Extension of the Abel et al. (1989) model to monopolistic competition

The setup is essentially the same as in the previous section, except that the consumption good is now a constant-elasticity of substitution aggregator of different varieties, and each firm produces one variety monopolistically. More precisely, both young and old consumption goods now are a CES of different varieties $\omega \in [0,1]$, such that:

$$c_t^y = \left[\int_0^1 c_t^y(\omega)^{\frac{\theta-1}{\theta}} d\omega\right]^{\frac{\theta}{\theta-1}}, \quad c_t^o = \left[\int_0^1 c_t^o(\omega)^{\frac{\theta-1}{\theta}} d\omega\right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

Dropping generation subscripts (everything is symmetric for old and young), the demand function for a generic good and the welfare-based price index are :

$$c_t(\omega) = \left(\frac{p_t(\omega)}{p_t}\right)^{-\theta} c_t.$$

$$p_t = \left[\int_0^1 p_t(\omega)^{1-\theta} d\omega\right]^{\frac{1}{1-\theta}}$$

The model also differs from that of the previous section in that the environment is no longer competitive, but that of monopolistic competition. That is, every variety is produced by a monopolist with a constant-returns-to-scale[24] production function, defining as previously $I_{t-n}^{t-1}(\omega) = (I_{t-1}(\omega), ..., I_{t-n}(\omega))$:

$$y_t(\omega) = F(I_{t-n}^{t-1}(\omega), L_t(\omega), \theta_t).$$

Note that θ_t is a productivity shock affecting all corporations equally. Denote the value function of minimizing the labor cost for an individual monopolistic firm ω:

$$\Lambda_t(y_t(\omega)) = \min_{L_t(\omega)} \left\{w_t L_t(\omega) \text{ s.t. } y_t(\omega) = F(I_{t-n}^{t-1}(\omega), L_t(\omega), \theta_t)\right\}.$$

Again defining aggregate profit as:

$$\pi_t = \int_0^1 \left(\frac{\theta}{\theta-1}\Lambda_t'(y_t(\omega))y_t(\omega) - \Lambda_t(y_t(\omega))\right) d\omega,$$

and monopoly profit as:

$$\pi_t^m = \int_0^1 \left(\frac{\theta}{\theta-1}\Lambda_t'(y_t(\omega))y_t(\omega) - \Lambda_t(y_t(\omega))\right) d\omega - \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i},$$

allows to state the following proposition:

Proposition 3. ***(Monopoly power.)*** *With monopolistic competition, a sufficient condition for dynamic inefficiency is $\exists\epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^m - I_t \leq -\epsilon V_t$. Moreover, $\exists\epsilon > 0, \forall t, \pi - I_t \geq \epsilon V_t$ is not sufficient for dynamic efficiency. A sufficient condition for dynamic efficiency is: $\exists\epsilon > 0, \forall t, \pi_t - \pi_t^m - I_t \geq \epsilon V_t$.*

[24] Here again, I could generalize to decreasing returns to scale, adding a "pure rent" component in addition to the "monopoly rents" profits, but this would only complicate the exposition.

Proof. See Appendix 1.2. □

Discussion of Tobin's q for the United States

Based on Figure 11 from Hall (2001) (updated by Philippon (2009)), one can notice that Tobin's q departs significantly from 1. Unfortunately, Tobin's q captures adjustment costs as well as potential monopoly rents and decreasing returns to scale.[25] Given Hall (2001)'s methodology, the starting value for Tobin's q is assumed to be 1. Moreover, given Hall (2001) adjustment cost model, which by construction minimizes the distance between Tobin's q and 1 (subject to constraints), average q is likely to be underestimated.

Note that in any case, Tobin's q cannot be lower than 1 for my purposes - that is, irreversibility of investment which potentially drives Tobin's q below 1 is of no interest. Decreasing returns to scale or monopoly power cannot be negative.

The discussion about Tobin's q, and why it can so persistently depart from 1, would lead us too far for the purpose of this paper. What is important is that while Hall (2001) work in the zero-rent framework, he repeatedly cites monopoly power as a potential microfundation for adjustment costs. Tobin's q might as well capture investment in intangibles that is not taken into account in official investment data - marketing costs are expensed for example, but they bring revenues in the future. In that case, it would be wrong to interpret the entirety of Tobin's q as consequential to the presence of rents. However, if investment was abnormally high in the 1990s and early 2000s as some have suggested to explain the "dotcom bubble", then most capital income would have materialized later, which would have dramatically increased the probability of dynamic inefficiency, at least in these years. Bond and Cummins (2000) question the importance of the intangible channel and instead point to irrational valuations from the part of investment. I believe there is more to the latter story than to the former, therefore I will never consider Tobin's q on the order of 3.[26]

Robustness to Tobin's q

Because of the difficulties outlined above, I do not want to take an affirmative stand on the value of Tobin's q. Given that Japan already provides us with the presumption that advanced economies may well be inefficient, I perform a sensitivity analysis using different parameters of Tobin's q.

The results for the United States are on Figure 16, for Japan, Anstralia and Canada on Figure 17. On Figure 12, I plot the fraction of years in which investment exceeds capital income. Graphically, dynamic inefficiency can be rejected for Tobin $q = 1$ in countries for which the line begins from the x axis. However, note that I here only use the thick line from previous graphs, which means I am missing many years for which those countries were actually inefficient, as suggested by the extrapolated thin line. Dynamic inefficiency cannot be rejected for Tobin $q = 1$ in countries for which the line begins from the y axis.

[25] Although adjustment costs can be microfounded by patent rights. (Hall (2001))

[26] Note however that the presence of rational bubbles is possible only with dynamic inefficiency. One could make the following reasoning: either rational bubbles exist and dynamic inefficiency is guaranteed, or they do not and Tobin's q should be taken at face value.

3.2 Other causes for overstatement of capital income

Capital income is likely to be overestimated for multiple other reasons not mentioned above because they are hard to quantify. However, some evidence suggests that some of them might be of first order significance. To get an idea of the orders of magnitudes involved, overstating capital income by 3% of GDP for the US leads to satisfy the sufficient condition for dynamic inefficiency assuming Tobin Q is equal to 1.

Incidence of corporate taxation. In the calculations above, taxes on profit are assumed to fall entirely on capital. Hence the revenue they raise is treated as capital income. If instead taxes on profit are borne by workers or consumers, as at least then taxes on profit are not capital income. Given the order of magnitude involved ($\simeq$ 2.3% of GDP), tax incidence is not a detail. In fact, if taxes on profit do not fall at all on capital income, then the sufficient condition for dynamic inefficiency is verified in the United States with Tobin q equal to 1.

Household production in financial services. Individual investors search for stocks, and put some effort in portfolio management. There is a cost to managing one's wealth, that is not recorded in the national accounts when it is not done professionally. Some individuals indeed spend a lot of time monitoring their financial intermediaries and finding more performing ones, doing their investment in stocks, etc. Other rent their real-estate assets, and provide the labor services of choosing tenants and collecting rents. All this implies that pure capital income tends to be over-estimated. As Piketty and Saez (2011) put it, these efforts should be viewed as informal financial services that are directly supplied and consumed by households; they estimate these financial services to be of the order of **2%-3% of GDP** at the very most.

Favorable tax treatment of capital income. As pointed out previously, it is very hard to measure the share of labor in mixed income. This is all the more true that capital income is less heavily taxed than labor income in most tax systems, so that entrepreneurs have a strong incentive to make their labor revenues appear as capital revenues. Hence, revenues from LBOs are usually treated as capital gains, or as revenue accruing to investment, while it usually employs a very qualified workforce to pick these investments and "beat the market".[27] In other words, management fees often understate the return to labor - part of financiers' wages are earned through capital gains. The public finance literature has only begun to investigate this issue (for example Piketty (2011) and Piketty and Saez (2011)).

Public debt and rational bubbles. Needless to say, advanced economies have very high levels of debt. To the extent that Ricardian equivalence does not hold in an overlapping generations model, private savings do not perfectly offset these public dissavings. Hence, dynamic inefficiency would likely even be more severe absent our extraordinary levels of public debt. Rational bubbles can similarly crowd out private savings and raise consumption. To the extent that they are a feature of the real world, rational bubbles lessen the severity of dynamic inefficiency.

Other rents. Only land rents have been taken out of capital income. But there are other physical rents, of which the World Bank maintains a data series (extensively used by Caselli

[27] Very often, changing the corporate finance structure of the company and loading it with debt helps benefit from the tax deductibility of debt interest.

and Feyrer (2007)). However, it might well be that countries have not reached a steady-state of their resource-extraction path (for example, US oil), so that excluding these rents would actually lead to an underestimation of capital income. For the sake of robustness, I do not attempt at such a calculation here, which would only strengthen my conclusions.

3.3 Robustness

Government investment. In Appendix 8, I take out government investment from Investment series, when available. I also take out Public Capital Consumption Allowances from Capital Income. Note however that such a calculation leads to be too sanguine about dynamic efficiency, as government investment mostly yields private benefits in the form of private capital income. Figures 21, 22, 23 and 24 give the results.

Data on land. One might worry that the "asset approach" yields to overstate the importance of land in value added, especially if land is subject to overvaluation due to animal spirits. This potential limitation applies only to 6 countries for which I use recent OECD data. I perform a number of robustness checks in this direction. In particular I assume that land shares stayed constant after 2000 - when the run-up in house prices began in most advanced economies.

4 Discussion

4.1 Consistent stylized facts

Feldstein and Horioka (1980) puzzle. A puzzle in international finance is that investment and savings are highly correlated in the cross section of countries. This puzzle has been named the Feldstein-Horioka puzzle: investment opportunities have no reason to be present where savings rate are also higher. Current accounts should make up for the differences, but they are relatively small compared to the differences in savings and investment rates across countries. All this theoretical analysis however assumes that capital is relatively scarce. With dynamic inefficiency, asset supply is in any case too low relative to asset demand, and so even an arbitrarily small amount of home bias translates into so high level of savings-investment correlation.

Crises. The severity of dynamic inefficiency seems to be strongly correlated with key macroeconomic events, like banking crises. For example, as the second and third quadrants of Figure 14 show, Norway and Sweden had a dynamic efficiency problem before their banking crises in 1991−1993. Japan was the most severely inefficient of all countries in the 80s, and the "lost decade" corresponds to a period of declining investment and increasing consumption.[28]

4.2 Potential caveats

Externalities to capital accumulation. An important factor for the validity of this analysis that there be no externalities, or that those are negligible. In a capital accumulation

[28] Note that going from an above-Golden rule steady state growth path to a Golden-Rule one mechanically decreases output and leads to a recession, even though consumption increases.

model with externalities (e.g. Saint-Paul (1992)), capital income could understate returns to capital, and increasing public debt in case of dynamic inefficiency could lower consumption. Note however that capital externalities are usually a feature of human capital rather than physical capital.

Conclusion

"Search for yield", "scarcity of assets", "abundant liquidity", all these phrases point to the fact that there might well be too many savings chasing too few assets in the world. Because of high levels of public debt, older people are usually accused of not caring enough about their children. In contrast, this paper suggests that they still save too much, relative to the relatively few investment opportunities present in our economies.

Series from the Bureau of Economic Analysis start in 1929, just before the Great Depression; before that, national accounting was very rudimentary. But the difference between capital income and investment was similar in 1929 than it is today. Was capital also overaccumulated at the eve of the Great Depression, after the long period of capital accumulation in the nineteenth century documented in particular by Piketty (2011)?

In any case, dynamic inefficiency invites us to revisit many policy questions. In a world of too much capital accumulation, capital taxation, which is often thought of as a deterrent to capital accumulation, is perhaps not a bad idea after all.[29] At the same time, future capital taxes decrease the value of assets today, and so increase the problem of asset scarcity. Dynamic inefficiency also makes rational bubbles possible (Tirole (1985), Santos and Woodford (1997)): when assets are scarce, any real, financial or even monetary asset can become a locus for bubbles. Other social contracts such as pay-as-you-go systems to replace funded systems can also be Pareto-improving.

Solving the problem of dynamic efficiency is certainly not as straightforward as Diamond (1965) suggested. In practice, the market may have a hard time coordinating on so high implied levels of debt. This is all the more true that this debt is not an infinitely lived asset, and has to be refinanced from time to time. Moreover, it always is subject to the risk of default, even if Hellwig and Lorenzoni (2009) show that the costs of defaulting (being further excluded from the financial markets) far exceeds the benefits with dynamic inefficiency. Moreover, I suspect that markets can very well coordinate on other rational bubbles that are equally fragile; and that public debt does not have a special status in this respect. I leave this very important issue for future research.

[29] However note that Piketty and Saez (2011) recommend in their Appendix to use public debt for dealing with dynamic inefficiency, to restore the Golden Rule level of capital accumulation; and use capital taxes for redistribution motives. There is a dichotomy with 2 instruments and 2 objectives.

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1 Proofs

1.1 Proof of Proposition 1

I will go over the sufficient condition for dynamic inefficiency in detail, since the condition is less restrictive as in Abel et al. (1989). The proof for the sufficient condition for dynamic efficiency is very similar.

Proof. Assume that : $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^p - I_t \leq -\epsilon V_t$. Then by definition

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i} - I_t \leq -\epsilon V_t \leq -\epsilon I_t$$

Therefore:

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon.$$

Let us consider now an increase in consumption financed by a decrease in investment (so that it is resource-feasible), and the size of this increase be $\delta > 0$, $dc_0 = \delta$ small $\Rightarrow dC_0 = \delta L_0, dI_0 = -\delta L_0$. To make this change Pareto-improving, one has to make up for the decrease in output in the following periods by reducing investment as well. From the production function, production in period 1 decreases by $dY_1 = \frac{\partial F}{\partial I_0} dI_0$, and so for unchanged consumption $dC_1 = 0$, $dI_1 = \frac{\partial F}{\partial I_0} dI_0$. More generally, today's output will be reduced by the n previous reductions in investment of all vintages. The general formula is:

$$dI_t = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} dI_{t-i}$$

$$\Rightarrow \frac{dI_t}{I_t} = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \frac{dI_{t-i}}{I_{t-i}}.$$

Of course, there is a limit to how much one can reduce investment in each period, since there is a non-negativity constraint on investment. Since $\forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon$, such a change is feasible for $\delta > 0$ sufficiently low. $\square$

Once can similarly extend Abel et al. (1989) to sufficient conditions for efficiency, redefining profit in an adequate manner.

1.2 Proof of Proposition 2

Again, I will only go over the proof for dynamic inefficiency.

Proof. Let us first calculate each firm's profit. Facing demand for its product as:

$$y_t(\omega) = \left(\frac{p_t(\omega)}{p_t} \right)^{-\theta} y_t,$$

a monopolistic firm chooses $p_t(\omega)$ so as to maximize its profit, that is:

$$\max_{p_t(\omega)} \left\{ p_t(\omega) \left(\frac{p_t(\omega)}{p_t} \right)^{-\theta} y_t - \Lambda \left(\left(\frac{p_t(\omega)}{p_t} \right)^{-\theta} y_t \right) - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}(\omega) \right\}.$$

The firm charges a markup over marginal labor cost:

$$p_t(\omega) = \frac{\theta}{\theta - 1} \Lambda_t'(Y_t(\omega)).$$

Monopoly profits for a single firm are therefore:

$$\pi_t^m(\omega) = \frac{\theta}{\theta - 1} \Lambda_t'(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)) - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}(\omega).$$

Total profits for a monopolistic firm, including returns to capital are:

$$\pi_t(\omega) = \frac{\theta}{\theta - 1} \Lambda_t'(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)).$$

Using that the returns on capital for each firm are equalized because it is supplied competitively, I can sum over ω to find:

$$\pi_t^m = \int_0^1 \pi_t^m(\omega) d\omega = \int_0^1 \left(\frac{\theta}{\theta - 1} \Lambda_t'(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)) \right) d\omega - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}.$$

Now as in the previous proof, assume that : $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^m - I_t \leq -\epsilon V_t$. Therefore:

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon.$$

Let us consider now an increase in consumption financed by a decrease in investment in each monopolistic firm (so that it is resource-feasible), and the size of this increase be $\delta > 0$, $dc_0 = \delta$ (per-capita consumption) small $\Rightarrow dC_0 = \delta L_0$, so that the aggregate decrease in investment must be $dI_0 = -\delta L_0$. Let us split this decrease in investment equally among firms so that $dI_0 = \int_0^1 dI_0(\omega) d\omega$. To make this change Pareto-improving, one has to make up for the decrease in output in each firm in the following periods by reducing investment as well. From the production function, production in period 1 decreases by $dY_1(\omega) = \frac{\partial F}{\partial I_0} dI_0(\omega)$ in each firm, and so for unchanged consumption $dC_1 = 0$, $dI_1(\omega) = \frac{\partial F}{\partial I_0} dI_0(\omega)$ in order to maintain the same level of production in each firm. More generally, today's output will be reduced by the n previous reductions in investment of all vintages. The general formula is:

$$\forall \omega \in [0, 1], dI_t(\omega) = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} dI_{t-i}(\omega).$$

Summing, and because marginal returns to capital are equalized across firms:

$$dI_t = \int_0^1 dI_t(\omega) d\omega = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \int_0^1 dI_{t-i}(\omega) d\omega = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} dI_{t-i}$$

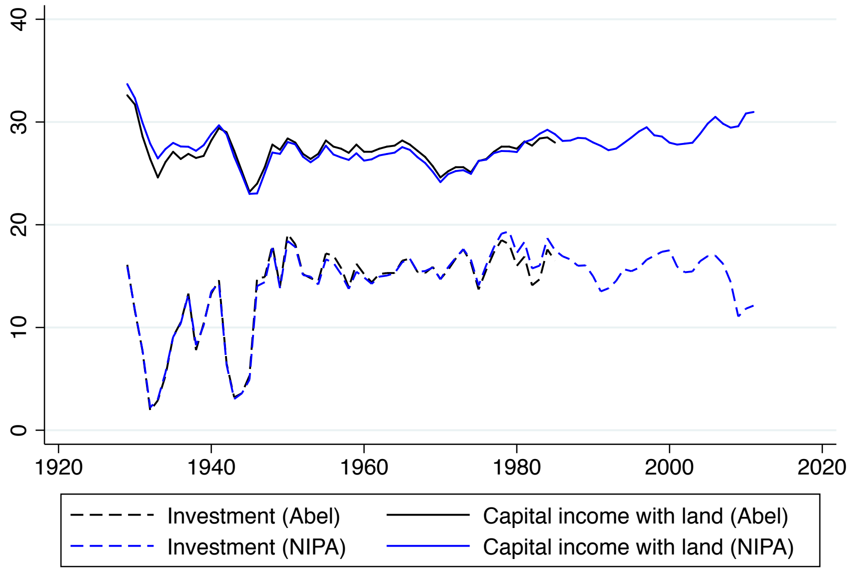
$$\Rightarrow \frac{dI_t}{I_t} = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \frac{dI_{t-i}}{I_{t-i}}.$$

Of course, there is a limit to how much one can reduce investment in each period, since there is a non-negativity constraint on investment. Since $\forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon$, such a change is feasible for $\delta > 0$ sufficiently low.

□

2 Reproducing Abel et al. (1989) step by step for the United States

Figure 2: REPRODUCING AND UPDATING ABEL ET AL. (1989)



Source: National Income and Product Account (NIPA) and Abel(1989)'s calculations

3 Data on land

Data on land comes mainly from two sources:

- OECD for 6 countries (Australia, Canada, Czech Republic, France, Japan, Korea). Data for OECD is plotted on Figure 7.
- Goldsmith (1985) *Comparative National Balance Sheets*, for 20 countries (with two dates 1973 and 1978).

For other countries, I take the lowest value in 1973 or 1978 from Goldsmith for all years. This is very conservative, especially for the last decade. Furthermore, Goldsmith underestimates relative to OECD (although it displays a similar evolution).

Figure 3: Removing land rents at 5% of GDP

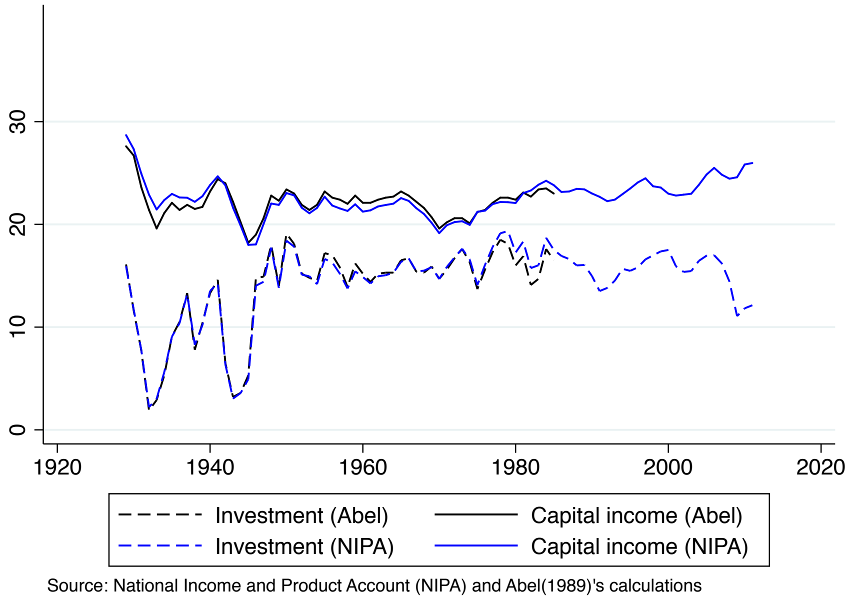
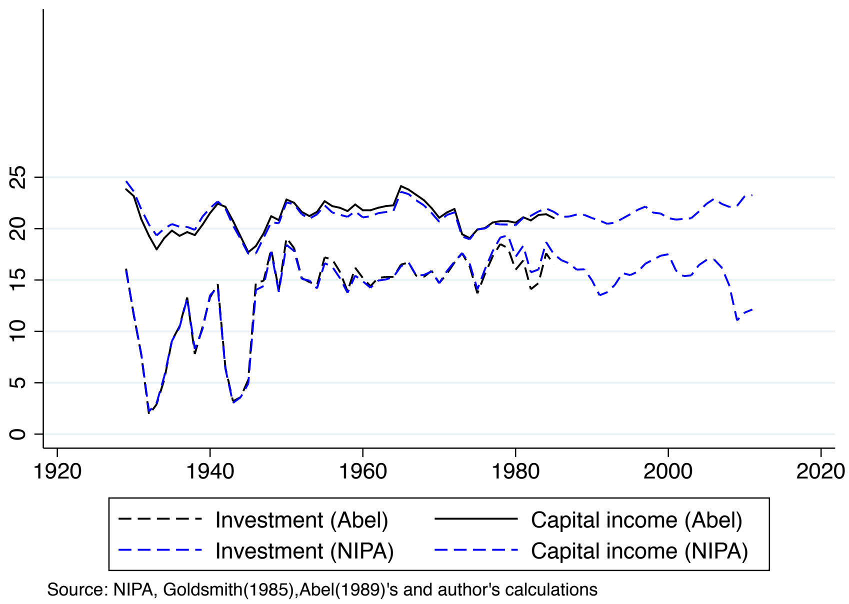


Figure 4: Using OECD data for land rents



In order to impute land income, I use an asset approach. That is, I assume that assets produce revenues in proportion to their relative importance in the capital stock. As discussed in Kravis

Figure 5: USING OECD DATA FOR MIXED INCOME

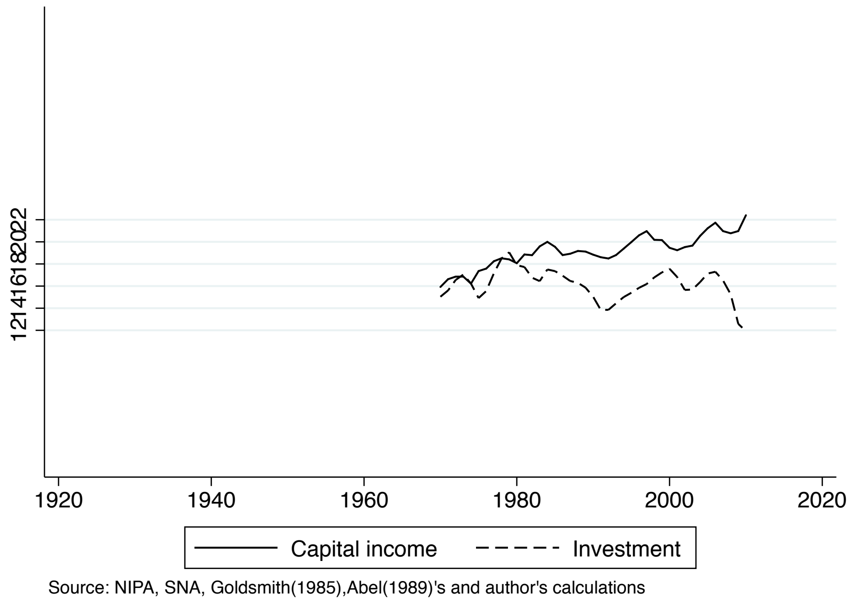
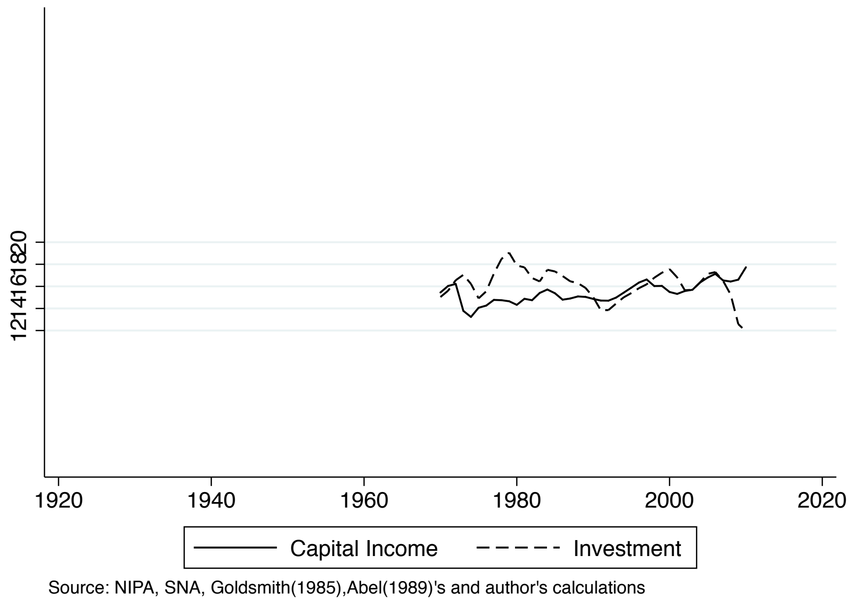


Figure 6: PREFERRED SPECIFICATION, ASSUMING TOBIN $q = 1$

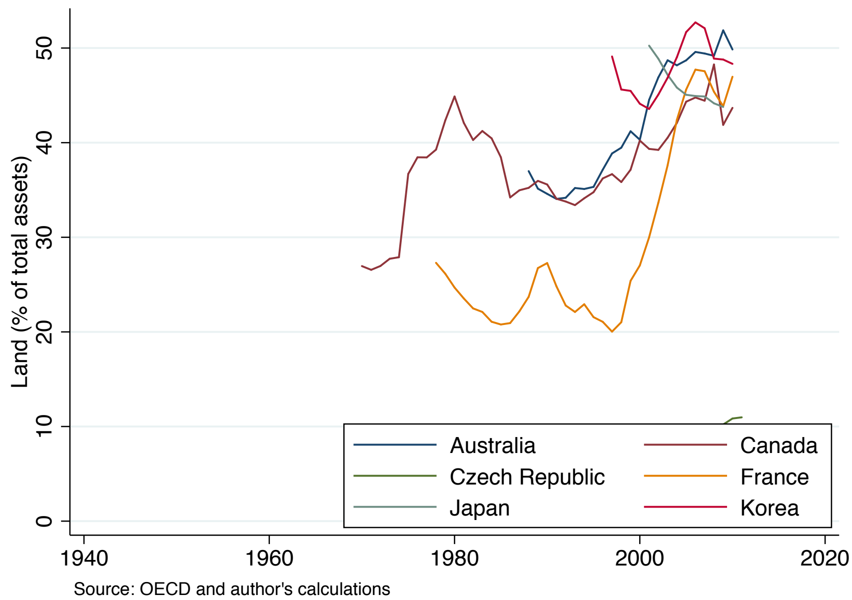


4 Reproducing Abel et al. (1989) step by step

4.1 United States

Abel et al. (1989) use data from the National Income and Products Accounts (NIPA). Raw data from Table 1 is reproduced in the upper left hand corner of Figure 8. Upper right hand corner updates this data with contemporaneous data given by the Bureau of Economic

Figure 7: Land as a % of total assets



Analysis. The numbers are very similar ; this confirms that I am using the same methodology for calculating capital income. In the bottom left hand corner, I use their estimate of land rents (5% of GDP), which I substract from capital income. Finally, I use more recent data on land rents in the bottom right hand corner.

4.2 UK, France, Germany, Canada, Italy, Japan

They then use gross investment and gross capital income coming from OECD database. Results from Table 3 are given in Figure 9. Note that even with their data, and taking an estimate of 5% of GDP for land rents and 4% of GDP for labor mixed income, dynamic efficiency could not be confirmed in Japan, Germany, United Kingdom, and Canada.

As shown on Figure 10, my estimates sometimes coincide, sometimes are higher for capital income than theirs. If anything, overstating capital income goes against the conclusions of this paper, and should only reinforce them.

5 Data on Tobin's q

6 Inefficiency as a function of Tobin q

This figure 12 summarizes the preceding graphs in $3D$. It plots the fraction of years for which investment exceeds capital income in the data, as a function of assumed Tobin q. When equal to 0%, the sufficient condition for efficiency is satisfied. When equal to 100%, the economy has unambiguously sinked resources into investment, and is dynamically inefficient.

Figure 8: Modifying Abel and al. (1989), step by step

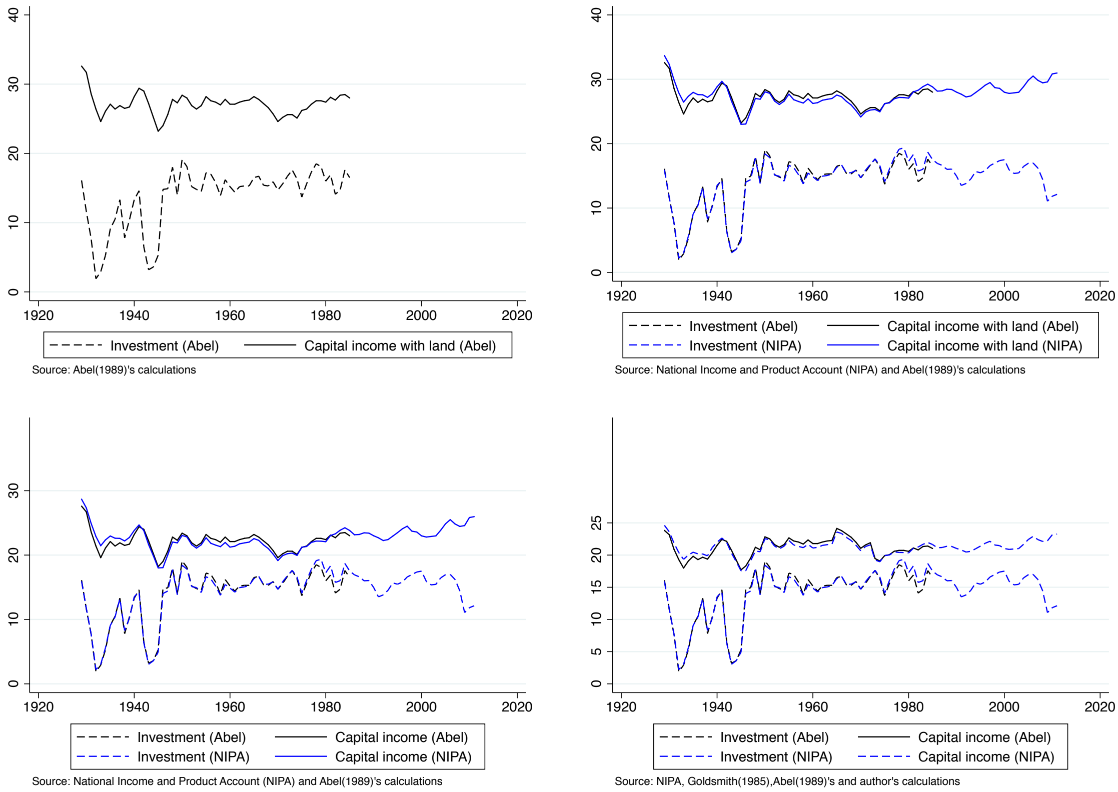


Figure 9: Plotting results from Table 3

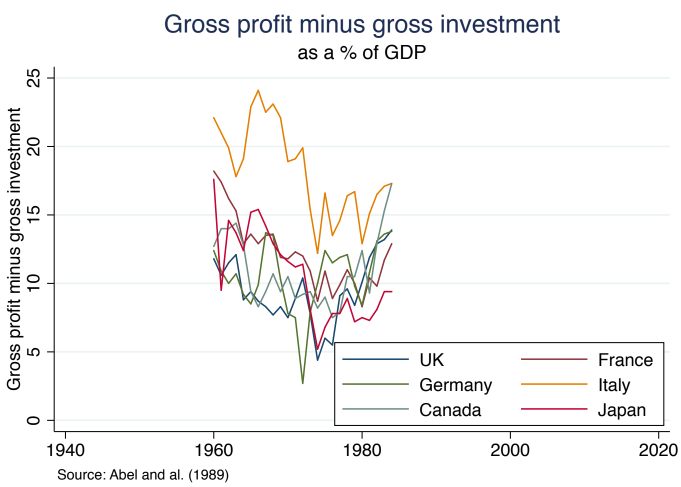


Figure 10: Reproducing for 6 OECD economies

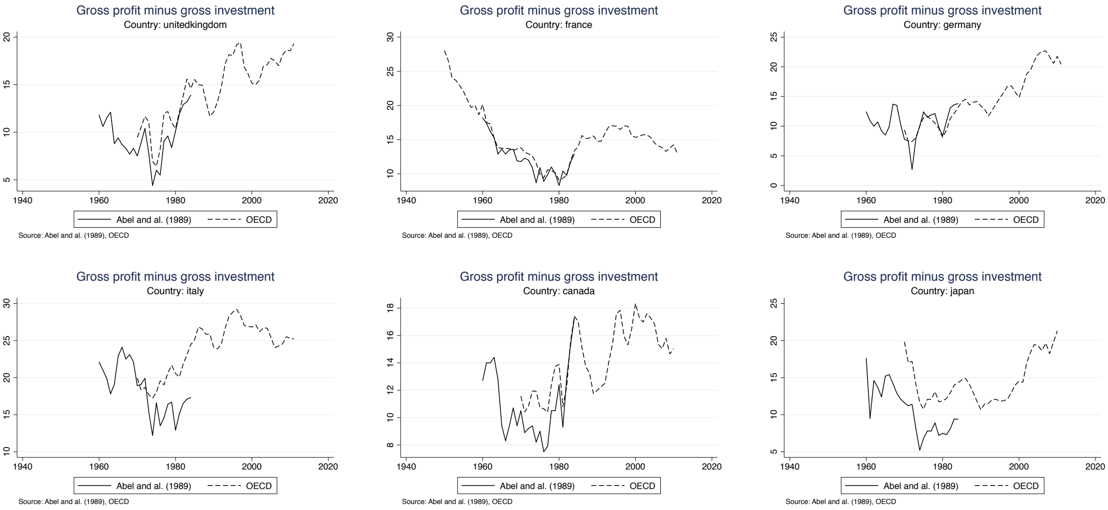
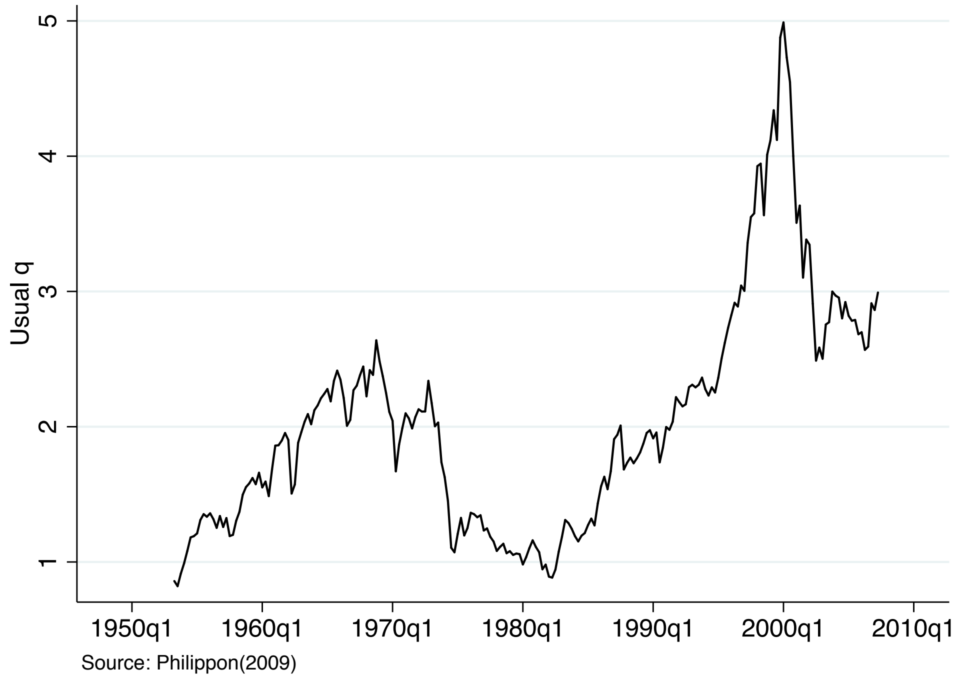


Figure 11: Tobin's q, using equity values.

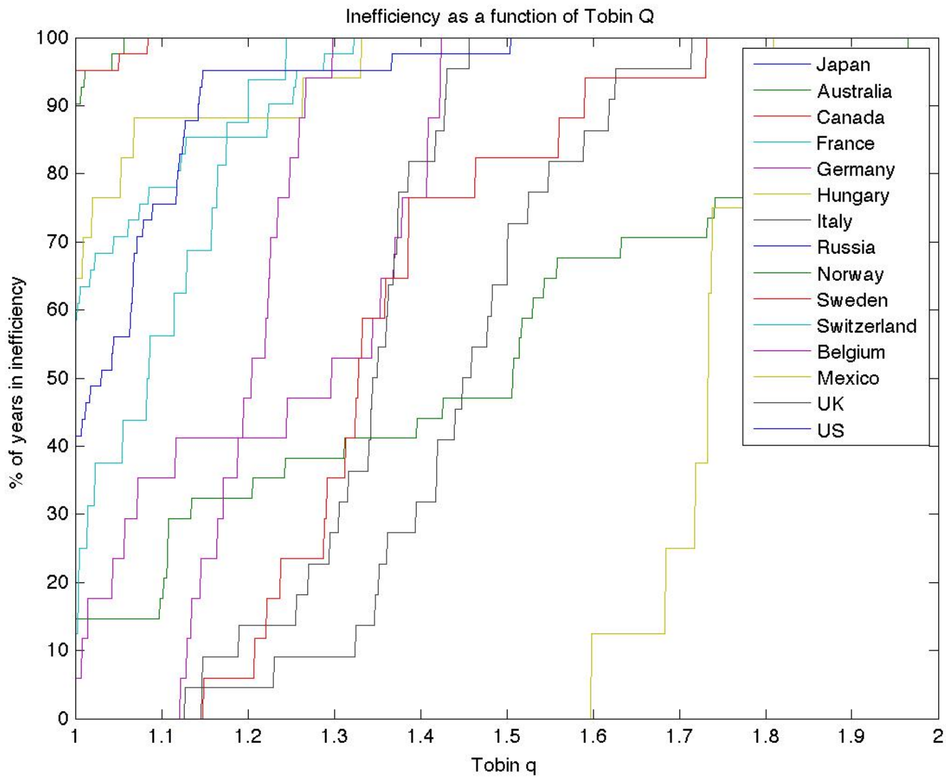


7 More countries

7.1 Tobin's q equal to 1

Capital income and investment in countries for which there is data are plotted in Figures 13, 14 and 15 (reminder: data for 4 inefficient economies is plotted in Figure 1).

Figure 12: FRACTION OF YEARS (IN %) IN WHICH INVESTMENT EXCEEDS CAPITAL INCOME



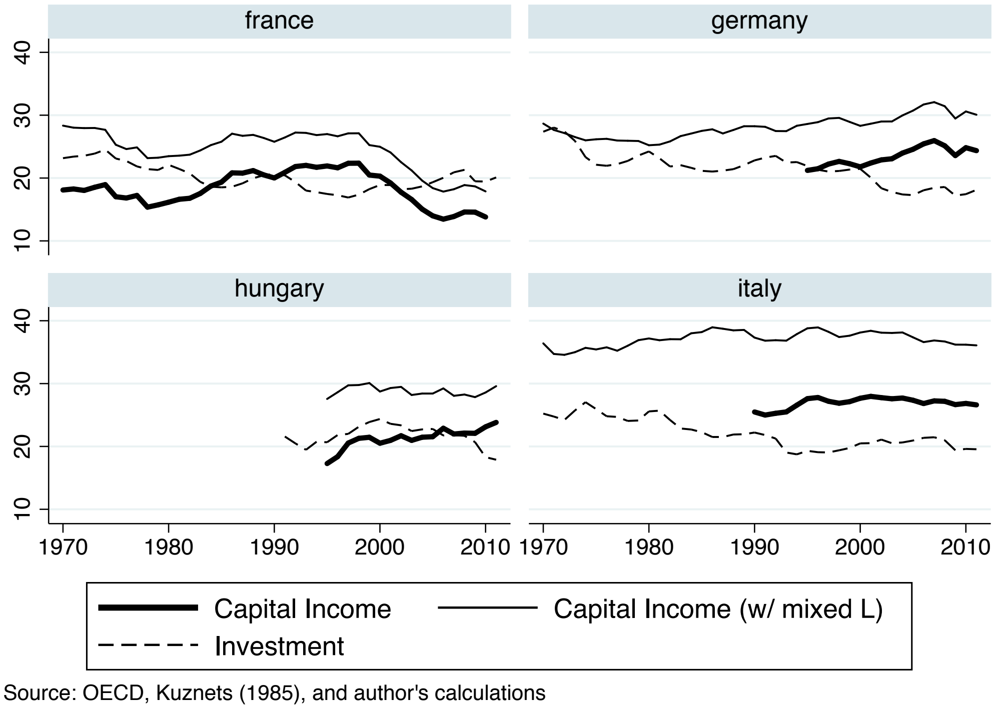
7.2 Varying the q of Tobin

Similarly, Tobin q is allowes to vary in Figures 18, 19 and 20. Reminder: inefficient economies are plotted in Figure 17.

8 Robustness: Government investment

As Abel et al. (1989), I have used private investment when using the Bureau of Economic Analysis (NIPA) data, and investment (including private and public) when using OECD data. This is because government investment series and capital consumption expenditures for government are not always available for all countries. However, I show here that government investment isn't driving dynamic inefficiency. In other words, it is not the case that government investment is so inefficient that it explains why capital income is low compared to investment. Moreover, there are many reasons to believe that private capital income includes some returns to government investment, as firms need roads to operate, for example. Results are show in Figures 21, 22, 23 and 24.

Figure 13: Assessing dynamic efficiency in France, Germany, Hungary and Italy



Notes: See Figure 1.

Figure 14: Assessing dynamic efficiency in Russia, Norway, Sweden, Switzerland

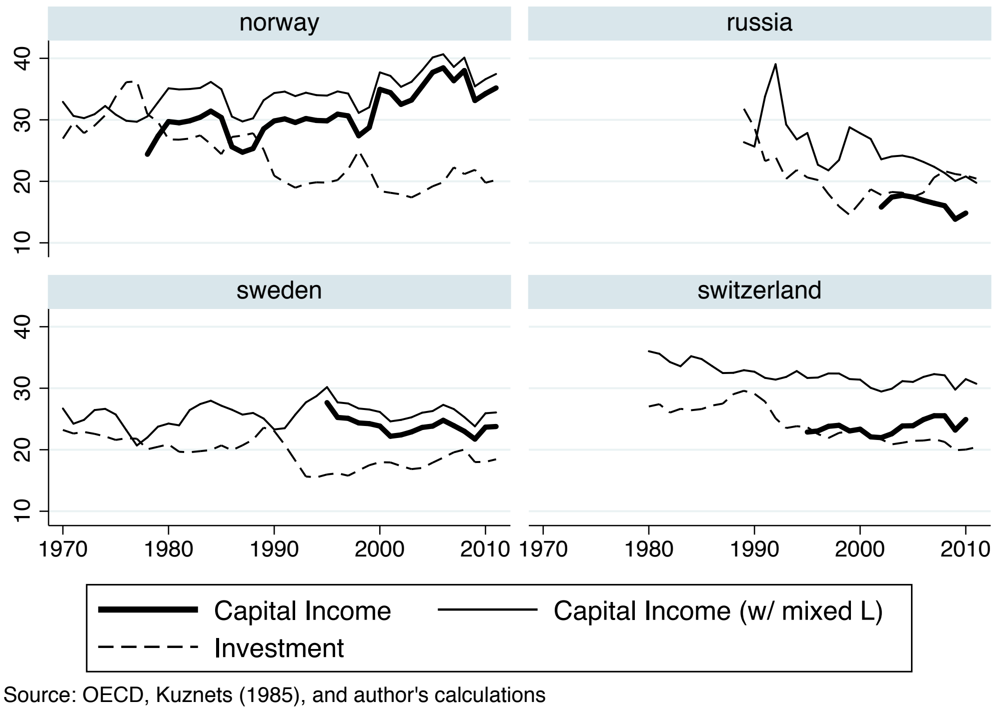
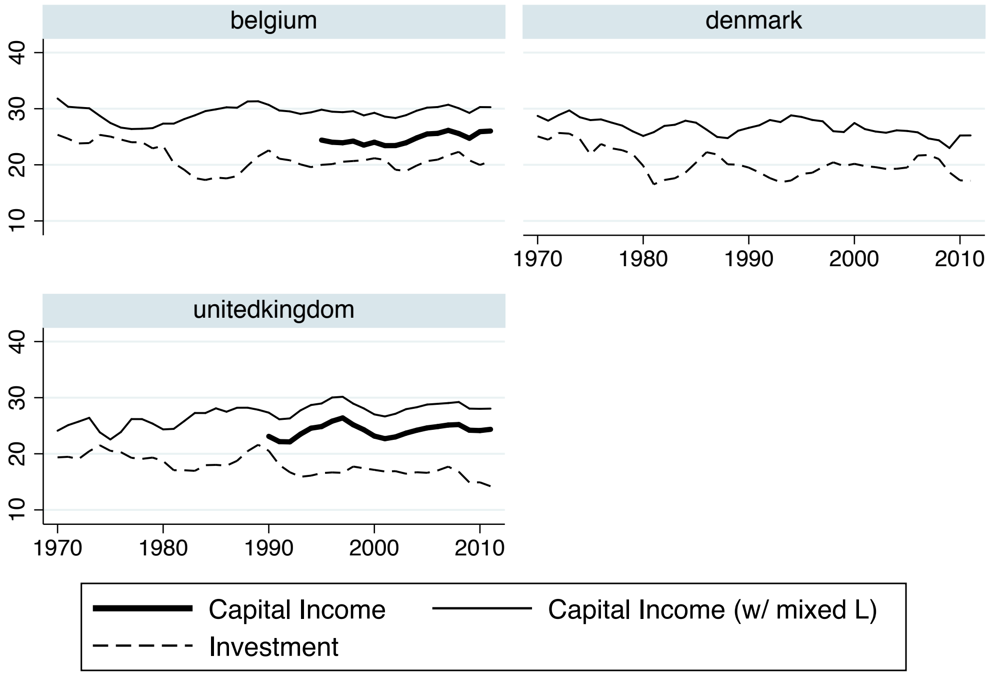
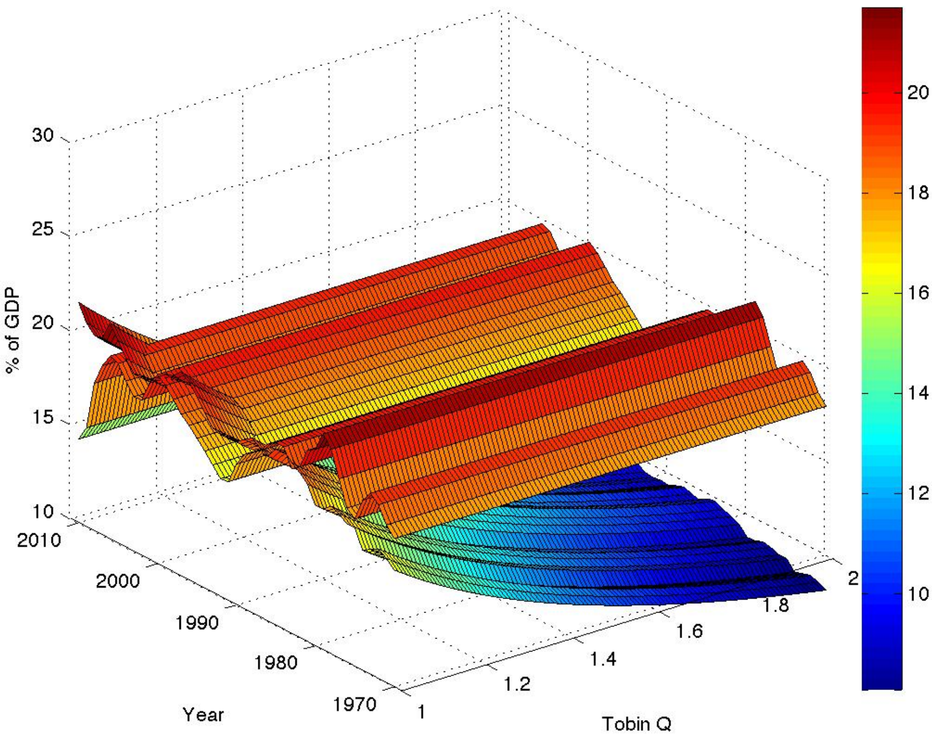


Figure 15: ASSESSING DYNAMIC EFFICIENCY IN BELGIUM, DENMARK, UNITED KINGDOM



Source: OECD, Kuznets (1985), and author's calculations

Figure 16: Dynamic efficiency as a function of Tobin Q in the US



Notes: This 3D graph represents capital income and investment (as a % of GDP), capital income coming from investment being an hyperbolic function of Tobin's q. If average Tobin q is superior to 1.5, then capital income is lower than investment for all years, and therefore the US economy is dynamically inefficient. The data is from OECD.

Figure 17: Dynamic efficiency as a function of Tobin Q in Japan, Australia and Canada

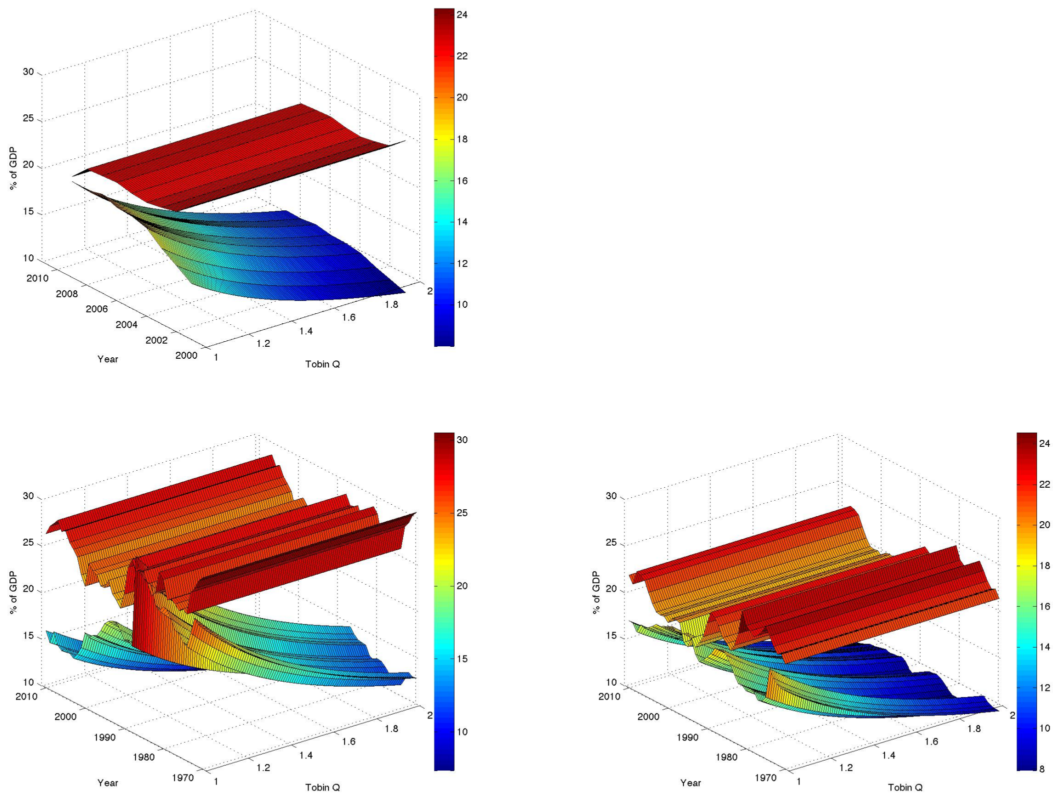


Figure 18: ASSESSING DYNAMIC EFFICIENCY IN FRANCE, GERMANY, HUNGARY AND ITALY

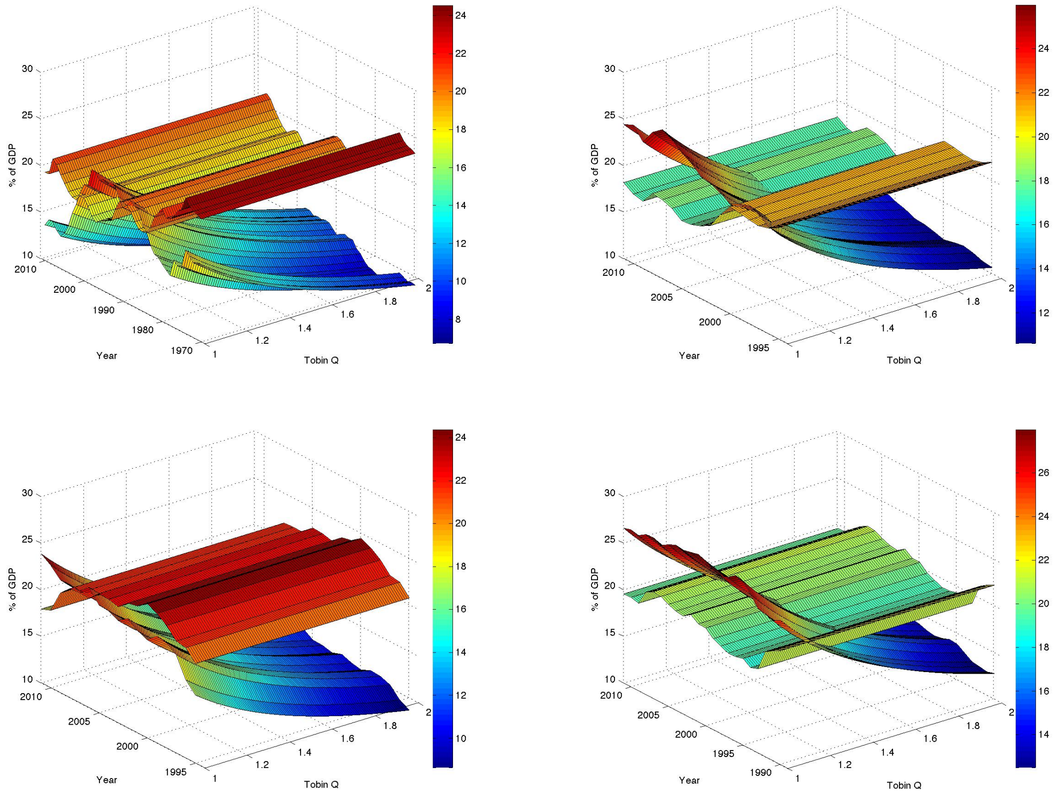


Figure 19: ASSESSING DYNAMIC EFFICIENCY IN RUSSIA, NORWAY, SWEDEN, SWITZERLAND

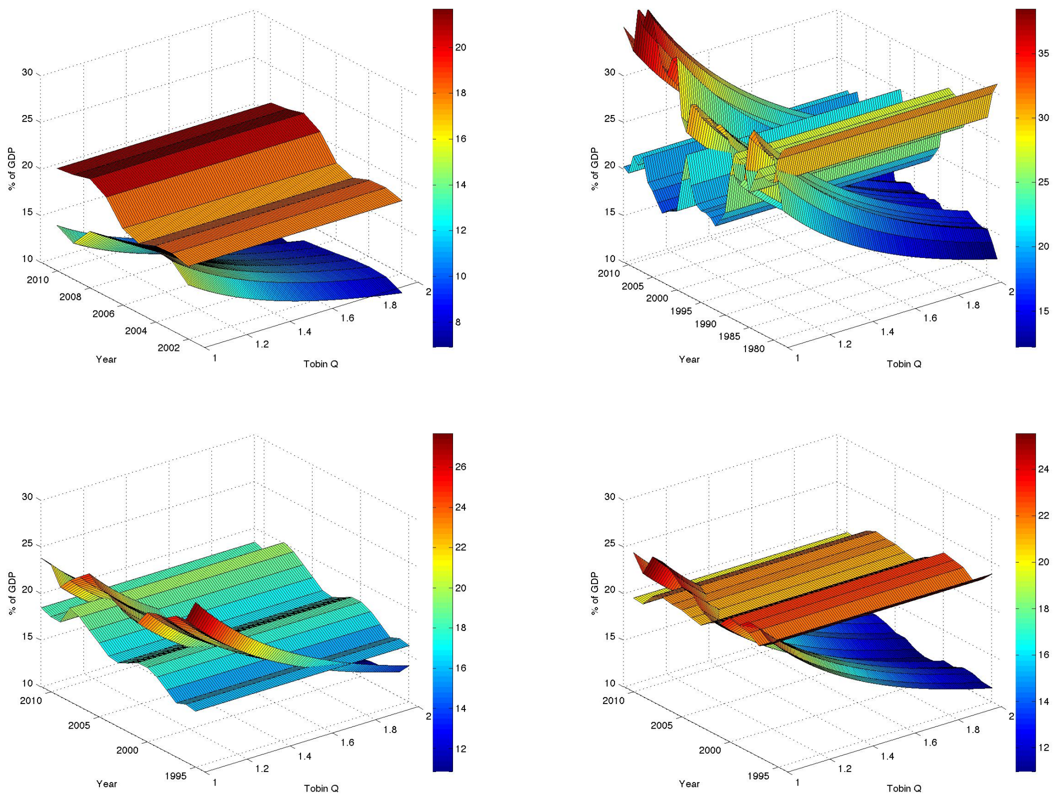


Figure 20: ASSESSING DYNAMIC EFFICIENCY IN BELGIUM, AND UNITED KINGDOM

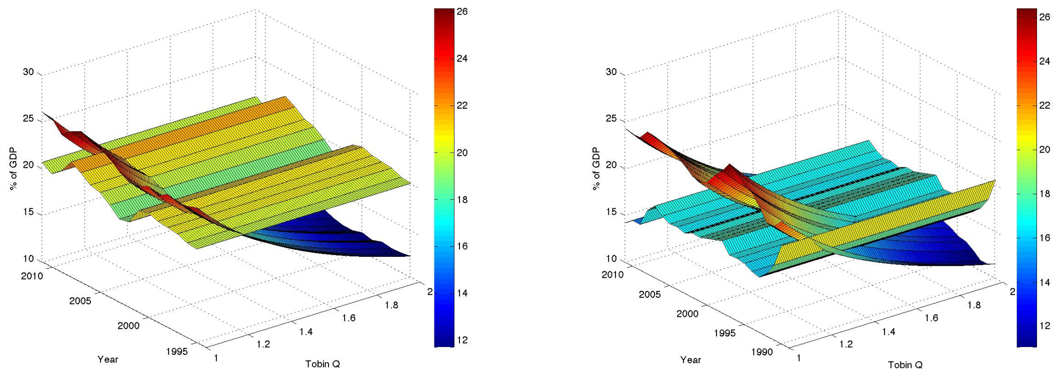
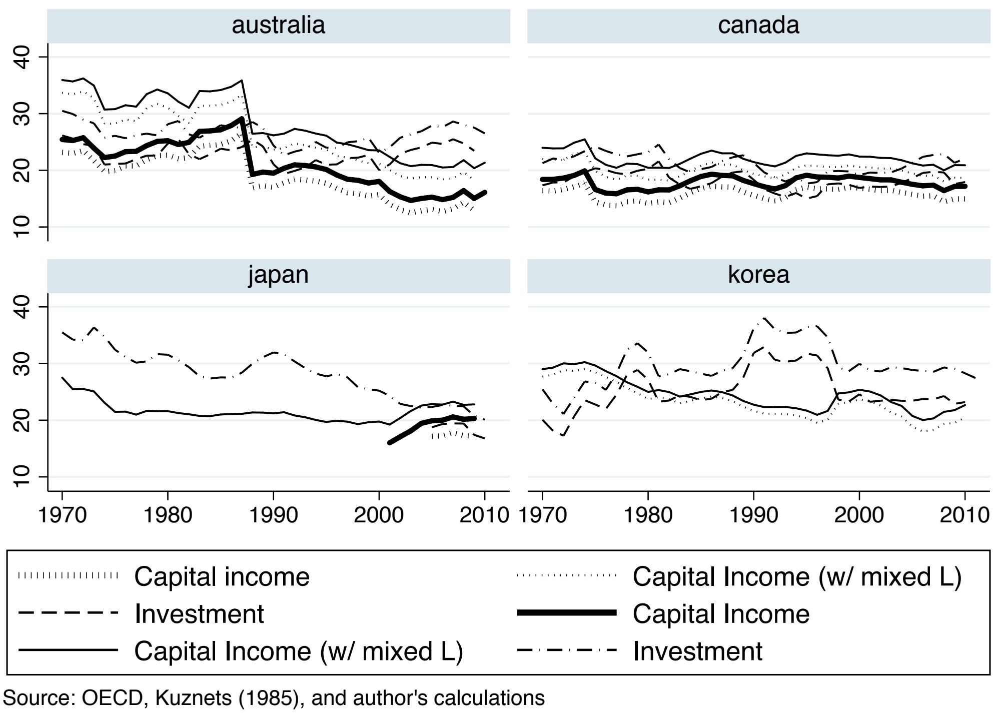
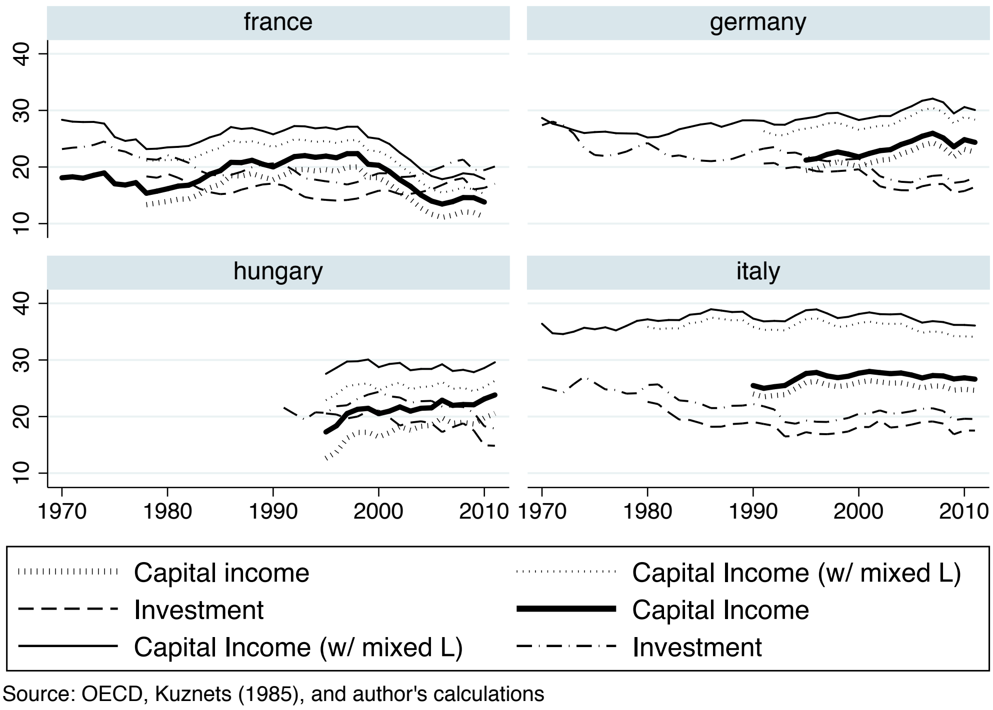


Figure 21: ASSESSING DYNAMIC EFFICIENCY IN JAPAN, SOUTH KOREA, AUSTRALIA, CANADA



Notes: The dotted line represents Gross Capital Formation as a % of GDP. The dash-dotted line is total investment, excluding public investment. The thick continuous line is Capital Income as a % of GDP, excluding land rents and mixed income coming from labor. The thin continuous line is Capital Income excluding land rents but including labor income in mixed income "Korea" is South Korea. The thick dotted line is Capital income as a % of GDP excluding Government Consumption Allowances, and the thin line as before includes labor income in mixed income. Data comes from OECD.

Figure 22: ASSESSING DYNAMIC EFFICIENCY IN FRANCE, GERMANY, HUNGARY AND ITALY



Notes: See Figure 21.

Figure 23: ASSESSING DYNAMIC EFFICIENCY IN RUSSIA, NORWAY, SWEDEN, SWITZERLAND

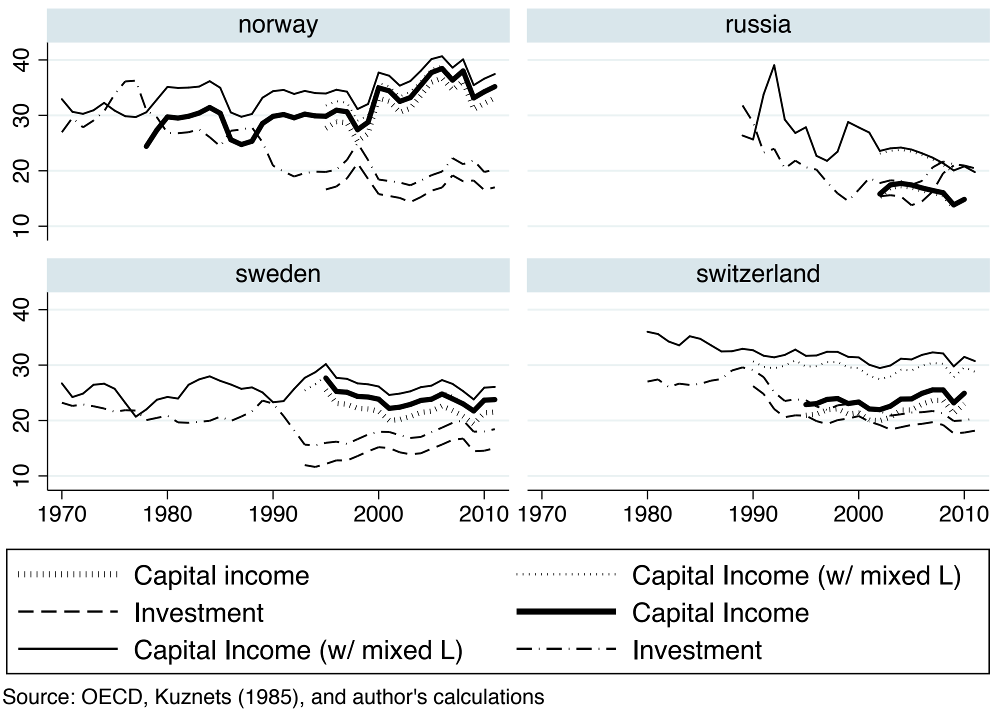
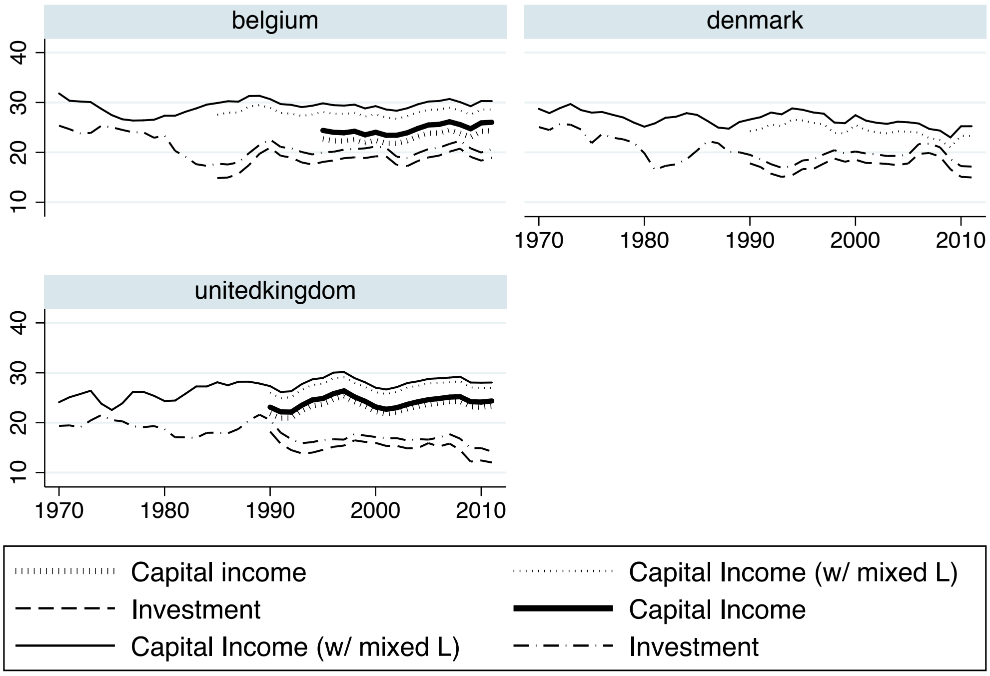


Figure 24: Assessing dynamic efficiency in Belgium, Denmark, United Kingdom



Source: OECD, Kuznets (1985), and author's calculations

Chapter 5

Dynamic Inefficiency and Capital Taxation

Abstract

Life cycle models of capital accumulation with land (or monopoly rents or decreasing returns) can only feature capital under accumulation, because of the extreme capital crowding out properties of capitalizable rents as one approaches the Golden Rule. In this paper, I show that it is no longer valid when a government levies property, wealth or estate/gift taxes. Contrary to infinite horizon models, positive capital taxes therefore help capital accumulation towards the Golden Rule in overlapping-generations models ; and no other set of transfers from young or old, public debt, or social security system can help achieve such a level of capital accumulation.

Keywords: Dynamic efficiency, land.

JEL classification: H55

Acknowledgements: I am deeply indebted to Guy Laroque for calling my attention to the dramatic capital crowding out properties of land when interest rates fall below the Golden-Rule level. I thank Emmanuel Farhi, Greg Mankiw, Philippe Martin, Thomas Piketty, Larry Summers and seminar participants at Harvard University, MIT and Paris School of Economics for comments and suggestions. I am extremely grateful to "Corps des Ponts" for funding this research and to Harvard University for their hospitality.

> "As soon as the land of any country has all become private property, the landlords, like all other men, love to reap where they never sowed, and demand a rent even for its natural produce. The wood of the forest, the grass of the field, and all the natural fruits of the earth, which, when land was in common, cost the labourer only the trouble of gathering them, come, even to him, to have an additional price fixed upon them. He must then pay for the licence to gather them; and must give up to the landlord a portion of what his labour either collects or produces. This portion, or, what comes to the same thing, the price of this portion, constitutes the rent of land ...." (Adam Smith, The Wealth of Nations)

Introduction

Capital taxes are usually viewed both in academic and policy circles as a deterrent to capital accumulation. Notwithstanding the fact that capital taxes distort savers' decisions, there is generally little arguing about the supply side effect of capital taxes : they lead to lower capital accumulation, and therefore lower output.[1] Even when capital taxes are not explicitely studied through the lens of capital accumulation, this wisdom is always somewhat present : among many other examples, New Dynamic Public Finance finds that positive capital taxes are optimal when agents are faced with idiosynchratic productivity shocks and governments can only tax in a distortive way, but that the positive welfare effects from those capital taxes are considerably diminished when one considers the general equilibrium effects taxation has on capital accumulation.[2]

This paper challenges this intuitive idea. In a nutshell, I recognize that savings do not only nourish capital accumulation but also serve to acquire non-reproducible assets such as land. Because of capitalization, this land is less valuable all the more that capital taxes are higher, so that capital taxes can actually help capital accumulation. To study the relative potency of these effects, I develop an Overlapping-Generations neoclassical growth model a la Diamond (1965) with an asset distributing rents each period (land rents are an example, but these rents can as well be monopoly rents, decreasing returns to scale as in Scheinkman (1980)). The dividends brought about by this asset endogenously grow at the rate of growth of output, which is meant to capture the idea that land is a constant (or growing) fraction of GDP; or, in the case of monopoly rents, that the economy does not become perfectly competitive asymptotically, which would happen if monopoly rents became negligible relative to GDP. As is very well known in this literature, the economy can then never reach an optimal level of capital accumulation corresponding to the Golden Rule.[3] The intuition is simple: land is a store of value for savings as capital, and therefore crowds out capital accumulation. As interest rates come closer to the Golden Rule, land becomes more and more valuable, infinitely valuable at the limit, thus making accumulation towards the Golden Rule level impossible. In this environment, capital taxes are a way to expropriate agents of these future dividends, thereby reducing the total supply of stores of value, and increasing the resources available for higher capital accumulation.

An important assumption maintained in all this paper is that governments cannot tax land (or monopoly or decreasing returns to scale) and productive capital separately: instead, they

[1] I voluntarily put aside here a discussion which was active in the eighties, about the fact that with an Elasticity of Intertemporal Substitution lower than 1 wealth effects could dominate over substitution effects, leading lower net of tax interest rates to increase savings, hence capital accumulation. However, I think that what comes out of this debate is that substitution effects dominate strongly once one takes into account realistic life cycle patterns for savings decisions. See for example Summers (1981). Moreover, there is a growing consensus that the inverse of the EIS and the coefficient of risk aversion - which is very high when backed out from the equity premium - are two different theoretical objects (Weil (1990), Epstein and Zin (1989)), so that the EIS is not so low.

[2] See Farhi and Werning (2012) for such a quantitative exploration. This idea also underlies Lucas (1990)' calculations that "supply-side economics", or removal of all capital taxes, would bring about.

[3] George J. Stigler recalls in his *Memoirs of an unregulated economist*: "Maurice Allais was a gifted engineer and economist, but at the time he believed that private ownership of land was untenable. (The reason need not occupy us; it turned on the fact that if the interest rate went to zero, as he feared it would, land would become infinitely valuable.)".

must tax both productive capital and rents are the same rate.[4] Rents and capital income are indeed not as easy to distinguish as in economists' models, because in practice, rents belong to the ownership of productive capital. For the purpose of taxing land rents for example (and them only), the government would need to charge homeowners a rent corresponding to the exact value of renting land, since land and residential structures are tied to one another. Needless to say, as the relative value of residential structures and land fluctuates over time[5], this would entail high administrative costs; the value of land could not be inferred from the resale value of a home either, as the value of home improvements (for example) is unobservable albeit to a very high administrative cost. In practice, the property tax therefore falls on both elements, hence, to quote Vickrey (1999),"The property tax is, economically speaking, a combination of one of the worst taxes, the part that is assessed on improvements and in some cases to a limited extent on personalty, and one of the best taxes, the tax on land or site value". [6] And although the model will only feature land rents, Scheinkman (1980) has shown that the crowding out properties of rents also apply to decreasing returns and monopoly rents, which are even harder to measure. Interestingly, this remark also suggests that one needs to go beyond Allais (1947)'s proposition that land should simply be nationalized: because land are not the only growing rents out there, all capital might in that case would need to be nationalized to the extreme. Adopting a market first, government second approach leads us to stay clear of such a proposition, and look for the lowest level of government ownership consistent with efficiency.[7]

Introducing capital taxation in a model with land rents allows to get several new results which were not known previously in the literature. A first result is that capital taxation has two opposite effects : it reduces the supply of savings for reasonable values of the Elasticity of Intertemporal Substitution[8], but it also decreases the demand for savings coming from land values, and competing with capital for savings. Second, a result in this literature is that even though capital taxation could be used to try and target an optimal level of capital accumulation[9], the government could use other tools to target the optimum level of capital: pay-as-you-go systems, or public debt. What comes out of this literature is that targeting the optimal level of capital accumulation does not provide a strong and convincing rationale for taxing capital. This paper shows in contrast that such schemes would not be sufficient to reach the Golden Rule, and that a strictly positive level of

[4]If this was not the case, then the government would optimally tax rents at a positive rate, but would not have to tax productive capital in the same way. Then, as in Ordover and Phelps (1979), capital taxation would be one of many instruments to target the optimal level of capital accumulation, together with public debt or assets and transfers to old and young generations.

[5]Economic geography and urban economics teach us that land values depend on the presence of a number of changing amenities.

[6]This point is discussed at length in the *Mirrlees Review* (IFS (2011)).

[7]For the sake of completeness, this would of course need to be justified in richer model. One can think that government planning requires much more information and understanding than it will ever have, and that a decentralized market economy is preferable to a planned one.

[8]As Summers (1981) in particular has shown, for realistic life cycle profiles one does not need a higher than 1 Elasticity of Intertemporal Substitution to get this result, because increased interest rates also reduces human wealth which increases the need for savings.

[9]See for example, Ordover and Phelps (1979). More in the following literature review. Positive capital taxation would be desirable in the case of too much capital accumulation, and the reverse in the case of too little. See Figure 2 for the inefficiency and Figure 3 for the efficiency case later in the text.

taxation is necessary to attain this social objective. The tax on capital making possible to reach the Golden Rule might be very small in practice (especially if further decreases in asset supply are made through an increase in public assets or in reverse pay-as-you-go systems); however the fact that capital *needs to be* taxed at a positive rate to favor capital accumulation is conceptually a very important result when the zero capital tax result is an important reference point both in academic and policy discussions. Moreover, high levels of public debt and of pay-as-you systems both rely on the possibility of commitment by the government, which it may be lacking in practice, to which taxes on capital can provide a good substitute.[10] Third, on the positive side, it shows that useful land and dynamic inefficiency (hence bubbles, Pareto-improving public debt, etc.) can coexist when some capital taxes are levied, a new result in the overlapping-generations literature. Fourth and perhaps more anecdotally, it allows to revisit other theoretical results which are taken for granted in the capital taxation literature, for example that taxes on flows and stocks of capital are equivalent.

Related Literature. The literature on life-cycle models and capital accumulation starts with Allais (1947) who remarks that there exists an optimal quantity of capital, or "Golden-Rule" level ("optimum capitalistique")[11] corresponding to a long run consumption-maximizing level of capital.[12] But as physical capital would be accumulating progressively, interest rates would go down and approach zero. This would drive the value of land to infinity, as it distributes constant dividends each period. Therefore, in order to get faster (and eventually) to the Golden-rule level of capital, Allais (1947) advocated complete nationalization of land, as George J. Stigler recalls in his memoirs quoted above.[13] This remark has later been used somewhat differently, to argue that dynamic inefficiency (capital accumulation above the Golden-rule level of capital) would be impossible, as it would make land impossible to transfer across generations.[14] Scheinkman (1980) has extended this argument to decreasing returns to scale technologies, which are another form of rent, to prove that economies with

[10]I have previously emphasized that the government could increase public assets or use reverse pay-as-you-go systems to try to mimic the imposition of a small capital tax. However, as I show in the rest of the paper, this effect on asset supply dominates for low values of the tax, while the effect on asset demand is more important when the value of the tax increases. The government will therefore want to use capital taxation to decrease asset demand if the small tax on capital has led to dynamic inefficiency.

[11]*En français dans le texte*: "Chaque jour l'ensemble de l'économie dispose d'un certain nombre d'heures de travail et la question se pose de savoir quelle est la répartition la meilleure qu'il y a lieu de faire de ce travail entre les différents stades de la production (biens directs et biens indirects)." Here is a tentative translation: "Every day an economy is given a certain number of hours of work, and the question is how we should use these hours as the different stages of production - direct or indirect." *Direct* production is the production of consumption goods and *indirect* production is the production of investment goods, which are later useful for producing consumption goods.

[12]For more on the Golden-Rule level of capital, see Ramsey (1928), Phelps (1961), Phelps (1965), Cass (1965), Diamond (1965).

[13]"En fait, il résulte de ce qui précède qu'une politique efficace d'annulation du taux de l'intérêt implique nécessairement comme conditions préalables la collectivisation de la propriété du sol et la dévalorisation de la monnaie circulante par rapport à l'unité de compte. Si de telles modifications de structure étaient apportées, non seulement l'annulation du taux d'intérêt deviendrait possible, mais encore on peut considérer que le taux d'intérêt qui tendrait à s'établir spontanément serait probablement nul, voire même légèrement négatif" (Allais (1947)).

[14]In fact, this argument is generalizable to a growing economy, no matter what the cause of this growth is (population or technological progress), as long as the share of land does not vanish relative to GDP. This is intuitive, as the Golden Rule level of capital accumulation verifies $r = f(k^{GR}) = n$ with n the rate of growth, and land dividends grow at rate n, so that when $r < n$, the value of land is again infinite.

a non-vanishing rent could not reach a dynamically inefficient state. Tirole (1985) examined this claim rigorously in a Diamond (1965) overlapping-generations growth model, and showed that rational bubbles could exist even when the economy wasn't asymptotically rentless. All that was needed was that rents be not capitalized ex-ante, so as not to be used by young generations as vehicles for savings.[15] However, as McCallum (1986) noted, land is a non-vanishing rent, capitalized ex-ante, hence ruling out the possibility of dynamic inefficiency and rational bubbles.[16] This problem was in fact also recognized as early as in Samuelson (1958), albeit somewhat indirectly.[17]

This paper also draws on the very large and developed literature dealing with capital taxation. The literature most directly linked to the present study is that concerning the link between capital taxation and accumulation. Ordover and Phelps (1979) showed that if the government had the policy tool to fix the capital stock at its optimal level, then the optimal capital tax rate should be zero, and Stiglitz (1985) gave a simpler proof of this same result. In the same line of thought, Atkinson and Sandmo (1980) show that whether capital taxation is of any use in this context depends a lot on the policy instruments of the government. This paper shows that with capitalizable rents, the government actually does not have any other tool to fix capital at its optimal level than to use strictly positive capital taxation. No level of finite public debt or of finite transfers from the young to the old can mimic the implementation of this outcome. By contrast, the supply-side calculations in both Lucas (1990) and Farhi and Werning (2012) (for example) take as a given that the capital stock is below the Golden Rule level, and that increasing the level of capital is welfare-enhancing. Moreover, because productive capital is the only asset in their models, the supply of stores of value is unaffected by the level of capital taxation. In contrast, in my model, the supply of stores of value decreases with higher capital taxes, which unambiguously mitigates the adverse effects on demand near the Golden Rule. Stiglitz (1978) shows that somewhat counterintuively, higher estate taxation can lead to higher inequality if the elasticity of capital and labor is less than 1 in the production function: in that case, higher capital taxation leading to lower capital accumulation can increase the factor share of capital which is more concentrated among high-income individuals. Once again, the negative effects of capital taxation on capital accumulation is here taken for granted. On the general subject of capital taxation, the literature puts forward other reasons to tax capital which I shall not consider here: unobservable wealth (Cremer et al. (2003)), indirect taxation of bequests and insurance of shocks on rates of returns (Piketty and Saez (2013)). In contrast, I will work under a paradigm of complete asset markets and perfect information, thus making the case for capital taxation even stronger. Compared to Piketty and Saez (2013), I do not leave the demand side effects of capital taxation as a free parameter: the elasticity of savings to the before tax rate are not infinite as with infinite horizon models, but they are endogenously determined (and

[15]For example, paintings, one type of rent, could not be sold by a painter's forebear.

[16]And, for his purposes, Pareto-improving money or other social contrivances. Rhee (1991) made the same argument and verified empirically that land rents were a non-decreasing share of GDP in the United States. Demange (2002) generalizes the argument to economies with uncertainty.

[17]"In it nothing kept. All ice melted, and so did all chocolates. (If non-depletable land existed, it must have been superabundant.)" (Samuelson (1958), p 481) Hence Paul Samuelson assumes that land is in infinite supply, so that it has no value.

finite) in models with finite lives.

The zero capital tax result remains a reference point in most academic work concerning capital taxation. Atkinson and Stiglitz (1976) show that if consumption is separable from leisure choices, and the economy deterministic, then savings decisions should not be distorted at the optimum. But they consider only life cycle savings, so that agents' preferences shall be respected: in contrast, both in Farhi and Werning (2010) and Piketty and Saez (2013), the social welfare function differs from individuals' objectives (for example, in Farhi and Werning (2012) the social planner puts more weight on future generations). Similarly, I will adopt a steady-state utility criterion a la Phelps (1965), so that individuals' decisions will not be optimal from the society's point of view. Moreover, I will consider a finite number of overlapping-generations, so that the first and second welfare theorems fail, while Atkinson and Stiglitz (1976)'s result relies crucially on the Pareto-optimality of the competitive equilibrium without taxes.Chamley (1986) and Judd (1985) show that Ramsey (linear) taxes on capital should be set to 0 in the long run, if individuals have infinite lives. Once again, this benchmark is one in which welfare theorems provide a reference point without taxes, and in which the economy is uniformly impatient. The utility criterion considered in this Chamley-Judd benchmark is implicitly the one used by the first generation to evaluate their children's utility. As is well known, such a perspective yields to the immiseration result, which is not very satisfactory from a welfare perspective.[18] Finally, capital mobility is another argument not to tax capital (see for example Gordon (1986)); though I will not consider capital mobility in the model, the results will go through with an higher elasticity of the supply of stores of value: because land is by definition immobile, the tax will not be completely shifted through lower accumulation.

Finally, it is important to understand that the research agenda carried out in this paper is somewhat orthogonal to the New Dynamic Public Finance literature (NDPF), though the latter also rationalizes strictly positive levels of capital taxation. I will consider only a deterministic environment, so that NDPF would prescribe zero capital taxation with my assumptions. Also, I will work with Ramsey (proportional) tax systems, in contrast to the NDPF which takes a Mirrlees (1971) approach to taxation. There are two reasons for this. The first one is that studying proportional taxes is a useful benchmark, on which some strong results are based (like Chamley (1986)-Judd (1985)) The second is that Atkinson and Stiglitz (1976) suggest that labor income and capital taxation are to a large extent orthogonal: mirrleesian taxation helps tax labor in the least distortive way, and capital taxes must be set to 0. In the case considered here, competitive equilibrium fails to be Pareto-optimal anyways and linear capital taxes are not necessarily distortive, but on the other hand can help restore efficiency.

The remainder of the paper proceeds as follows. Section 1 presents a Diamond (1965) model of overlapping generations with land rents. Section 2 gives the results. Section 3 discusses the limitations of the analysis and possible future work.

[18] In particular, Atkeson and Lucas (1992) show the importance of taking into account the welfare of children directly rather than only in the welfare of parents.

1 Model

In this section, I will develop a standard Diamond (1965) model with land. For concreteness, land will be a useful input in the production function, available in fixed supply. Note that this corresponds more to an "old" (agricultural) use of land (though businesses arguably need some amount of land to operate, also nowadays); a more "modern" use of land would correspond to an inclusion in the utility function, with two goods, a consumption good and a good corresponding to the utility for living in the business district:

$$U(C,L) = C^{1-\gamma} L^{\gamma} \quad \text{with} \quad L = 1.$$

The elasticity of substitution would be assumed to be one, so that the rental price of land would grow at the rate of growth of consumption g. Finally, the price of land could also be derived by recognizing the existence of some form of increasing returns (this could be done in an urban economics model - like the Alonso-Muth-Mills monocentric model for example).

1.1 Agents

There are overlapping-generations of agents. The generation born at time t consumes c_t^y when young and c_{t+1}^o when old. Work occurs only when young, and labor is supplied inelastically. Agents then earn a wage w_t, save s_t, on which they earn return r_{t+1}. They receive a transfer T_t^y from the government when young and T_t^o when old, so that:

$$\begin{cases} c_t^y + s_t = w_t + T_t^y \\ c_{t+1}^o = (1+r_{t+1})s_t + T_t^o \end{cases}$$

Therefore, their intertemporal budget constraint sums up to:

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}} \tag{1}$$

They consume in both periods of their lives, and therefore they solve:

$$\max_{c_t^y, c_{t+1}^o} U(c_t^y, c_{t+1}^o)$$
$$\text{s.t. } c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} \leq w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}$$

This gives a demand function for consumption when young and when old, and an implied savings demand, depending on wages and the interest rates. For simplicity, let us assume that the utility function exhibits Constant Relative Risk Aversion (CRRA) with risk aversion σ (and intertemporal elasticity of substitution $1/\sigma$):

$$U(c_t^y, c_{t+1}^o) = a\frac{(c_t^y)^{1-\sigma}}{1-\sigma} + (1-a)\frac{(c_{t+1}^o)^{1-\sigma}}{1-\sigma}.$$

The solution of this problem yields the consumption decision of the young (see Appendix

1.1 for detail) :

$$c_t^y = \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma} (1+r)^{1/\sigma - 1}}.$$

And hence, savings are given by:

$$s_t = w_t + T_t^y - \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma} (1+r)^{1/\sigma - 1}}.$$

1.2 Production

Factor incomes are not exogenous. The supply of land is fixed to $L_t = 1$ for every t. On the production side, firms hire labor and use capital with a constant returns to scale technology with respect to joint labor and capital such that $Y_t = f(K_t, N_t, 1) = N_t F(k_t, 1, 1) = N_t f(k_t)$. Capital and labor earn their marginal returns expressed with the intensive form of the production function as:

$$\begin{cases} w_t = f(k_t) - k_t f'(k_t) \\ r_t = f'(k_t) \end{cases}$$

With a Cobb-Douglas production function, $F(K_t, N_t, L_t) = K_t^\alpha N_t^{1-\alpha} L_t^\beta$ and $f(k_t) = k_t^\alpha$, so that $w_t = (1-\alpha) k_t^\alpha$ and $r_t = \alpha k_t^{\alpha - 1}$. Note that the rental price of land is also given by its marginal return, which gives the demand for land:

$$r_t^* = \frac{\partial Y_t}{\partial L_t} = \beta f(K_t, L_t, 1).$$

In equilibrium, r_t^* must be such that demand is consistent with supply $L_t = 1$. It is easy to see that capital will in that case receive r_t^* which increases like the rate of growth of output (and K_t and L_t on a steady-state growth path). Of course, the fact that land appears as having a unit elasticity of substitution with both labor and capital in the production function comes from the reverse-engineering of the share of land in total value added, which was first noted by Rhee (1991).[19] β parametrizes the constant relative importance of land with respect to other inputs. Except in the base where $\beta = 0$, land will turn out to play a very important role in the determination of asset supply.

[19] This was discussed more precisely in the introduction. The original argument, for example by Allais (1947), is formulated in an economy which does not grow. Hence, he considers constant dividends each period; and negative interest rates for dynamic inefficiency. shows using US data that land does not vanish relative to GDP, so that the same argument can be made with growth, g land rents growing at g, and interest rates below g for dynamic inefficiency. In contrast, Tirole (1985) rules out those growing with GDP capitalized rents by assumption, and only allows non-capitalized rents to grow at the same rate as GDP.

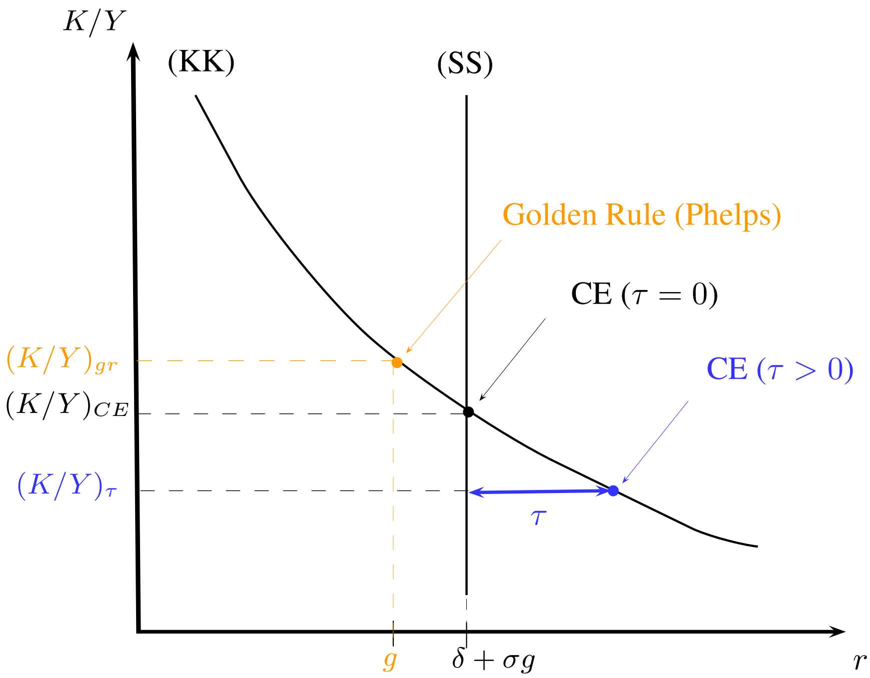
1.3 Discussion

Infinite horizon models. In the Fisher (1930) theory of interest, the after-tax interest rate is always equal to the so-called modified Golden Rule:

$$r = \delta + \sigma g.$$

This after-tax steady-state interest rate comes from a condition for agents' consumption at infinity; but the theory predicts very large variations of capital stock accumulation in response to capital taxes. In fact, the elasticity of savings to after tax interest rates is infinite in this model, as shown in Figure 1.

Figure 1: Capital Taxation in Fisher (1930) Theory of Interest



Note: In Fisher (1930)'s theory of interest, the after tax rate of return is pinned down. Therefore, all the adjustment to tax rates goes through a reduction in the demand for capital by firms.

However, this theory is somewhat at odds with the data for at least two reasons. First, long-term interest rates move a lot over the business cycle (see for example Figure 5 for a plot of long term interest rates), thus explaining those movements would require very high "patience" shocks on the part of consumers (such shocks are indeed used in monetary models to generate movements in the natural rate of interest). Second, this theory predicts always higher than growth interest rates (in principle, the coefficient of relative risk aversion could be lower than 1, but it is both empirically implausible and theoretically problematic, as this would mean that the consumers' infinite horizon optimization program would not be well defined). Finally, as Piketty and Saez (2013) remark, it predicts very large movements of wealth/income ratios in relation with capital tax rates, which do not seem to be there in the data.

OLG models. In contrast, I use a finite life model, in which savings have a finite interest elasticity. Note that agents are in a sense inherently impatient in OLG models, as they don't care about after death consumption, and so their objective functions is always well-defined. At the same time, they are in many respects much less impatient than infinitely-lived optimizers, as they by assumption cannot borrow against future generations' income (they could not repay). For the finite live assumption to hold it is important that agents do not leave bequests a la Barro (1974); however they could well have some other form of dynastic altruism, like a warm-glow of giving bequests for example. The analysis naturally generalizes to this case.

2 Results

2.1 Some heuristics

No land

As is well known, with no land the competitive outcome hence will not correspond to the steady-state maximizing outcome, albeit for very particular values of the parameters. When there is capital under-accumulation, one might need to reduce some generations' consumption in order to improve future generations' welfare, and reach the Golden-rule level of steady-state consumption (this is likely to be the rationale behind savings-enhancing policies). However, under capital over-accumulation, that is $k^* > k^g$ there are allocations that unambiguously improve on the welfare of all generations. If $k^* - k^g$ is added to consumption when steady state has been reached[20], then consumption in period T is

$$\begin{aligned} f(k^*) + k^* - k^g - nk^g = f(k^*) &- nk^* + (n+1)(k^* - k^g) \\ &> f(k^*) - nk^* \end{aligned}$$

Subsequently quantity available for consumption is

$$f(k^g) - nk^g > f(k^*) - nk^*.$$

This case therefore leads to a well known result: dynamic efficiency or inefficiency depends upon the parameters of the economy. The economy can by itself accumulate too much capital or too little. However, as Ordover and Phelps (1979) note for example, there is no clear role for capital taxation in this model as pay as you go systems and public debt can as well help the economy reach the Golden Rule of capital accumulation.

[20]It might be that this level is never reached, but only asymptotically. Yet the argument remains considering a large enough T above which one lies arbitrarily close to steady-state values.

Land with no taxes

Land substantially change the analysis of the previous model. Denote by R the steady state share of land in GDP (corresponding to the limit of r_t^*/Y_t as t approaches ∞). Then land distributes $\frac{R(1+n)^t}{1+r}$ in period t, and its value at time t hence is $\frac{R}{r-n}(1+n)^t$ or $R/(r-n)$ per capita.[21] Note that at date $t+1$, each generation born in period t gets a value of the rent per capita that decomposes in the following way:

$$\frac{R}{r-n}(1+r) = \underbrace{\overbrace{R}^{\text{land dividend}} + \overbrace{\frac{R(1+n)}{r-n}}^{\text{capital gains}}}_{\text{total return}}.$$

I then state the following proposition, generalizing Allais (1947)'s fears that an economy with land will never reach the Golden Rule level of capital accumulation, let alone dynamic inefficiency:

Proposition 7. *Whenever land remains productive (that is $r_t^*/Y_t \to R > 0$), the competitive allocation always displays under-accumulation relative to Golden-Rule level of capital, that is $k^* < k^g$. This is true no matter what the steady-state levels of transfers T_o or T_y between old and young, or finite levels of government debt (positive or negative).*

Proof. Let $(b, T^y, T^o) \in \mathbb{R}^3$ per-capita public debt, per-capita transfers to the old and per-capita transfers to the young such that

$$T^y + \frac{T^o}{1+n} = 0.$$

For levels of public debt per capita b ($b < 0$ denotes assets):

$$(1+n)k^* + \frac{R}{\alpha(k^*)^{\alpha-1} - n} = (1-a)(1-\alpha)(k^*)^\alpha + (1-a)T^{y'} - \frac{aT^{o'}}{1+\alpha(k^*)^{\alpha-1}}$$

with:

$$\begin{cases} T^{y'} = T^y - b(1+r) \\ T^{o'} = T^o + (1+r)(1+n)b \end{cases}$$

Hence, Golden-rule or above Golden-rule capital accumulation are impossible, no matter what the values of b, T^y, and T^o are. $\square$

[21]The value of land at date t is

$$p_t = \sum_{i=t}^{+\infty} \frac{R(1+n)^i}{(1+r)^{i+1-t}} = \frac{R(1+n)^t}{1+r} \sum_{i=0}^{+\infty} \left(\frac{1+n}{1+r}\right)^i = \frac{R(1+n)^t}{r-n}.$$

.

Land with taxes

For simplicity, let us assume that the government cannot tax land and productive capital differently, as discussed in the Introduction. In that case, the value of land at date t is

$$p_t = \sum_{i=t}^{+\infty} \frac{R}{1+r}(1+n)^i \left(\frac{1-\tau}{1+r}\right)^{i-t} = \frac{R(1+n)^t}{1+r} \sum_{i=0}^{+\infty} \left(\frac{(1+n)(1-\tau)}{1+r}\right)^i = \frac{R(1+n)^t}{r-n(1-\tau)+\tau}.$$

This time at $t+1$, each generation born in period t gets a value of the rent per capita that yields return r by arbitrage decomposing in the following way:

$$\frac{R}{r-n(1-\tau)+\tau}(1+r) = \underbrace{\overbrace{R}^{\text{land dividend}} + \overbrace{\frac{R(1+n)}{r-n(1-\tau)+\tau}}^{\text{capital gains}} - \overbrace{\frac{R\tau(1+n)}{r-n(1-\tau)+\tau}}^{\text{wealth tax}}}_{\text{total (net of tax) return}}.$$

Proposition 8. *With positive wealth taxes $\tau > 0$, there exist values for $(a, \alpha) \in [0,1]$ and $R \in \mathbb{R}_+^*$ such that the economy is inefficient or $k^* > k^g$.*

The intuition is straightforward: because the condition for finiteness of land values is now $r - n + \tau(1+n) > 0$, or $\alpha(k^*)^{\alpha-1} - n + \tau(1+n) > 0$, which does not rule out that $k^* > k^g$.

2.2 Quantitative results

No land

Let me assume for the moment that land is not valuable in production, that is: $\beta = 0$. It that case:

$$\forall t > 0, r_t^* = 0.$$

Then, at the steady state, the (SS) curve writes:

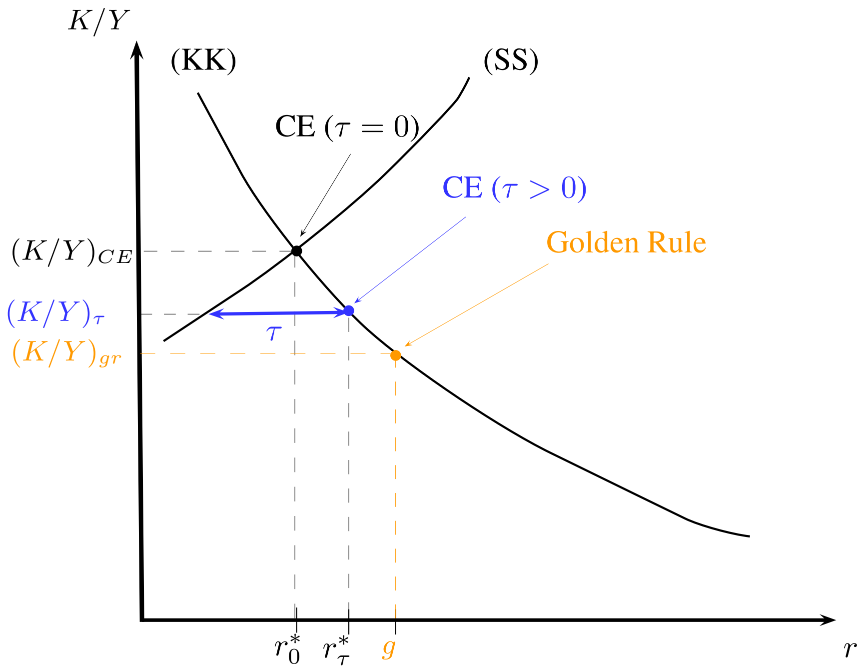
$$ss(r,0) = (1-\alpha)\left(1 - \frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}\right)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Under the assumptions outlined above, this defines an upward sloping locus in the $(r, K/Y)$ plane. That is, the assumptions are equivalent to assuming that:

$$ss'(r,0) = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{\frac{1}{\sigma}} \frac{(1+r)^{\frac{1}{\sigma}-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}} \frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} > 0.$$

The (KK) curve is standard, and defines a downward sloping curve in the $(r, K/Y)$ plane. As is well known in this case, there can be dynamic inefficiency like in the case depicted in Figure 2 or dynamic efficiency like in Figure 3.

Figure 2: OLG model - Dynamic Inefficiency



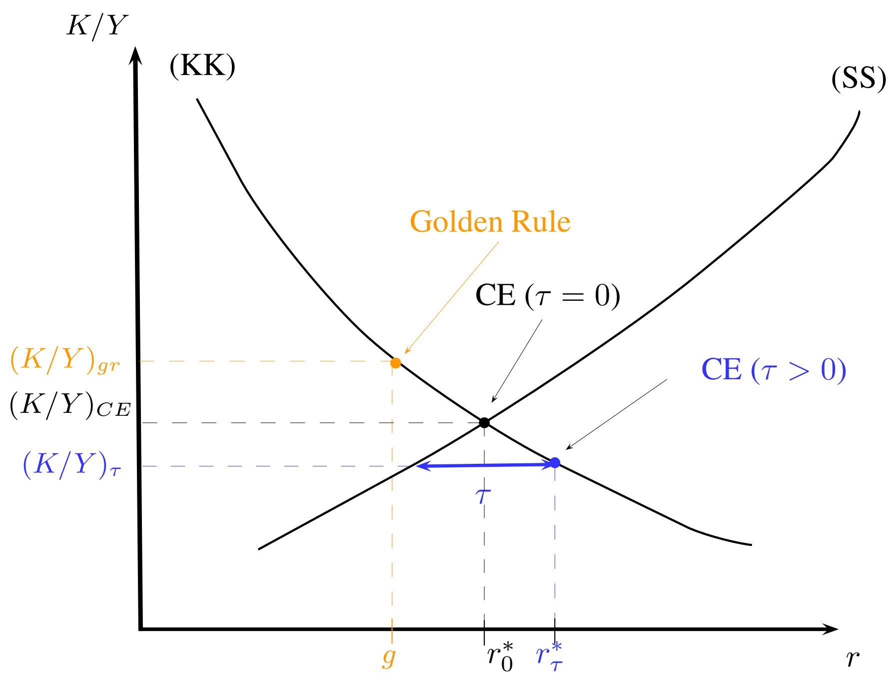
Note: Steady-state of an OLG neoclassical growth model without land. As Diamond (1965) showed, the competitive equilibrium may feature too much capital accumulation. In that case, capital taxes can help discourage capital accumulation and hence help reach the Golden Rule. However, as Ordover and Phelps (1979) have shown, public debt or pay-as-you-go systems can as well be used for this matter.

The effects of capital taxation on capital accumulation are unambiguous on those two figures. If the competitive equilibrium of the undistorted economy leads to too much capital accumulation as on Figure 2, then capital taxation allows to get closer to the Golden-Rule. Denoting by $(K/Y)_{gr}$ the capital/output ratio at the Golden Rule, $(K/Y)_\tau$ the same ratio at the distorted allocation (with capital taxed at rate τ), and $(K/Y)_{CE}$ the undistorted capital/output ratio, it is clear that:

$$(K/Y)_{gr} < (K/Y)_\tau < (K/Y)_{CE},$$

so that imposing a capital tax leads to an unambiguously better outcome (note however that this holds for a small enough value for τ). However, as Ordover and Phelps (1979) suggested, the government could just as well target a higher level of public debt or put in place pay-as-you go systems to remedy this dynamic inefficiency problem. Conversely, the case with dynamic efficiency depicted on Figure 3 would suffer from capital taxation, as the resulting capital/output ratio would be even lower. With the same notations, it is clear from the figure or from basic algebra that:

Figure 3: OLG MODEL - DYNAMIC EFFICIENCY



Note: Steady-state of an OLG neoclassical growth model without land. Here the competitive equilibrium is efficient. As with the Fisher theory of interest, capital taxation discourages capital accumulation away from the Golden Rule. Note that this effect is however dampened by the fact that the elasticity of savings to the rate of interest is finite.

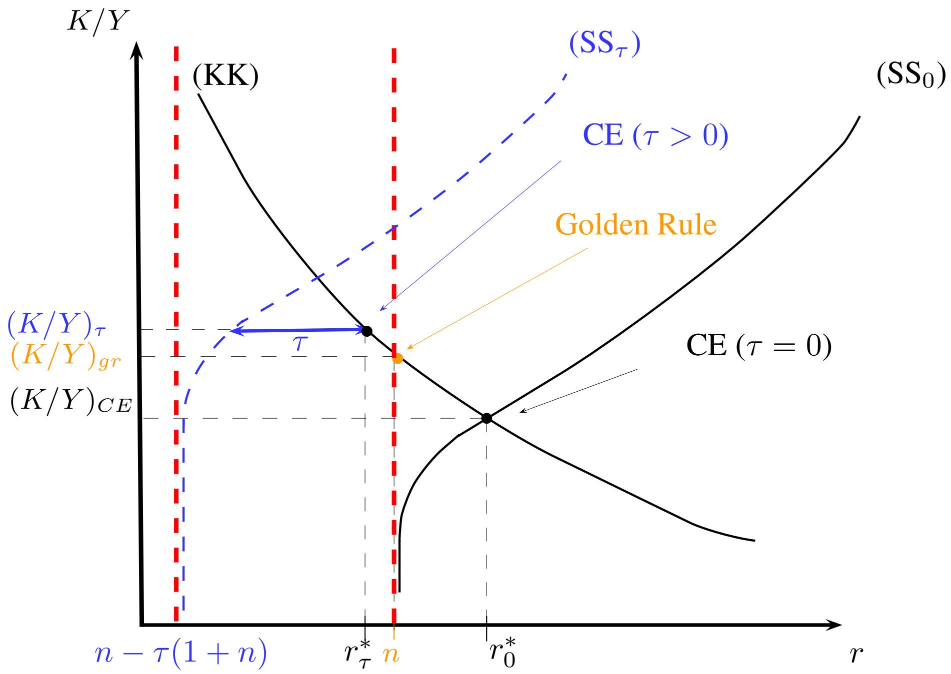
$$(K/Y)_\tau < (K/Y)_{CE} < (K/Y)_{gr}.$$

With land

In contrast, in the more realistic case where $R \neq 0$, there are two opposing forces. On the one hand, holding land values constant, capital taxes also have the effect of driving a wedge between the return earned by savers and that paid by entrepreneurs, decreasing overall capital accumulation. However, capital taxes also lead to a decrease in land values, thus increasing the amount of resources available from savers to productive capital (which I call "free savings" in the following).

These two opposing forces are visible on Figure 4. The latter effect is shown as a leftward shift in the $(r, K/Y)$ plane of the free-savings curve from (SS_0) to (SS_τ): with increases in capital taxation, the resources available for productive capital K increase for a given level of interest rates. Note how initially, because of the extreme capital crowding out properties of land when the interest rate approaches the Golden Rule, the curve (SS_0) was always to the right of the $r = n$ curve. In contrast, with positive levels of capital taxation, the curve (SS_τ) is totally consistent with a Golden-Rule level of capital accumulation.

Figure 4: OLG model - With land



Note: Steady-state of an OLG neoclassical growth model, incorporating land rents. Capital taxation has two effects on capital accumulation. First, it increases the supply of free savings for capital accumulation, by reducing land values. The curve (SS) shifts from $(\text{SS})_0$ to $(\text{SS})_\tau$. Second, it discourages capital accumulation by driving a wage between the rate of return for investors and the rate of return for savers. In the case pictured on this graph, the net effect of a capital tax on capital accumulation is positive.

Algebraically, free savings (that is, savings available for productive capital investment) are equal to:

$$ss(r,R) = (1-\alpha)\left(1 - \frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}\right)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} - \frac{R}{r-n(1-\tau)+\tau}.$$

Note that this "free savings" curve is steeper than the previous one without land. Under the previous assumptions, the (SS) curve is therefore unambiguously increasing in the $(r, K/Y)$ plane. This is because:

$$\begin{aligned}\frac{\partial ss(r,R)}{\partial r} &= (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{\frac{1}{\sigma}}\frac{(1+r)^{\frac{1}{\sigma}-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}}\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} + \frac{R}{(r-n(1-\tau)+\tau)^2}\\ &= \frac{\partial ss(r,0)}{\partial r} + \frac{R}{(r-n(1-\tau)+\tau)^2} > \frac{\partial ss(r,0)}{\partial r}.\end{aligned}$$

The fact that $(SS)_0$ is always to the right of the $r = n$ vertical line, that a positive level of capital taxation corresponds to a leftward shift in the curve, etc. follows immediately from the examination of these functions. In the case shown in Figure 4, we have that:

$$(K/Y)_{gr} < (K/Y)_{CE} < (K/Y)_{\tau}.$$

In that case, positive capital taxation allows for dynamic inefficiency.

3 Discussion

Before making a few side remarks on the model, let me give a fairly simple intuition for why increasing capital taxes can lead to higher capital accumulation. In fact, committing to taxing land in the future amounts for the government to take on a (possibly very large) positive asset position. Just as public debt crowds out capital accumulation, public assets encourages it. And since Ricardian equivalence does not always hold in overlapping-generations models, this very high increase in government savings is not necessarily matched by a corresponding decrease in private savings. As was the case on Figure 4, it can even be that a sufficient amount of stores of value is no longer available once the capital tax has been imposed, so that the economy is dynamically inefficient.

3.1 Non-equivalence between stock and flow taxes on capital

A side result mentioned in the introduction is that in this framework wealth taxes and dividend taxes are not equivalent. In the model, very different would be to impose a positive capital income tax τ, which would tax dividend uniformly across time, and effectively expropriate private agents of a fraction τ of their capital stock (thus, previous arguments, which require only an arbitrarily small yet positive amount of land, would still hold). Therefore, allowing for a pre-existing factor of production like land allows to break down a well-known equivalence result between capital income taxation and wealth taxation. This result only holds *in equilibrium*, as taxation on capital τ_K and taxation on wealth τ_W are linked by :

$$1 - \tau_W)(1+R) = 1 + (1-\tau_K)R \quad \Leftrightarrow \quad \tau_K = \frac{\tau_W(1+R)}{R}.$$

3.2 Government's liquidity and solvency

A side result from the theory developed above is that if there is dynamic inefficiency because the government uses capital taxation (on land in particular), then governments are solvent since they potentially have infinite assets. These assets could be used to back potentially very large levels of public debt. However, there are several reasons why those infinite positive assets cannot be used to obtain financing. A first solution would be to sell those assets, but it is not possible since the government does not legally own future land rents. It only has the right to levy a tax on the value of this land in the future, and the right to collect taxes is not transferable to private agents. This asset therefore is not redeployable, and therefore cannot be sold nor directly collateralized. A second, more intermediate solution would be to promise the revenue from wealth or property taxes to new bondholders. However, this overlooks that property taxes, which represent the bulk of land taxes in most countries, are usually levied by local governments.[22]

Conclusion

This paper is only a first attempt at incorporating land and monopoly rents into a life cycle theory of capital accumulation. It has shown that the relationship between capital accumulation and taxation is far from one-directional in such an environment. Some positive level of capital taxes is even necessary to achieve the Golden Rule level.

There are many other elements which have been left out here, but which could be fruitfully integrated in future work. In this analysis, the government did not have any financing needs on its own. Needless to say, the introduction of a need for financing public expenditures would likely strengthen the case for capital taxation since land and monopoly rents represent pure profits. The alternative would be to tax labor income, which is distortive with flexible labor supply, even when it is done in a Mirrleesian way.

If the zero capital tax benchmark is not a reference point, then this paper naturally calls for quantitative evaluation of the effects it presents. In particular, Figures 6 and 7 show that there is a high cross-sectional variation of wealth, property and bequest/gift taxes over time, which might be fruitfully used for empirical analysis.

[22] And in effect, such public debt financing schemes has already been used by the city of San José in the United States to finance municipal debt. As Michael Lewis (2013) anecdotally puts it: "It is one of the few cities in America with a triple-A rating from Moody's and Standard & Poor's, but only because its bondholders have the power to comel the city to levy a tax on property owners to pay off the bonds."

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1 Omitted proofs

1.1 Agents' optimization

Agents solve the following two period standard optimization problem:

$$\max_{c_t^y, c_{t+1}^o} a\frac{(c_t^y)^{1-\sigma}}{1-\sigma} + (1-a)\frac{(c_{t+1}^o)^{1-\sigma}}{1-\sigma}$$
$$\text{s.t. } c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} \leq w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}.$$

Eliminating the multiplier on the budget constraint gives:

$$\left(\frac{c_{t+1}^o}{c_t^y}\right)^\sigma = (1+r_{t+1})\frac{1-a}{a}.$$

Replacing the expression for c_{t+1}^o in the resource constraint gives:

$$\left[1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}\right]c_t^y = w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}} \quad \Rightarrow \quad c_t^y = \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}$$

1.2 Monotonicity of the steady-state "free savings" function

Denoting by $ss(.)$ the function associating free savings to the equilibrium interest rate,

$$ss(r) = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{1/\sigma}\frac{(1+r)^{1/\sigma-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}}\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}.$$

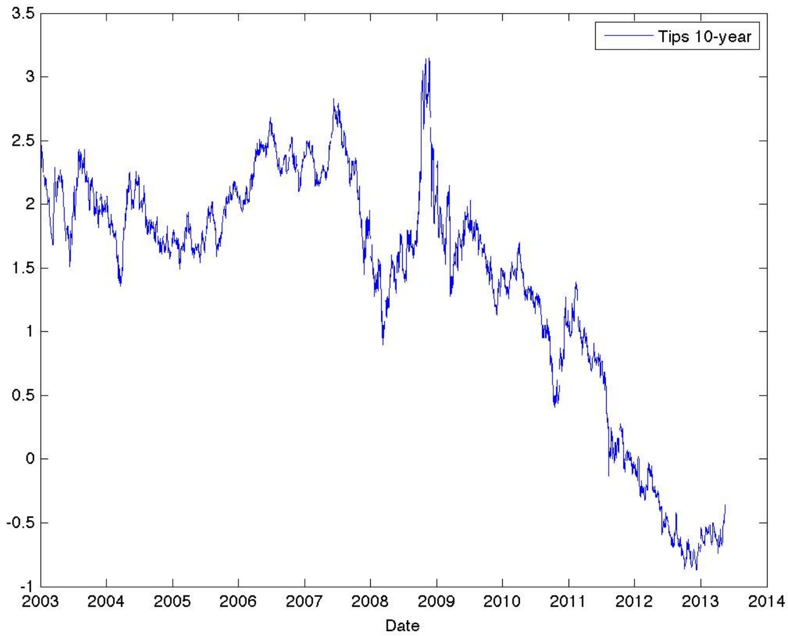
It follows that $ss(.)$ is non monotone since:

$$ss'(r) = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(1+r)^{1/\sigma-1}\left(\frac{1-a}{a}\right)^{1/\sigma}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}\left(1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}\right)}\left[\left(\frac{1}{\sigma}-1\right)\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} - \frac{\alpha}{1-\alpha}\frac{1+r}{r+\delta}\right],$$

but for $1/\sigma - 1$ large enough, and low enough values for r it will be monotonically increasing. Note that this assumption is akin to assuming that increases in interest rates do not have too much adverse impacts on wages which end up to depress savings. A similar assumption is actually made in Diamond (1965) where interest rates also appear on both sides of an equation and some single-crossing and stability assumptions must be made.

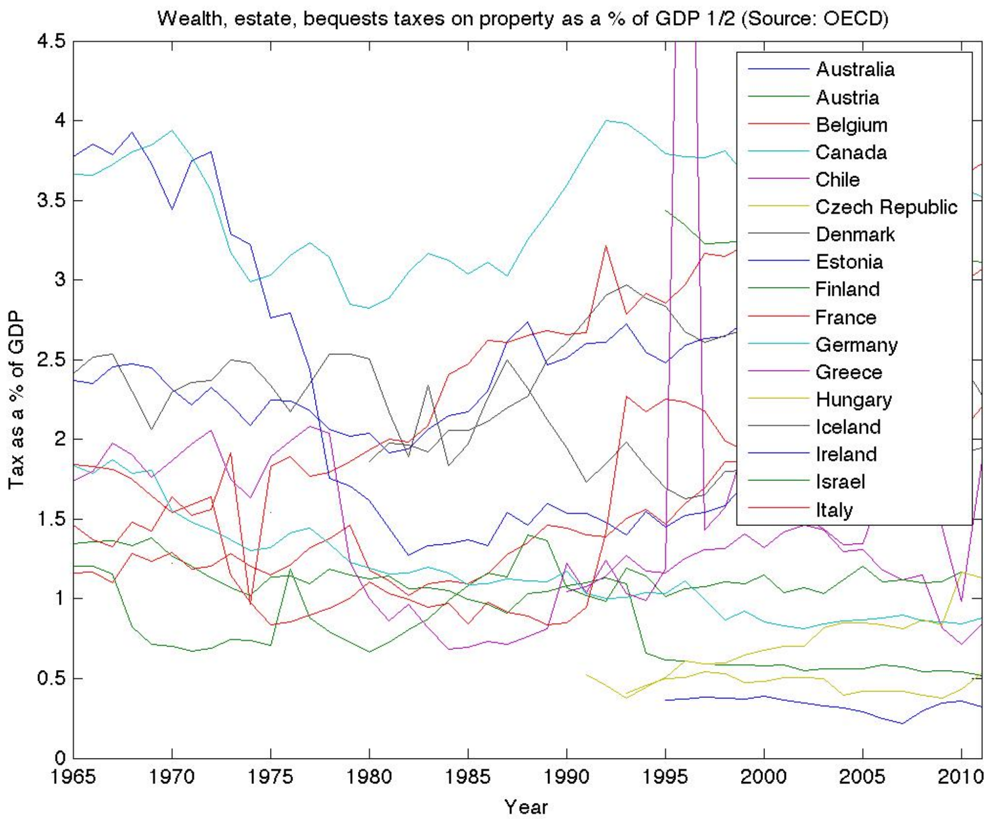
2 Figures

Figure 5: Treasury Inflation Protected Securities 10-Year



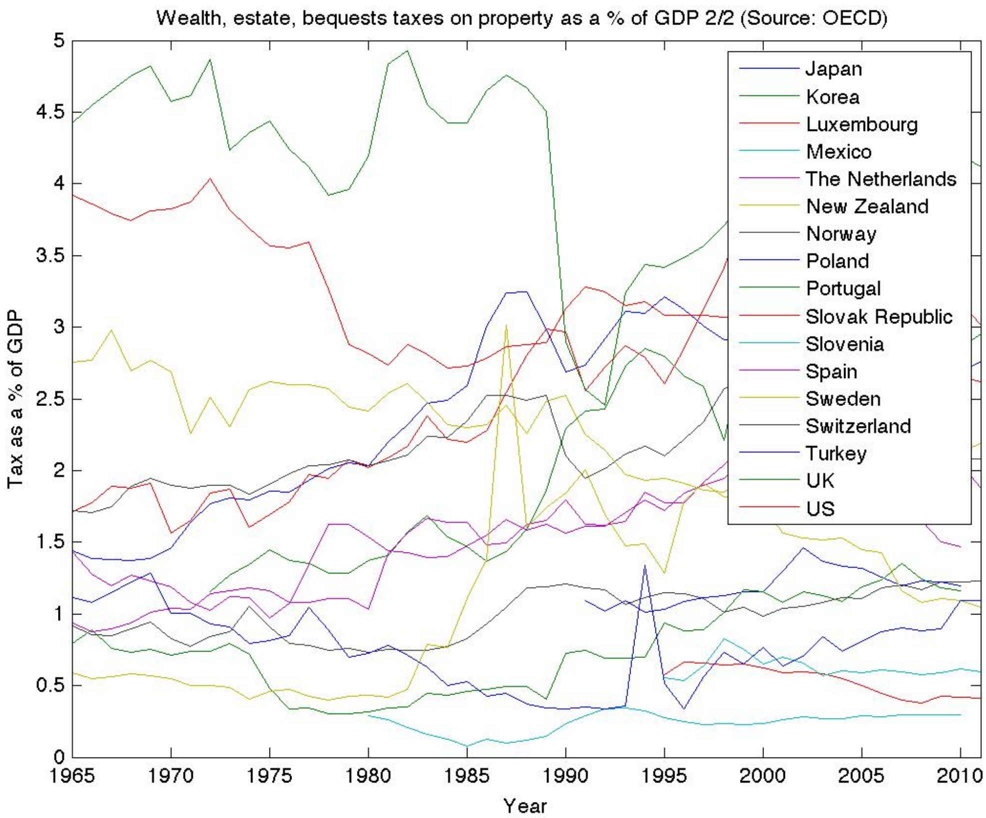
Note: Yield on Treasury Inflation Protected Securities (TIPS) with a maturity of 10 years. These are a proxy for expected real returns on assets. TIPS do not have much of a liquidity value.

Figure 6: Property taxes around the world 1/2



Note: The data is for Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Israel, Italy. This figure shows that property, wealth and bequests/gifts taxes are always present in the world. This suggests that nowhere can land or monopoly rents prevent accumulation of capital towards the Golden Rule.

Figure 7: Property taxes around the world 2/2



Note: The data is for Japan, Korea, Luxembourg, Mexico, The Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. This figure shows that property, wealth and bequests/gifts taxes are always present in the world. This suggests that nowhere can land or monopoly rents prevent accumulation of capital towards the Golden Rule.