Dynamic Inefficiency and Capital Taxation*

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Abstract

Life cycle models of capital accumulation with land (or monopoly rents or decreasing returns) can only feature capital under accumulation, because of the extreme capital crowding out properties of capitalizable rents as one approaches the Golden Rule. In this paper, I show that it is no longer valid when a government levies property, wealth or estate/gift taxes. Contrary to infinite horizon models, positive capital taxes therefore help capital accumulation towards the Golden Rule in overlapping-generations models ; and no other set of transfers from young or old, public debt, or social security system can help achieve such a level of capital accumulation.

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"As soon as the land of any country has all become private property, the landlords, like all other men, love to reap where they never sowed, and demand a rent even for its natural produce. The wood of the forest, the grass of the field, and all the natural fruits of the earth, which, when land was in common, cost the labourer only the trouble of gathering them, come, even to him, to have an additional price fixed upon them. He must then pay for the licence to gather them; and must give up to the landlord a portion of what his labour either collects or produces. This portion, or, what comes to the same thing, the price of this portion, constitutes the rent of land ...." (Adam Smith, *The Wealth of Nations*)

Introduction

Capital taxes are usually viewed both in academic and policy circles as a deterrent to capital accumulation. Notwithstanding the fact that capital taxes distort savers' decisions, there is generally little arguing about the supply side effect of capital taxes : they lead to lower capital accumulation, and therefore lower output.[1] Even when capital taxes are not explicitely studied through the lens of capital accumulation, this wisdom is always somewhat present : among many other examples, New Dynamic Public Finance finds that positive capital taxes are optimal when agents are faced with idiosynchratic productivity shocks and governments can only tax in a distortive way, but that the positive welfare effects from those capital taxes are considerably diminished when one considers the general equilibrium effects taxation has on capital accumulation.[2]

This paper challenges this intuitive idea. In a nutshell, I recognize that savings do not only nourish capital accumulation but also serve to acquire non-reproducible assets such as land. Because of capitalization, this land is less valuable all the more that capital taxes are higher, so that capital taxes can actually help capital accumulation. To study the relative potency of these effects, I develop an Overlapping-Generations neoclassical growth model a la Diamond (1965) with an asset distributing rents each period (land rents are an example, but these rents can as well be monopoly rents, decreasing returns to scale as in Scheinkman (1980)). The dividends brought about by this asset endogenously grow at the rate of growth of output, which is meant to capture the idea that land is a constant (or growing) fraction of GDP; or, in the case of monopoly rents, that the economy does not become perfectly competitive asymptotically, which would happen

[1] I voluntarily put aside here a discussion which was active in the eighties, about the fact that with an Elasticity of Intertemporal Substitution lower than 1 wealth effects could dominate over substitution effects, leading lower net of tax interest rates to increase savings, hence capital accumulation. However, I think that what comes out of this debate is that substitution effects dominate strongly once one takes into account realistic life cycle patterns for savings decisions. See for example Summers (1981). Moreover, there is a growing consensus that the inverse of the EIS and the coefficient of risk aversion - which is very high when backed out from the equity premium - are two different theoretical objects (Weil (1990), Epstein and Zin (1989)), so that the EIS is not so low.

[2] See Farhi and Werning (2012) for such a quantitative exploration. This idea also underlies Lucas (1990)' calculations that "supply-side economics", or removal of all capital taxes, would bring about.

if monopoly rents became negligible relative to GDP. As is very well known in this literature, the economy can then never reach an optimal level of capital accumulation corresponding to the Golden Rule.[3] The intuition is simple: land is a store of value for savings as capital, and therefore crowds out capital accumulation. As interest rates come closer to the Golden Rule, land becomes more and more valuable, infinitely valuable at the limit, thus making accumulation towards the Golden Rule level impossible. In this environment, capital taxes are a way to expropriate agents of these future dividends, thereby reducing the total supply of stores of value, and increasing the resources available for higher capital accumulation.

An important assumption maintained in all this paper is that governments cannot tax land (or monopoly or decreasing returns to scale) and productive capital separately: instead, they must tax both productive capital and rents are the same rate. If this was not the case, then the government would optimally tax rents at a positive rate, but would not have to tax productive capital in the same way. Then, as in Ordover and Phelps (1979), capital taxation would be one of many instruments to target the optimal level of capital accumulation, together with public debt or assets and transfers to old and young generations. Rents and capital income are indeed not as easy to distinguish as in economists' models, because in practice, rents belong to the ownership of productive capital. For the purpose of taxing land rents for example (and them only), the government would need to charge homeowners a rent corresponding to the exact value of renting land, since land and residential structures are tied to one another. Needless to say, as the relative value of residential structures and land fluctuates over time[4], this would entail high administrative costs; the value of land could not be inferred from the resale value of a home either, as the value of home improvements (for example) is unobservable albeit to a very high administrative cost. In practice, the property tax therefore falls on both elements, hence, to quote Vickrey (1999),"The property tax is, economically speaking, a combination of one of the worst taxes, the part that is assessed on improvements and in some cases to a limited extent on personalty, and one of the best taxes, the tax on land or site value", a point discussed at length in the *Mirrlees Review* (IFS (2011)). And although the model will only feature land rents, Scheinkman (1980) has shown that the crowding out properties of rents also apply to decreasing returns and monopoly rents, which are even harder to measure. Interestingly, this remark also suggests that one needs to go beyond Allais (1947)'s proposition that land should simply be nationalized: because land are not the only growing rents out there, all capital might in that case would need to be nationalized to the extreme. Adopting a market first, government second approach leads us to stay clear of such a proposition, and look for the lowest level of government ownership consistent with efficiency.

Introducing capital taxation in a model with land rents allows to get several new results

[3]George J. Stigler recalls in his *Memoirs of an unregulated economist*: "Maurice Allais was a gifted engineer and economist, but at the time he believed that private ownership of land was untenable. (The reason need not occupy us; it turned on the fact that if the interest rate went to zero, as he feared it would, land would become infinitely valuable.)".

[4]Economic geography and urban economics teach us that land values depend on the presence of a number of changing amenities.

which were not known previously in the literature. A first result is that capital taxation has two opposite effects : it reduces the supply of savings for reasonable values of the Elasticity of Intertemporal Substitution[5], but it also decreases the demand for savings coming from land values, and competing with capital for savings. Second, a result in this literature is that even though capital taxation could be used to try and target an optimal level of capital accumulation[6], the government could use other tools to target the optimum level of capital: pay-as-you-go systems, or public debt. What comes out of this literature is that targeting the optimal level of capital accumulation does not provide a strong and convincing rationale for taxing capital. This paper shows in contrast that such schemes would not be sufficient to reach the Golden Rule, and that a strictly positive level of taxation is necessary to attain this social objective. The tax on capital making possible to reach the Golden Rule might be very small in practice (especially if further decreases in asset supply are made through an increase in public assets or in reverse pay-as-you-go systems); however the fact that capital *needs to be* taxed at a positive rate to favor capital accumulation is conceptually a very important result when the zero capital tax result is an important reference point both in academic and policy discussions. Moreover, high levels of public debt and of pay-as-you systems both rely on the possibility of commitment by the government, which it may be lacking in practice, to which taxes on capital can provide a good substitute.[7] Third, on the positive side, it shows that useful land and dynamic inefficiency (hence bubbles, Pareto-improving public debt, etc.) can coexist when some capital taxes are levied, a new result in the overlapping-generations literature. Fourth and perhaps more anecdotally, it allows to revisit other theoretical results which are taken for granted in the capital taxation literature, for example that taxes on flows and stocks of capital are equivalent.

Related Literature. The literature on life-cycle models and capital accumulation starts with Allais (1947) who remarks that there exists an optimal quantity of capital, or "Golden-Rule" level ("optimum capitalistique")[8] corresponding to a long run consumption-maximizing level of

[5]As Summers (1981) in particular has shown, for realistic life cycle profiles one does not need a higher than 1 Elasticity of Intertemporal Substitution to get this result, because increased interest rates also reduces human wealth which increases the need for savings.

[6]See for example, Ordover and Phelps (1979). More in the following literature review. Positive capital taxation would be desirable in the case of too much capital accumulation, and the reverse in the case of too little. See Figure 2 for the inefficiency and Figure 3 for the efficiency case later in the text.

[7]I have previously emphasized that the government could increase public assets or use reverse pay-as-you-go systems to try to mimic the imposition of a small capital tax. However, as I show in the rest of the paper, this effect on asset supply dominates for low values of the tax, while the effect on asset demand is more important when the value of the tax increases. The government will therefore want to use capital taxation to decrease asset demand if the small tax on capital has led to dynamic inefficiency.

[8]*En franÃ§ais dans le texte*: "Chaque jour l'ensemble de l'Ã©conomie dispose d'un certain nombre d'heures de travail et la question se pose de savoir quelle est la rÃ©partition la meilleure qu'il y a lieu de faire de ce travail entre les diffÃ©rents stades de la production (biens directs et biens indirects)." Here is a tentative translation: "Every day an economy is given a certain number of hours of work, and the question is how we should use these hours as the different stages of production - direct or indirect." *Direct* production is the production of consumption goods and *indirect* production is the production of investment goods, which are later useful for

capital.[9] But as physical capital would be accumulating progressively, interest rates would go down and approach zero. This would drive the value of land to infinity, as it distributes constant dividends each period. Therefore, in order to get faster (and eventually) to the Golden-rule level of capital, Allais (1947) advocated complete nationalization of land, as George J. Stigler recalls in his memoirs quoted above.[10] This remark has later been used somewhat differently, to argue that dynamic inefficiency (capital accumulation above the Golden-rule level of capital) would be impossible, as it would make land impossible to transfer across generations.[11] Scheinkman (1980) has extended this argument to decreasing returns to scale technologies, which are another form of rent, to prove that economies with a non-vanishing rent could not reach a dynamically inefficient state. Tirole (1985) examined this claim rigorously in a Diamond (1965) overlapping-generations growth model, and showed that rational bubbles could exist even when the economy wasn't asymptotically rentless. All that was needed was that rents be not capitalized ex-ante, so as not to be used by young generations as vehicles for savings.[12] However, as McCallum (1986) noted, land is a non-vanishing rent, capitalized ex-ante, hence ruling out the possibility of dynamic inefficiency and rational bubbles.[13] This fact is a reminded as a problem in the theory of rational bubbles up to this day (see, for example Kocherlakota (2008)).[14] This problem was in fact also recognized as early as in Samuelson (1958), albeit somewhat indirectly.[15]

This paper also draws on the very large and developed literature dealing with capital taxation. The literature most directly linked to the present study is that concerning the link between capital taxation and accumulation. Ordover and Phelps (1979) showed that if the government had the policy tool to fix the capital stock at its optimal level, then the optimal capital tax rate should be zero, and Stiglitz (1985) gave a simpler proof of this same result. In the same line

producing consumption goods.

[9]For more on the Golden-Rule level of capital, see Ramsey (1928), Phelps (1961), Phelps (1965), Cass (1965), Diamond (1965).

[10]"En fait, il rÃ©sulte de ce qui prÃ©cÃ¨de qu'une politique efficace d'annulation du taux de l'intÃ©rÃªt implique nÃ©cessairement comme conditions prÃ©alables la collectivisation de la propriÃ©tÃ© du sol et la dÃ©valorisation de la monnaie circulante par rapport Ã l'unitÃ© de compte. Si de telles modifications de structure Ã©taient apportÃ©es, non seulement l'annulation du taux d'intÃ©rÃªt deviendrait possible, mais encore on peut considÃ©rer que le taux d'intÃ©rÃªt qui tendrait Ã s'Ã©tablir spontanÃ©ment serait probablement nul, voire mÃªme lÃ©gÃ¨rement nÃ©gatif" (Allais (1947)).

[11]In fact, this argument is generalizable to a growing economy, no matter what the cause of this growth is (population or technological progress), as long as the share of land does not vanish relative to GDP. This is intuitive, as the Golden Rule level of capital accumulation verifies $r = f(k^{GR}) = n$ with n the rate of growth, and land dividends grow at rate n, so that when $r < n$, the value of land is again infinite.

[12]For example, paintings, one type of rent, could not be sold by a painter's forebear.

[13]And, for his purposes, Pareto-improving money or other social contrivances. Rhee (1991) made the same argument and verified empirically that land rents were a non-decreasing share of GDP in the United States. Demange (2002) generalizes the argument to economies with uncertainty.

[14]To cite Kocherlakota (2008): "This means in turn that bubbles cannot exist in any economy in which there is some infinitely lived asset with a dividend that grows at the same long-run rate as aggregate consumption: such an asset would have an infinite price."

[15]"In it nothing kept. All ice melted, and so did all chocolates. (If non-depletable land existed, it must have been superabundant.)" (Samuelson (1958), p 481) Hence Paul Samuelson assumes that land is in infinite supply, so that it has no value.

of thought, Atkinson and Sandmo (1980) show that whether capital taxation is of any use in this context depends a lot on the policy instruments of the government. This paper shows that with capitalizable rents, the government actually does not have any other tool to fix capital at its optimal level than to use strictly positive capital taxation. No level of finite public debt or of finite transfers from the young to the old can mimic the implementation of this outcome. By contrast, the supply-side calculations in both Lucas (1990) and Farhi and Werning (2012) (for example) take as a given that the capital stock is below the Golden Rule level, and that increasing the level of capital is welfare-enhancing. Moreover, because productive capital is the only asset in their models, the supply of stores of value is unaffected by the level of capital taxation. In contrast, in my model, the supply of stores of value decreases with higher capital taxes, which unambiguously mitigates the adverse effects on demand near the Golden Rule. Stiglitz (1978) shows that somewhat counterintuively, higher estate taxation can lead to higher inequality if the elasticity of capital and labor is less than 1 in the production function: in that case, higher capital taxation leading to lower capital accumulation can increase the factor share of capital which is more concentrated among high-income individuals. Once again, the negative effects of capital taxation on capital accumulation is here taken for granted. On the general subject of capital taxation, the literature puts forward other reasons to tax capital which I shall not consider here: unobservable wealth (Cremer et al. (2003)), indirect taxation of bequests and insurance of shocks on rates of returns (Piketty and Saez (2013)). In contrast, I will work under a paradigm of complete asset markets and perfect information, thus making the case for capital taxation even stronger. Compared to Piketty and Saez (2013), I do not leave the demand side effects of capital taxation as a free parameter: the elasticity of savings to the before tax rate are not infinite as with infinite horizon models, but they are endogenously determined (and finite) in models with finite lives.

The zero capital tax result remains a reference point in most academic work concerning capital taxation. Atkinson and Stiglitz (1976) show that if consumption is separable from leisure choices, and the economy deterministic, then savings decisions should not be distorted at the optimum. But they consider only life cycle savings, so that agents' preferences shall be respected: in contrast, both in Farhi and Werning (2010) and Piketty and Saez (2013), the social welfare function differs from individuals' objectives (for example, in Farhi and Werning (2012) the social planner puts more weight on future generations). Similarly, I will adopt a steady-state utility criterion a la Phelps (1965), so that individuals' decisions will not be optimal from the society's point of view. Moreover, I will consider a finite number of overlapping-generations, so that the first and second welfare theorems fail, while Atkinson and Stiglitz (1976)'s result relies crucially on the Pareto-optimality of the competitive equilibrium without taxes.Chamley (1986) and Judd (1985) show that Ramsey (linear) taxes on capital should be set to 0 in the long run, if individuals have infinite lives. Once again, this benchmark is one in which welfare theorems provide a reference point without taxes, and in which the economy is uniformly impatient. The utility criterion considered in this Chamley-Judd benchmark is implicitly the one used by the first generation to evaluate their children's utility. As is well known, such a perspective yields to the immiseration result, which is not very satisfactory from a welfare perspective.[16] Finally, capital mobility is another argument not to tax capital (see for example Gordon (1986)); though I will not consider capital mobility in the model, the results will go through with an higher elasticity of the supply of stores of value: because land is by definition immobile, the tax will not be completely shifted through lower accumulation.

Finally, it is important to understand that the research agenda carried out in this paper is somewhat orthogonal to the New Dynamic Public Finance literature (NDPF), though the latter also rationalizes strictly positive levels of capital taxation. I will consider only a deterministic environment, so that NDPF would prescribe zero capital taxation with my assumptions. Also, I will work with Ramsey (proportional) tax systems, in contrast to the NDPF which takes a Mirrlees (1971) approach to taxation. There are two reasons for this. The first one is that studying proportional taxes is a useful benchmark, on which some strong results are based (like Chamley (1986)-Judd (1985)) The second is that Atkinson and Stiglitz (1976) suggest that labor income and capital taxation are to a large extent orthogonal: mirrleesian taxation helps tax labor in the least distortive way, and capital taxes must be set to 0. In the case considered here, competitive equilibrium fails to be Pareto-optimal anyways and linear capital taxes are not necessarily distortive, but on the other hand can help restore efficiency.

The remainder of the paper proceeds as follows. Section 1 presents a Diamond (1965) model of overlapping generations with land rents. Section 2 gives the results. Section 3 discusses the limitations of the analysis and possible future work.

1 Model

In this section, I will develop a standard Diamond (1965) model with land. For concreteness, land will be a useful input in the production function, available in fixed supply. Note that this corresponds more to an "old" (agricultural) use of land (though businesses arguably need some amount of land to operate, also nowadays); a more "modern" use of land would correspond to an inclusion in the utility function, with two goods, a consumption good and a good corresponding to the utility for living in the business district:

$$U(C, L) = C^{1-\gamma} L^{\gamma} \quad \text{with} \quad L = 1.$$

The elasticity of substitution would be assumed to be one, so that the rental price of land would grow at the rate of growth of consumption g. Finally, the price of land could also be derived by recognizing the existence of some form of increasing returns (this could be done in an urban economics model - like the Alonso-Muth-Mills monocentric model for example).

[16] In particular, Atkeson and Lucas (1992) show the importance of taking into account the welfare of children directly rather than only in the welfare of parents.

1.1 Agents

There are overlapping-generations of agents. The generation born at time t consumes c_t^y when young and c_{t+1}^o when old. Work occurs only when young, and labor is supplied inelastically. Agents then earn a wage w_t, save s_t, on which they earn return r_{t+1}. They receive a transfer T_t^y from the government when young and T_t^o when old, so that:

$$\begin{cases} c_t^y + s_t = w_t + T_t^y \\ c_{t+1}^o = (1 + r_{t+1})s_t + T_t^o \end{cases}$$

Therefore, their intertemporal budget constraint sums up to:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t + T_t^y + \frac{T_t^o}{1 + r_{t+1}} \tag{1}$$

They consume in both periods of their lives, and therefore they solve:

$$\max_{c_t^y, c_{t+1}^o} U(c_t^y, c_{t+1}^o)$$
$$\text{s.t. } c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} \leq w_t + T_t^y + \frac{T_t^o}{1 + r_{t+1}}$$

This gives a demand function for consumption when young and when old, and an implied savings demand, depending on wages and the interest rates. For simplicity, let us assume that the utility function exhibits Constant Relative Risk Aversion (CRRA) with risk aversion σ (and intertemporal elasticity of substitution $1/\sigma$):

$$U(c_t^y, c_{t+1}^o) = a\frac{(c_t^y)^{1-\sigma}}{1-\sigma} + (1-a)\frac{(c_{t+1}^o)^{1-\sigma}}{1-\sigma}.$$

The solution of this problem yields the consumption decision of the young (see Appendix A.1 for detail) :

$$c_t^y = \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma - 1}}.$$

And hence, savings are given by:

$$s_t = w_t + T_t^y - \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma - 1}}.$$

1.2 Production

Factor incomes are not exogenous. The supply of land is fixed to $L_t = 1$ for every t. On the production side, firms hire labor and use capital with a constant returns to scale technology with respect to joint labor and capital such that $Y_t = f(K_t, N_t, 1) = N_t F(k_t, 1, 1) = N_t f(k_t)$. Capital and labor earn their marginal returns expressed with the intensive form of the production function as:

$$\begin{cases} w_t = f(k_t) - k_t f'(k_t) \\ r_t = f'(k_t) \end{cases}$$

With a Cobb-Douglas production function, $F(K_t, N_t, L_t) = K_t^\alpha N_t^{1-\alpha} L_t^\beta$ and $f(k_t) = k_t^\alpha$, so that $w_t = (1-\alpha)k_t^\alpha$ and $r_t = \alpha k_t^{\alpha-1}$. Note that the rental price of land is also given by its marginal return, which gives the demand for land:

$$r_t^* = \frac{\partial Y_t}{\partial L_t} = \beta f(K_t, L_t, 1).$$

In equilibrium, r_t^* must be such that demand is consistent with supply $L_t = 1$. It is easy to see that capital will in that case receive r_t^* which increases like the rate of growth of output (and K_t and L_t on a steady-state growth path). Of course, the fact that land appears as having a unit elasticity of substitution with both labor and capital in the production function comes from the reverse-engineering of the share of land in total value added, which was first noted by Rhee (1991).[17] β parametrizes the constant relative importance of land with respect to other inputs. Except in the base where $\beta = 0$, land will turn out to play a very important role in the determination of asset supply.

1.3 Discussion

Infinite horizon models. In the Fisher (1930) theory of interest, the after-tax interest rate is always equal to the so-called modified Golden Rule:

$$r = \delta + \sigma g.$$

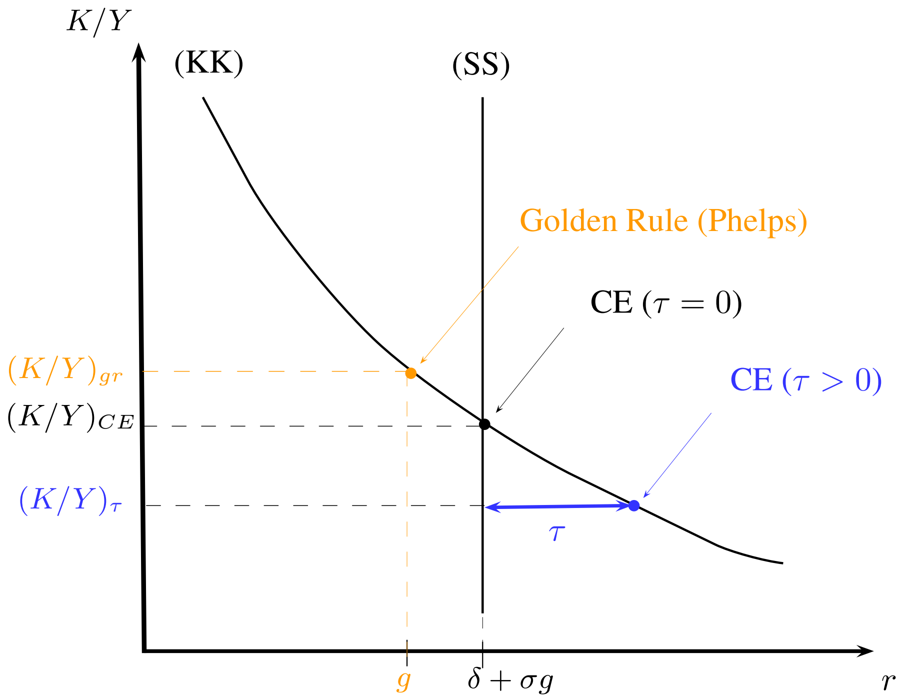
This after-tax steady-state interest rate comes from a condition for agents' consumption at infinity; but the theory predicts very large variations of capital stock accumulation in response to capital taxes. In fact, the elasticity of savings to after tax interest rates is infinite in this model, as shown in Figure 1.

However, this theory is somewhat at odds with the data for at least two reasons. First, long-term interest rates move a lot over the business cycle (see for example Figure 5 for a plot of long term interest rates), thus explaining those movements would require very high "patience" shocks on the part of consumers (such shocks are indeed used in monetary models to generate movements in the natural rate of interest). Second, this theory predicts always higher than growth interest rates (in principle, the coefficient of relative risk aversion could be lower than 1, but it is both empirically implausible and theoretically problematic, as this would mean that the consumers' infinite horizon optimization program would not be well defined). Finally, as Piketty and Saez (2013) remark, it predicts very large movements of wealth/income ratios in relation with capital tax rates, which do not seem to be there in the data.

OLG models. In contrast, I use a finite life model, in which savings have a finite interest elasticity. Note that agents are in a sense inherently impatient in OLG models, as they don't

[17] This was discussed more precisely in the introduction. The original argument, for example by Allais (1947), is formulated in an economy which does not grow. Hence, he considers constant dividends each period; and negative interest rates for dynamic inefficiency. shows using US data that land does not vanish relative to GDP, so that the same argument can be made with growth, g land rents growing at g, and interest rates below g for dynamic inefficiency. In contrast, Tirole (1985) rules out those growing with GDP capitalized rents by assumption, and only allows non-capitalized rents to grow at the same rate as GDP.

Figure 1: Capital Taxation in Fisher (1930) Theory of Interest



Note: In Fisher (1930)'s theory of interest, the after tax rate of return is pinned down. Therefore, all the adjustment to tax rates goes through a reduction in the demand for capital by firms.

care about after death consumption, and so their objective functions is always well-defined. At the same time, they are in many respects much less impatient than infinitely-lived optimizers, as they by assumption cannot borrow against future generations' income (they could not repay). For the finite live assumption to hold it is important that agents do not leave bequests a la Barro (1974); however they could well have some other form of dynastic altruism, like a warm-glow of giving bequests for example. The analysis naturally generalizes to this case.

2 Results

2.1 Some heuristics

2.1.1 No land

As is well known, with no land the competitive outcome hence will not correspond to the steady-state maximizing outcome, albeit for very particular values of the parameters. When there is capital under-accumulation, one might need to reduce some generations' consumption in order to improve future generations' welfare, and reach the Golden-rule level of steady-state consumption (this is likely to be the rationale behind savings-enhancing policies). However, under capital over-accumulation, that is $k^* > k^g$ there are allocations that unambiguously improve on the welfare of all generations. If $k^* - k^g$ is added to consumption when steady state has been reached[18], then consumption in period T is

$$\begin{aligned} f(k^*) + k^* - k^g - nk^g = & f(k^*) - nk^* + (n+1)(k^* - k^g) \\ & > f(k^*) - nk^* \end{aligned}$$

Subsequently quantity available for consumption is

$$f(k^g) - nk^g > f(k^*) - nk^*.$$

This case therefore leads to a well known result: dynamic efficiency or inefficiency depends upon the parameters of the economy. The economy can by itself accumulate too much capital or too little. However, as Ordover and Phelps (1979) note for example, there is no clear role for capital taxation in this model as pay as you go systems and public debt can as well help the economy reach the Golden Rule of capital accumulation.

2.1.2 Land with no taxes

Land substantially change the analysis of the previous model. Denote by R the steady state share of land in GDP (corresponding to the limit of r_t^*/Y_t as t approaches ∞). Then land distributes $\frac{R(1+n)^t}{1+r}$ in period t, and its value at time t hence is $\frac{R}{r-n}(1+n)^t$ or $R/(r-n)$ per capita.[19] Note that at date $t+1$, each generation born in period t gets a value of the rent per capita that decomposes in the following way:

$$\frac{R}{r-n}(1+r) = \underbrace{\overbrace{R}^{\text{land dividend}} + \overbrace{\frac{R(1+n)}{r-n}}^{\text{capital gains}}}_{\text{total return}}.$$

I then state the following proposition, generalizing Allais (1947)'s fears that an economy with land will never reach the Golden Rule level of capital accumulation, let alone dynamic inefficiency:

Proposition. *Whenever land remains productive (that is $r_t^*/Y_t \to R > 0$), the competitive allocation always displays under-accumulation relative to Golden-Rule level of capital, that is $k^* < k^g$. This is true no matter what the steady-state levels of transfers T_o or T_y between old and young, or finite levels of government debt (positive or negative).*

[18] It might be that this level is never reached, but only asymptotically. Yet the argument remains considering a large enough T above which one lies arbitrarily close to steady-state values.

[19] The value of land at date t is

$$p_t = \sum_{i=t}^{+\infty} \frac{R(1+n)^i}{(1+r)^{i+1-t}} = \frac{R(1+n)^t}{1+r} \sum_{i=0}^{+\infty} \left(\frac{1+n}{1+r}\right)^i = \frac{R(1+n)^t}{r-n}.$$

.

Proof. Let $(b, T^y, T^o) \in \mathbb{R}^3$ per-capita public debt, per-capita transfers to the old and per-capita transfers to the young such that

$$T^y + \frac{T^o}{1+n} = 0.$$

For levels of public debt per capita b ($b < 0$ denotes assets):

$$(1+n)k^* + \frac{R}{\alpha(k^*)^{\alpha-1} - n} = (1-a)(1-\alpha)(k^*)^\alpha + (1-a)T^{y'} - \frac{aT^{o'}}{1+\alpha(k^*)^{\alpha-1}}$$

with:

$$\begin{cases} T^{y'} = T^y - b(1+r) \\ T^{o'} = T^o + (1+r)(1+n)b \end{cases}$$

Hence, Golden-rule or above Golden-rule capital accumulation are impossible, no matter what the values of b, T^y, and T^o are. □

2.1.3 Land with taxes

For simplicity, let us assume that the government cannot tax land and productive capital differently, as discussed in the Introduction. In that case, the value of land at date t is

$$p_t = \sum_{i=t}^{+\infty} \frac{R}{1+r}(1+n)^i \left(\frac{1-\tau}{1+r}\right)^{i-t} = \frac{R(1+n)^t}{1+r} \sum_{i=0}^{+\infty} \left(\frac{(1+n)(1-\tau)}{1+r}\right)^i = \frac{R(1+n)^t}{r - n(1-\tau) + \tau}.$$

This time at $t+1$, each generation born in period t gets a value of the rent per capita that yields return r by arbitrage decomposing in the following way:

$$\frac{R}{r-n(1-\tau)+\tau}(1+r) = \underbrace{\overbrace{R}^{\text{land dividend}} + \overbrace{\frac{R(1+n)}{r-n(1-\tau)+\tau}}^{\text{capital gains}} - \overbrace{\frac{R\tau(1+n)}{r-n(1-\tau)+\tau}}^{\text{wealth tax}}}_{\text{total (net of tax) return}}.$$

Proposition. *With positive wealth taxes $\tau > 0$, there exist values for $(a, \alpha) \in [0,1]$ and $R \in \mathbb{R}_+^*$ such that the economy is inefficient or $k^* > k^g$.*

The intuition is straightforward: because the condition for finiteness of land values is now $r - n + \tau(1+n) > 0$, or $\alpha(k^*)^{\alpha-1} - n + \tau(1+n) > 0$, which does not rule out that $k^* > k^g$.

2.2 Quantitative results

2.2.1 No land

Let me assume for the moment that land is not valuable in production, that is: $\beta = 0$. It that case:

$$\forall t > 0, r_t^* = 0.$$

Then, at the steady state, the (SS) curve writes:

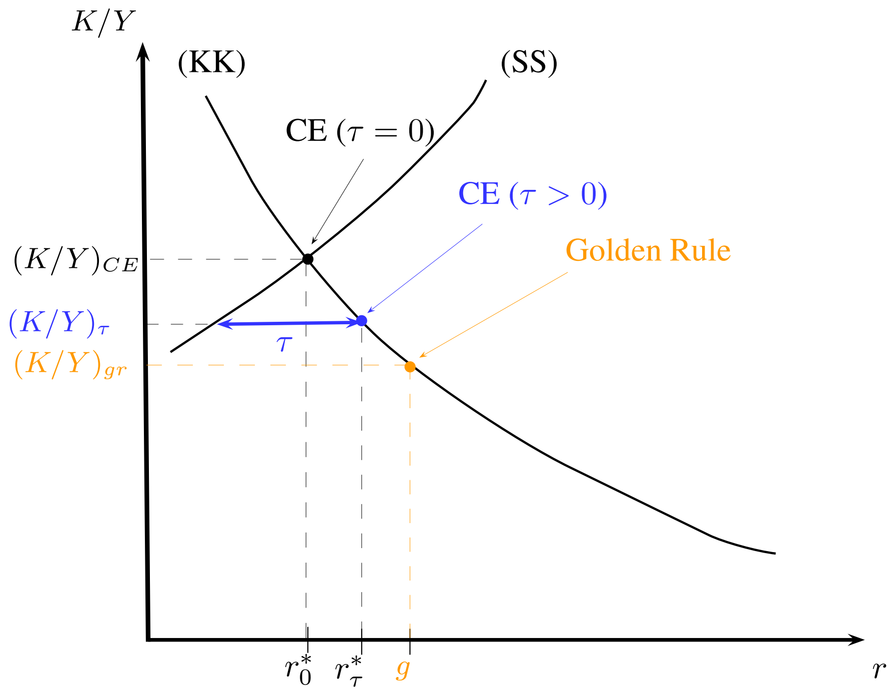
$$ss(r,0) = (1-\alpha)\left(1 - \frac{1}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}\right)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Under the assumptions outlined above, this defines an upward sloping locus in the $(r, K/Y)$ plane. That is, the assumptions are equivalent to assuming that:

$$ss'(r,0) = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{\frac{1}{\sigma}} \frac{(1+r)^{\frac{1}{\sigma}-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}} \frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} > 0.$$

The (KK) curve is standard, and defines a downward sloping curve in the $(r, K/Y)$ plane. As is well known in this case, there can be dynamic inefficiency like in the case depicted in Figure 2 or dynamic efficiency like in Figure 3.

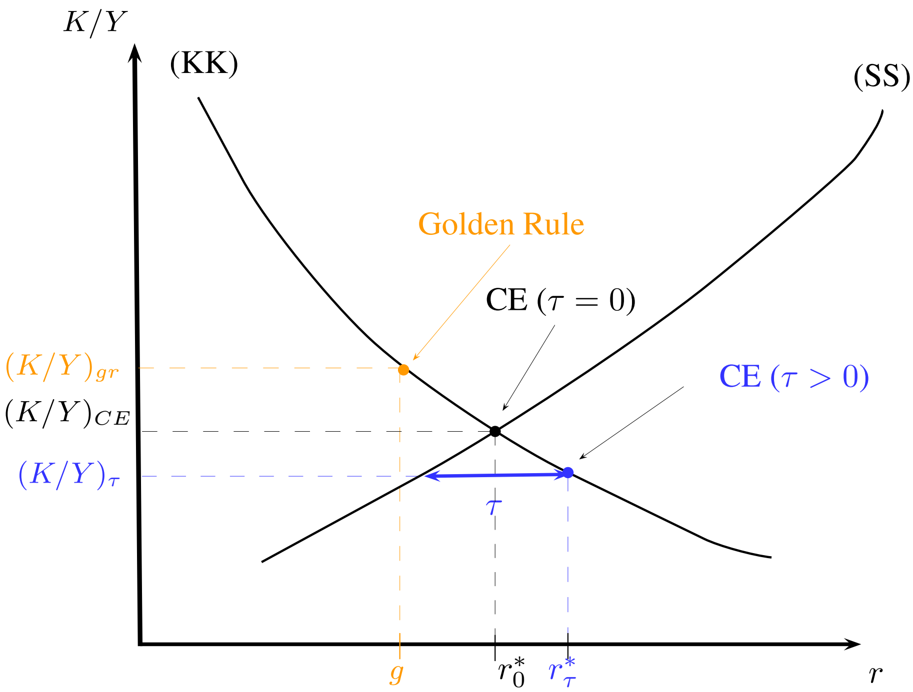
Figure 2: OLG model - Dynamic Inefficiency



Note: Steady-state of an OLG neoclassical growth model without land. As Diamond (1965) showed, the competitive equilibrium may feature too much capital accumulation. In that case, capital taxes can help discourage capital accumulation and hence help reach the Golden Rule. However, as Ordover and Phelps (1979) have shown, public debt or pay-as-you-go systems can as well be used for this matter.

The effects of capital taxation on capital accumulation are unambiguous on those two figures. If the competitive equilibrium of the undistorted economy leads to too much capital accumulation as on Figure 2, then capital taxation allows to get closer to the Golden-Rule. Denoting by

Figure 3: OLG model - Dynamic Efficiency



Note: Steady-state of an OLG neoclassical growth model without land. Here the competitive equilibrium is efficient. As with the Fisher theory of interest, capital taxation discourages capital accumulation away from the Golden Rule. Note that this effect is however dampened by the fact that the elasticity of savings to the rate of interest is finite.

$(K/Y)_{gr}$ the capital/output ratio at the Golden Rule, $(K/Y)_\tau$ the same ratio at the distorted allocation (with capital taxed at rate τ), and $(K/Y)_{CE}$ the undistorted capital/output ratio, it is clear that:

$$(K/Y)_{gr} < (K/Y)_\tau < (K/Y)_{CE},$$

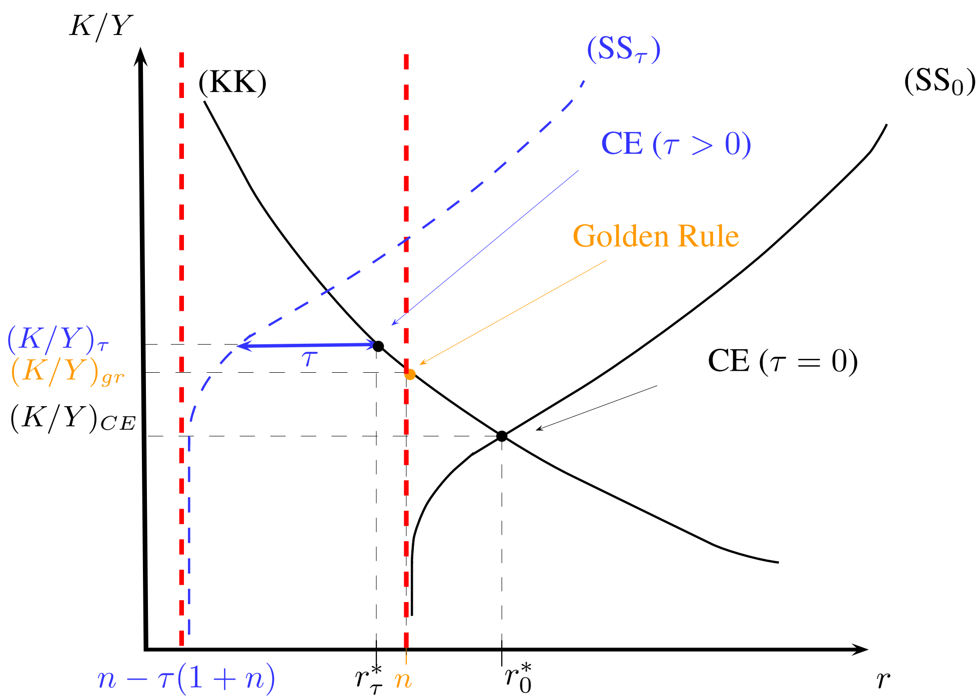
so that imposing a capital tax leads to an unambiguously better outcome (note however that this holds for a small enough value for τ). However, as Ordover and Phelps (1979) suggested, the government could just as well target a higher level of public debt or put in place pay-as-you go systems to remedy this dynamic inefficiency problem. Conversely, the case with dynamic efficiency depicted on Figure 3 would suffer from capital taxation, as the resulting capital/output ratio would be even lower. With the same notations, it is clear from the figure or from basic algebra that:

$$(K/Y)_\tau < (K/Y)_{CE} < (K/Y)_{gr}.$$

2.2.2 With land

In contrast, in the more realistic case where $R \neq 0$, there are two opposing forces. On the one hand, holding land values constant, capital taxes also have the effect of driving a wedge between the return earned by savers and that paid by entrepreneurs, decreasing overall capital accumulation. However, capital taxes also lead to a decrease in land values, thus increasing the amount of resources available from savers to productive capital (which I call "free savings" in the following).

Figure 4: OLG MODEL - WITH LAND



Note: Steady-state of an OLG neoclassical growth model, incorporating land rents. Capital taxation has two effects on capital accumulation. First, it increases the supply of free savings for capital accumulation, by reducing land values. The curve (SS) shifts from $(SS)_0$ to $(SS)_\tau$. Second, it discourages capital accumulation by driving a wage between the rate of return for investors and the rate of return for savers. In the case pictured on this graph, the net effect of a capital tax on capital accumulation is positive.

These two opposing forces are visible on Figure 4. The latter effect is shown as a leftward shift in the $(r, K/Y)$ plane of the free-savings curve from (SS_0) to (SS_τ): with increases in capital taxation, the resources available for productive capital K increase for a given level of interest rates. Note how initially, because of the extreme capital crowding out properties of land when the interest rate approaches the Golden Rule, the curve (SS_0) was always to the right of the $r = n$ curve. In contrast, with positive levels of capital taxation, the curve (SS_τ) is totally consistent with a Golden-Rule level of capital accumulation.

Algebraically, free savings (that is, savings available for productive capital investment) are equal to:

$$ss(r,R) = (1-\alpha)\left(1 - \frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}\right)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} - \frac{R}{r-n(1-\tau)+\tau}.$$

Note that this "free savings" curve is steeper than the previous one without land. Under the previous assumptions, the (SS) curve is therefore unambiguously increasing in the $(r, K/Y)$ plane. This is because:

$$\begin{aligned}\frac{\partial ss(r,R)}{\partial r} &= (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{\frac{1}{\sigma}}\frac{(1+r)^{\frac{1}{\sigma}-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}}\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} + \frac{R}{(r-n(1-\tau)+\tau)^2}\\ &= \frac{\partial ss(r,0)}{\partial r} + \frac{R}{(r-n(1-\tau)+\tau)^2} > \frac{\partial ss(r,0)}{\partial r}.\end{aligned}$$

The fact that $(SS)_0$ is always to the right of the $r=n$ vertical line, that a positive level of capital taxation corresponds to a leftward shift in the curve, etc. follows immediately from the examination of these functions. In the case shown in Figure 4, we have that:

$$(K/Y)_{gr} < (K/Y)_{CE} < (K/Y)_{\tau}.$$

In that case, positive capital taxation allows for dynamic inefficiency.

3 Discussion

Before making a few side remarks on the model, let me give a fairly simple intuition for why increasing capital taxes can lead to higher capital accumulation. In fact, committing to taxing land in the future amounts for the government to take on a (possibly very large) positive asset position. Just as public debt crowds out capital accumulation, public assets encourages it. And since Ricardian equivalence does not always hold in overlapping-generations models, this very high increase in government savings is not necessarily matched by a corresponding decrease in private savings. As was the case on Figure 4, it can even be that a sufficient amount of stores of value is no longer available once the capital tax has been imposed, so that the economy is dynamically inefficient.

3.1 Non-equivalence between stock and flow taxes on capital

A side result mentioned in the introduction is that in this framework wealth taxes and dividend taxes are not equivalent. In the model, very different would be to impose a positive capital income tax τ, which would tax dividend uniformly across time, and effectively expropriate private agents of a fraction τ of their capital stock (thus, previous arguments, which require only an arbitrarily small yet positive amount of land, would still hold). Therefore, allowing for a pre-existing factor of production like land allows to break down a well-known equivalence result between capital income taxation and wealth taxation. This result only holds *in equilibrium*, as taxation on capital τ_K and taxation on wealth τ_W are linked by :

$$1 - \tau_W)(1 + R) = 1 + (1 - \tau_K)R \quad \Leftrightarrow \quad \tau_K = \frac{\tau_W(1 + R)}{R}.$$

3.2 Government's liquidity and solvency

A side result from the theory developed above is that if there is dynamic inefficiency because the government uses capital taxation (on land in particular), then governments are solvent since they potentially have infinite assets. These assets could be used to back potentially very large levels of public debt. However, there are several reasons why those infinite positive assets cannot be used to obtain financing. A first solution would be to sell those assets, but it is not possible since the government does not legally own future land rents. It only has the right to levy a tax on the value of this land in the future, and the right to collect taxes is not transferable to private agents. This asset therefore is not redeployable, and therefore cannot be sold nor directly collateralized. A second, more intermediate solution would be to promise the revenue from wealth or property taxes to new bondholders. However, this overlooks that property taxes, which represent the bulk of land taxes in most countries, are usually levied by local governments.[20]

Conclusion

This paper is only a first attempt at incorporating land and monopoly rents into a life cycle theory of capital accumulation. It has shown that the relationship between capital accumulation and taxation is far from one-directional in such an environment. Some positive level of capital taxes is even necessary to achieve the Golden Rule level.

There are many other elements which have been left out here, but which could be fruitfully integrated in future work. In this analysis, the government did not have any financing needs on its own. Needless to say, the introduction of a need for financing public expenditures would likely strengthen the case for capital taxation since land and monopoly rents represent pure profits. The alternative would be to tax labor income, which is distortive with flexible labor supply, even when it is done in a Mirrleesian way.

If the zero capital tax benchmark is not a reference point, then this paper naturally calls for quantitative evaluation of the effects it presents. In particular, Figures 6 and 7 show that there is a high cross-sectional variation of wealth, property and bequest/gift taxes over time, which might be fruitfully used for empirical analysis.

[20] And in effect, such public debt financing schemes has already been used by the city of San Jose in the United States to finance municipal debt. As Michael Lewis (2013) anecdotally puts it: "It is one of the few cities in America with a triple-A rating from Moody's and Standard & Poor's, but only because its bondholders have the power to comel the city to levy a tax on property owners to pay off the bonds."

References

Allais, M. (1947), *Economie et intérêt*, Imprimerie Nationale et Librairie des Publications Officielles, Paris.

Atkeson, A. and Lucas, R. (1992), 'On efficient distribution with private information', *The Review of Economic Studies* **59**(3), 427–453.

Atkinson, A. and Sandmo, A. (1980), 'Welfare implications of the taxation of savings', *The Economic Journal* **90**(359), 529–549.

Atkinson, A. and Stiglitz, J. (1976), 'The Design of Tax Structure : Direct versus Indirect Taxation', *Journal of Public Economics* **6**, 55–75.

Barro, R. (1974), 'Are government bonds net wealth?', *The Journal of Political Economy* **82**(6), 1095–1117.

Cass, D. (1965), 'Optimum Growth of Capital in an Aggregative Model Accumulation', *Review of Economic Studies* **32**(3), 233–240.

Chamley, C. (1986), 'Optimal taxation of capital income in general equilibrium with infinite lives', *Econometrica: Journal of the Econometric Society* **54**(3), 607–622.

Cremer, H., Pestieau, P. and Rochet, J.-C. (2003), 'Capital income taxation when inherited wealth is not observable', *Journal of Public Economics* **87**(11), 2475–2490.

Demange, G. (2002), 'On optimality in intergenerational risk sharing', *Economic Theory* **20**(1), 1–27.

Diamond, P. (1965), 'National debt in a neoclassical growth model', *The American Economic Review* **55**(5), 1126–1150.

Epstein, L. and Zin, S. (1989), 'Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns : A Theoretical Framework', *Econometrica: Journal of the Econometric Society* **57**(4), 937–969.

Farhi, E. and Werning, I. (2010), 'Progressive estate taxation', *The Quarterly Journal of Economics* (May), 635–673.

Farhi, E. and Werning, I. (2012), 'Capital Taxation : Quantitative Explorations of the Inverse Euler Equation', *Journal of Political Economy* **120**(3), 398–445.

Fisher, I. (1930), *The theory of interest*, Augustus M. Kelley.

Gordon, R. (1986), 'Taxation of investment and savings in a world economy', *The American Economic Review* **76**(5), 1086–1102.

IFS (2011), *The Mirrless Review - Tax By Design*, Vol. i, Oxford University Press, Oxford.

Judd, K. (1985), 'Redistributive taxation in a simple perfect foresight model', *Journal of public Economics* **177**, 1–2.

Kocherlakota, N. (2008), 'Injecting rational bubbles', *Journal of Economic Theory* **142**(1), 218–232.

Lucas, R. (1990), 'Supply side economics: an analytical review', *Oxford Economic Papers* **42**(2), 293–316.

McCallum, B. (1986), 'The optimal inflation rate in an overlapping-generations economy with land', *NBER Working Paper* .

Mirrlees, J. (1971), 'An Exploration in the Theory of Optimum Income Taxation', *The Review of Economic Studies* **38**(2), 175–208.

Ordover, J. and Phelps, E. S. (1979), 'The concept of optimal taxation in the overlapping-generations model of capital and wealth', *Journal of Public Economics* **12**.

Phelps, E. (1961), 'The golden rule of accumulation: a fable for growthmen', *The American Economic Review* **151**(3712), 867–8.

Phelps, E. (1965), 'Second Essay on the Golden Rule of Accumulation', *The American Economic Review* **55**(4), 793–814.

Piketty, T. and Saez, E. (2013), 'A Theory of Optimal Inheritance Taxation', *Econometrica* .

Ramsey (1928), 'A mathematical theory of saving', *The Economic Journal* **38**(152), 543–559.

Rhee, C. (1991), 'Dynamic inefficiency in an economy with land', *The Review of Economic Studies* **58**(4), 791–797.

Samuelson, P. (1958), 'An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money', *The Journal of Political Economy* **66**(6), 467–482.

Scheinkman, J. (1980), 'Notes on asset trading in an overlapping generations model', *Dept. of Economics, University of Chicago,* .

Stiglitz, J. (1978), 'Notes on estate taxes, redistribution, and the concept of balanced growth path incidence', *The Journal of Political Economy* **86**(2).

Stiglitz, J. (1985), 'Inequality and capital taxation', *mimeo Stanford University* **IMSSS Tech**(457), 1985.

Summers, L. (1981), 'Capital taxation and accumulation in a life cycle growth model', *The American Economic Review* **71**(4), 533–544.

Tirole, J. (1985), 'Asset Bubbles and Overlapping Generations', *Econometrica* **53**(5), 1071.

Vickrey, W. (1999), Simplification, Progression, and a Level Playing Field, *in* K. Wenzer, ed., 'Land-Value Taxation: The Equitable and Efficient Source of Public Finance', m.e sharpe edn, number 9596, Armonk, NY.

Weil, P. (1990), 'Nonexpected utility in macroeconomics', *The Quarterly Journal of Economics* **105**(1), 29.

A Omitted proofs

A.1 Agents' optimization

Agents solve the following two period standard optimization problem:

$$\max_{c_t^y, c_{t+1}^o} a\frac{(c_t^y)^{1-\sigma}}{1-\sigma} + (1-a)\frac{(c_{t+1}^o)^{1-\sigma}}{1-\sigma}$$
$$\text{s.t. } c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} \leq w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}.$$

Eliminating the multiplier on the budget constraint gives:

$$\left(\frac{c_{t+1}^o}{c_t^y}\right)^\sigma = (1+r_{t+1})\frac{1-a}{a}.$$

Replacing the expression for c_{t+1}^o in the resource constraint gives:

$$\left[1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}\right]c_t^y = w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}} \quad \Rightarrow \quad c_t^y = \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}$$

A.2 Monotonicity of the steady-state "free savings" function

Denoting by $ss(.)$ the function associating free savings to the equilibrium interest rate,

$$ss(r) = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{1/\sigma}\frac{(1+r)^{1/\sigma-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}}\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}.$$

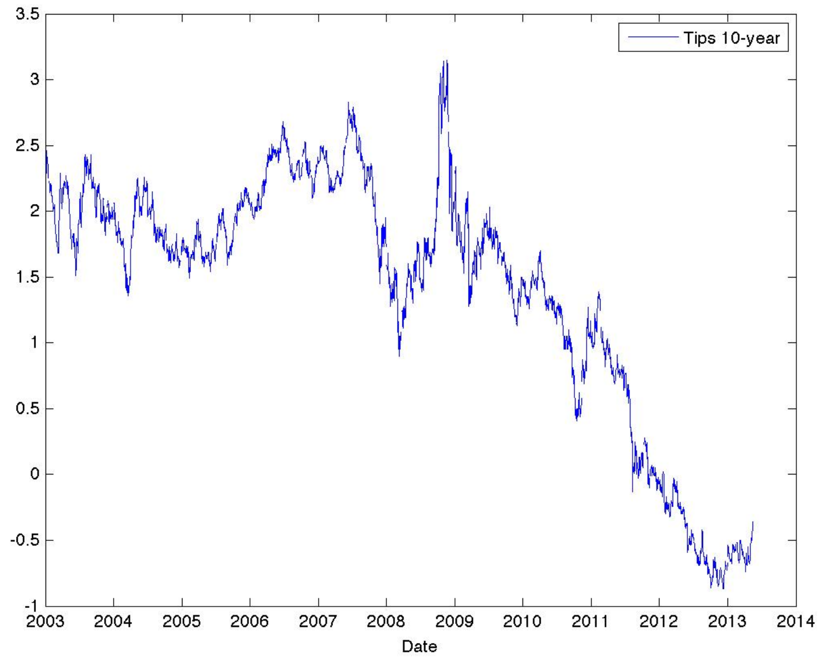
It follows that $ss(.)$ is non monotone since:

$$ss'(r) = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(1+r)^{1/\sigma-1}\left(\frac{1-a}{a}\right)^{1/\sigma}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}\left(1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}\right)}\left[\left(\frac{1}{\sigma}-1\right)\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} - \frac{\alpha}{1-\alpha}\frac{1+r}{r+\delta}\right],$$

but for $1/\sigma - 1$ large enough, and low enough values for r it will be monotonically increasing. Note that this assumption is akin to assuming that increases in interest rates do not have too much adverse impacts on wages which end up to depress savings. A similar assumption is actually made in Diamond (1965) where interest rates also appear on both sides of an equation and some single-crossing and stability assumptions must be made.

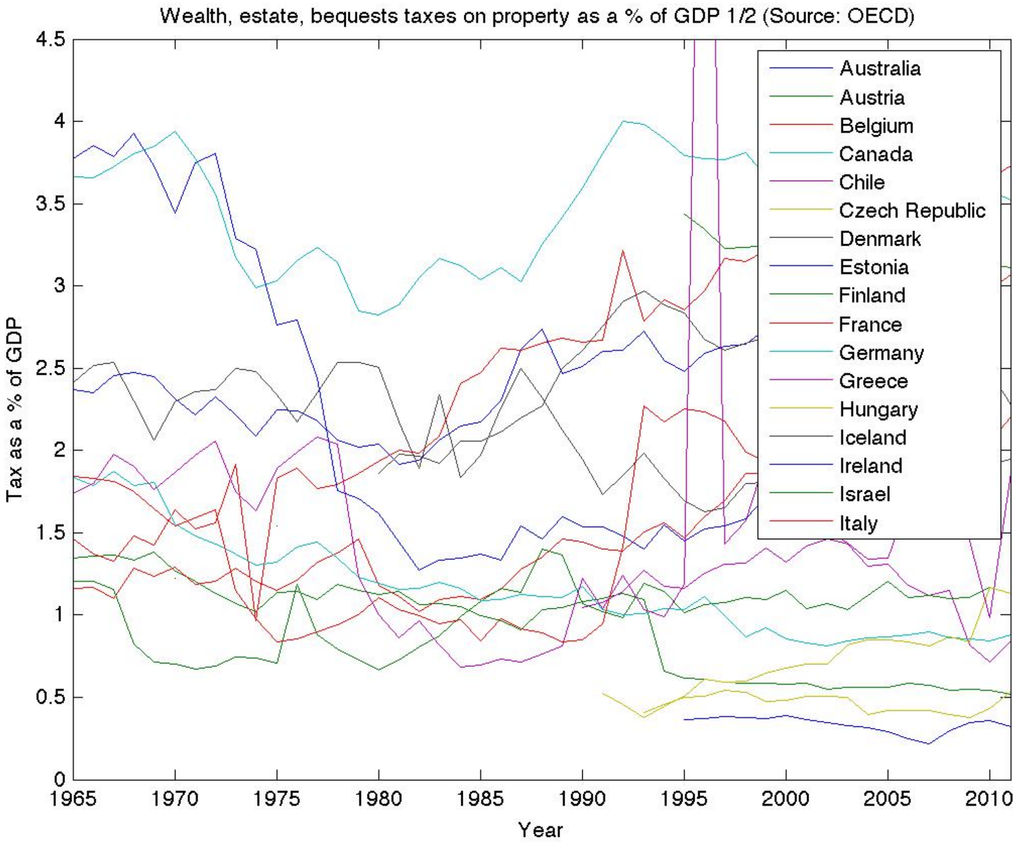
B Figures

Figure 5: Treasury Inflation Protected Securities 10-Year



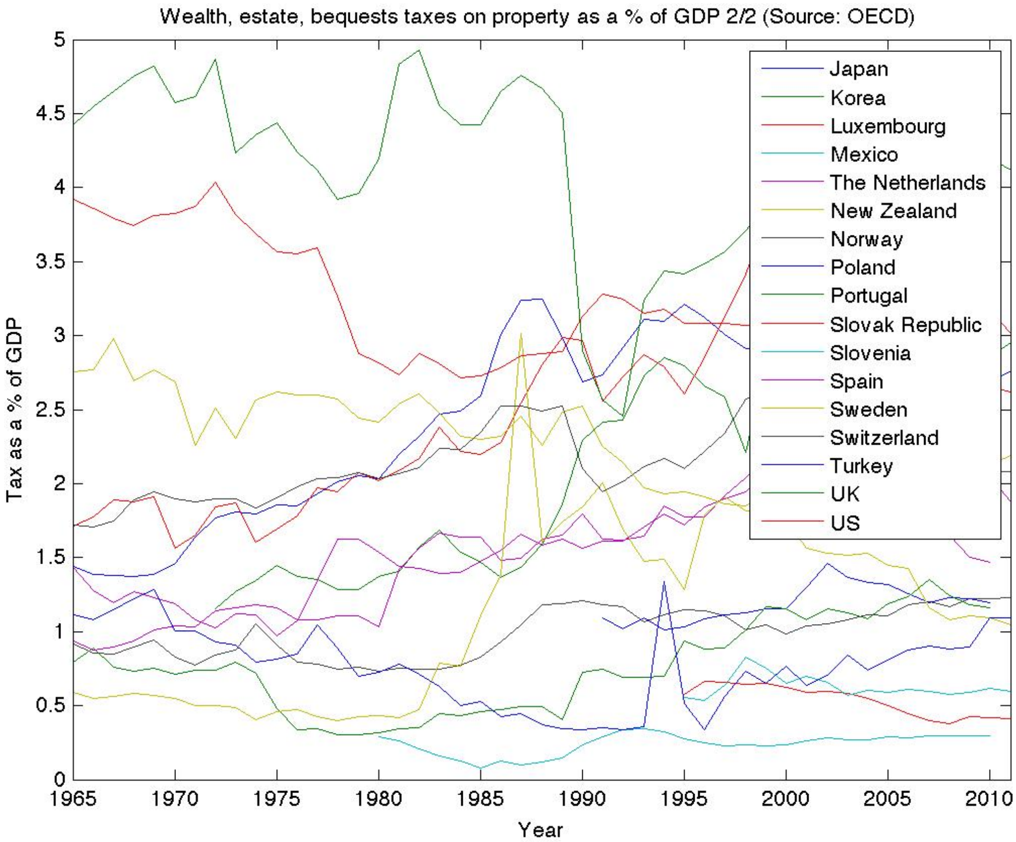
Note: Yield on Treasury Inflation Protected Securities (TIPS) with a maturity of 10 years. These are a proxy for expected real returns on assets. TIPS do not have much of a liquidity value.

Figure 6: Property taxes around the world 1/2



Note: The data is for Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Israel, Italy. This figure shows that property, wealth and bequests/gifts taxes are always present in the world. This suggests that nowhere can land or monopoly rents prevent accumulation of capital towards the Golden Rule.

Figure 7: Property taxes around the world 2/2



Note: The data is for Japan, Korea, Luxembourg, Mexico, The Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. This figure shows that property, wealth and bequests/gifts taxes are always present in the world. This suggests that nowhere can land or monopoly rents prevent accumulation of capital towards the Golden Rule.