A Theory of Demand Side Secular Stagnation*

François Geerolf[†]

UCLA

June 30, 2019
Last Version

Abstract

I study a standard growth model with a superelliptic production function, a generalization of the Cobb-Douglas with satiation of capital. When saving is scarce or investment demand is high, the growth model has a neoclassical regime, where output is determined only by supply factors, more saving fosters capital accumulation and growth, and labor is fully employed. When saving is abundant or investment demand is low, the model has a secular stagnation regime, where output is determined by aggregate demand in the long-run, even though prices are fully flexible. Through the lens of this model, secular stagnation is interpreted as an advanced state of dynamic inefficiency. There is a paradox of thrift: efforts to save more are self-defeating, and lead to lower saving, lower capital accumulation and growth, and underemployment. Deficit-financed government expenditures, as well as permanent tax cuts increase output, employment, consumption and investment permanently. Quantitatively, the model accounts for large long-run average government or tax multipliers, which depend on propensities to consume out of permanent income, and permanent redistribution schemes. In the closed economy, monetary policy works by redistributing purchasing power from lenders to borrowers, which alleviates the excess saving problem: monetary policy has real effects despite perfectly flexible prices and an absentee Phillips curve.

Keywords: Secular Stagnation, Aggregate Demand, Multipliers, Unemployment
JEL classification: E1, E2, E3, E6.

Introduction

Mainstream Keynesian and classical macroeconomic models differ on whether aggregate demand can affect output in the short run, depending on how fast prices adjust to economic disturbances. However, both agree that in the long run, output is determined only by supply forces - technology, capital, and the quality of the workforce. In this paper, I challenge this conventional wisdom by developing a model where output can be driven by aggregate demand also in the long run. I formalize Alvin Hansen (1939)'s and Larry Summers (2013)'s hypothesis of demand side secular stagnation, driven by the scarcity of investment possibilities, and the abundance of saving.[1] To that end, I propose a superelliptic production function, which is an extension of the Cobb-Douglas production function, with satiation of capital. I introduce this production function in an otherwise standard growth model. This apparently benign change turns out to have dramatic consequences for the growth process. When saving is abundant or investment demand is low, output is determined by aggregate demand in the long-run, even though prices are fully flexible. Efforts to save more can be self-defeating, and lead to lower saving, lower capital accumulation and growth, and underemployment. Deficit-financed government expenditures, as well as tax cuts increase output, employment, consumption and investment permanently. Thus, allowing for satiation in the production function leads to Keynesian policy prescriptions, but without any stickiness of prices. Therefore, aggregate demand shocks can have lasting effects, when prices

*This is a preliminary draft. Comments are welcomed. Please read the most recent version of the draft here.

[†]Email: fgeerolf@econ.ucla.edu

[1]Other theories of secular stagnation include Michau (2018), Benigno and Fornaro (2018) and Eggertsson, Mehrotra, and Robbins (2019).

are probably not sticky at a horizon of more than 1 to 2 years (Nakamura and Steinsson (2008)). Moreover, permanent shocks to aggregate demand coming from propensities to consume out of permanent income, and permanent tax shocks, matter greatly to determine the level of output. Quantitatively, the model can account for large long-run lump-sum tax multipliers, without behavioral biases or incomplete markets, solely coming from disposable income effects.

The standard growth model with a superelliptic (generalized Cobb-Douglas) production function leads to an intermediary case between Solow (1956)'s neoclassical and Harrod (1939)- Domar (1946)'s growth models. In the neoclassical regime with scarce saving, output is determined only by supply factors, more saving fosters capital accumulation and growth, and labor is fully employed. The neoclassical investment model applies, investment is determined by the cost of capital, as in Solow (1956)'s model. More saving always leads to more output in the future (although not necessarily higher consumption, in the case of dynamic inefficiency); higher public deficits crowd out capital accumulation and growth. However, when saving is abundant or investment demand is low, output is determined by aggregate demand in the long-run, even though prices are fully flexible. Efforts to save more are self-defeating, and lead to lower saving, lower capital accumulation and growth, and underemployment. Deficit-financed government expenditures, as well as tax cuts increase output, employment, consumption and investment permanently. In the closed economy, monetary policy works solely through a redistribution channel. Quantitatively, the model accounts for large long-run average government or tax multipliers, which depend on propensities to consume out of permanent income, and permanent redistribution schemes. The model provides an alternative to sticky prices to microfound why aggregate demand shocks have real effects.

I first define and study the superelliptic production function. In particular, I emphasize that this production function implies two very different regimes with respect to the relation of investment to the cost of capital. When capital intensity is low, the neoclassical model applies: a lower cost of capital increases investment demand and increases capital accumulation. When capital intensity is high, the accelerator model applies: investment does not at all depend on the cost of capital, but it only depends on output. For a very long time, the empirical literature on investment has in fact found that investment responds very little to the cost of capital, and that it responds a lot to sales.[2] At early as in 1979, Dornbusch and Fischer's were writing in the 3rd edition of their textbook, *Macroeconomics*: "At least on evidence through 1979, it seems that the cost of capital empirically does not much affect investment and that accordingly the simple accelerator model does as well as the neoclassical model at explaining investment". I conclude that the economy probably finds itself in the region where the production function is already satiated with capital, although this is not inconsistent with the fact that capital accumulation indeed played an important part in U.S. output growth in earlier phases of its capitalist history.

I then introduce this production function inside an otherwise standard growth model. I show that allowing for this slight modification of the production function (see Figure 2) allows to generate many of the Keynesian apparent "paradoxes" when capital intensity is sufficiently high: the multiplier, the paradox of thrift, the violation of Say (1803)'s law that "supply creates its own demand", the crowding in effect of government spending and tax cuts (contrary to the Treasury view). The economic intuition for this main result is as follows. With limited investment demand, supply does not "create its own demand", in the following sense. If saving propensities are higher than investment propensities, then the sum of consumption and investment demand created by additional labor, can be lower than the extra income it creates. Such a situation does not occur, almost by assumption, with a neoclassical production function, where non-satiation of capital implies that any amount of saving generates an equal amount of investment (implicitly, through a fall in the interest rate which implies that firms demand this extra investment). This interpretation also explains the multiplier intuition: one extra dollar of government spending generated an extra dollar of income, a fraction of which only is saved and invested, and so on. With a superelliptic production function, the "paradox of thrift" also arises when capital intensity is high. Efforts to save more can lead to less saving in equilibrium: everything else equal, a rise in saving does not generate any increase in investment, and on the contrary lowers the consumption component of demand. This leads to more underemployment, and as a consequence

[2] For example, Shapiro, Blanchard, and Lovell (1986), Fazzari et al. (1988), Cummins et al. (1994), Oliner, Rudebusch, and Sichel (1995), Caballero (1999), Yagan (2015). Section 1.3 discusses the empirical literature on the aggregate investment function.

lower investment demand, and lower equilibrium saving. Similarly, the Treasury view that government deficits crowd out investment demand is wrong with high capital intensity: government spending increases demand, and therefore investment demand. My theory of aggregate-demand driven output does not rely on frictions (such as sticky prices, or sticky information), but on technology. As a consequence, it can provide an explanation for why aggregate demand matters for long run output, as has been shown in a growing literature (Fatás and Summers (2018)).

I then extend the model to an economy populated with two types of heterogeneous agents, and different propensities to save out of permanent income. The reason is twofold. First, there has been a rise in both income and wealth inequality, so that heterogeneity is important to understand the sources of the excess of saving propensities over investment propensities. I show that the multipliers for tax cuts to low income earners are much higher than multipliers for tax cuts on high income earners, which is consistent with Zidar (2019), who finds close to no effect of tax cuts to high income earners, and very large multipliers for tax cuts to low income earners. Of course, this argument only takes into account the disposable income effects of taxation, and their impact on aggregate saving, not at all incentives effects, which would mitigate the advantages from redistributing income. Second, monetary policy plays an important role in thinking about stabilization policy. I show that even though prices are completely flexible in my model, monetary policy can have real effects on the economy by redistributing resources from creditors to borrowers, who on average have a higher marginal propensity to consume. As a consequence, monetary policy works similarly to fiscal policy. It decreases the average propensity to save, which makes the secular stagnation problem less severe, and increase labor utilization.

Using the concepts of undergraduate macroeconomics teaching, the superelliptic production function can be thought of as an alternative to the Phillips curve for the determination of Aggregate Supply. This theory of aggregate supply crucially relies on the special form of the production function: a superelliptic production function, with which there can be too much saving compared to investment demand. As long as labor is not fully employed, the economy can accommodate any level of demand, depending on the balance between saving and investment it creates. However, macroeconomic management would probably be too simple if this was the only relevant constraint. I have already alluded to incentive effects which constrain the possibility of redistribution to increase the average propensity to consume. In previous work (Geerolf (2018)), I have shown that the Phillips curve does not represent a trade-off between inflation and unemployment, but that it rather appears as a relationship between real exchange rates and economic activity. This highlights that negative consequences of uncoordinated aggregate demand stimulating policies sometimes include a loss in competitiveness, trade deficits and an over-sized non-traded sector. This trade-off is an important theme in the discussions between the United States, China and Germany, as well as inside the Euro area between surplus and deficit countries.

Whether oversaving (or underconsumption) is a theoretical possibility is a long standing theme in the economics literature. Most classical authors, at least since Smith (1776), denied that this even was a possibility, and accepted what is now referred to as Say's law that "supply creates its own demand." Implicitly, what Say's law implies is that firms always have a sufficiently high demand for investment, so that what is not consumed, and saved, is invested in the purchase of capital goods, and therefore also creates a demand for goods. However, many authors have expressed the opposite view. Malthus (1820) thought that increasing saving to an indefinite extent would result in a contradiction: "Adam Smith has stated that capitals are increased by parsimony, that every frugal man is a public benefactor, and that the increase of wealth depends upon the balance of produce above consumption. That these propositions are true to a great extent is perfectly unquestionable... But it is quite obvious that they are not true to an indefinite extent, and that the principles of saving, pushed to excess, would destroy the motive to production. If every person were satisfied with the simplest food, the poorest clothing, and the meanest houses, it is certain that no other sort of food, clothing, and lodging would be in existence." Underconsumptionist theories have a long history and are related to mercantilism (Mandeville (1732), Malthus (1820)) and theories of imperialism (Hobson (1902))

The rest of the paper proceeds as follows. In section 1, I first define and study the properties of the superelliptic production function: I show that the implied marginal product of capital, elasticity of substitution, and interest elasticity of investment have specific patterns which explain the results that follow. In section 2, I introduce this production function in the standard growth model with a representative agent, in order to present the main concepts of the paper. I present the paradox of thrift, discuss the Treasury View, discuss the permanent nature of shocks to aggregate demand, and compute expenditures and tax multipliers as a function of propensities to consume out of permanent income. In section 3, I allow for two types of agents with different marginal propensities to consume out of permanent income, which allows to discuss fiscal policy, as well as the real effects of monetary policy, despite perfectly flexible prices.

1 A Superelliptic Production Function

The key innovation in this paper is to propose a production function which can be satiated with capital, of which the superelliptic production function is an example. In section 1.1, I first define the superelliptic production function, and show that it is a generalization of the Cobb-Douglas production function. In section 1.2, I study its properties: the marginal product of capital, the elasticity of substitution, the interest elasticity of investment implied by this superelliptic production function. I show that there exists a region of the parameter space where the neoclassical model of investment applies, while in another region of the parameter space the accelerator model of investment applies. Finally, I discuss in section 1.3 where the evidence stands on the interest elasticity of investment, and conclude that the relevant region of the parameter space relevant to today's economy may be that of the secular stagnation regime.

1.1 Definition

1.1.1 Production Function

The key contribution of the paper is to propose a production function which can be satiated with capital. In order to keep as close as possible to the neoclassical growth model, I propose the **superelliptic** production function, which corresponds to a generalization of the Cobb-Douglas production function in the limit.[3] Production Y is given as a function of labor inputs L and capital inputs K by the following superelliptic (generalized Cobb-Douglas) production function, defined piecewise:

$$Y = F(K,L) = \begin{cases} K^\alpha \left(AL\right)^{1-\alpha} \left(1 - \dfrac{1}{2\kappa}\dfrac{K}{AL}\right)^\alpha & \text{if} \quad 0 < \dfrac{K}{AL} \leq \kappa \\ \left(\dfrac{\kappa}{2}\right)^\alpha AL & \text{if} \quad \dfrac{K}{AL} > \kappa, \end{cases} \tag{1}$$

where α and A are parameters. α measures the importance of decreasing returns to capital, as in the Cobb-Douglas case. A is labor-augmenting productivity growth.

This production function is a clear generalization of the Cobb-Douglas, as shown on Figure 1. As parameter κ in the production function becomes large in equation (1), the Cobb-Douglas production function obtains in the limit when $\kappa \to \infty$, as then $Y = K^\alpha \left(AL\right)^{1-\alpha}$. It will be useful to sometimes work with β, defined as the maximum possible capital-over-output ratio, which is a monotone transformation of κ and α:

$$\beta \equiv \left(\frac{K}{Y}\right)_{k=\kappa} = \left(\frac{\kappa}{y}\right)_{k=\kappa} = \frac{\kappa}{\left(\dfrac{\kappa}{2}\right)^\alpha} = 2^\alpha \kappa^{1-\alpha}. \tag{2}$$

As a consequence, κ should really be interpreted as proxying for investment demand, as the maximum possible capital over output ratio β is a positive function of κ. To fix ideas, $\beta = 2 = 200\%$ of GDP (the approximate book value of capital in the United States) corresponds to an investment demand equal to $\kappa = 2$.

Another useful formulation of the production function is given by equation (3) below, which shows the superelliptic nature of the production function (and uses β defined in equation (2) above):

$$Y = F(K,L) = \begin{cases} AL \cdot \dfrac{\kappa}{\beta} \left(1 - \left(1 - \dfrac{k}{\kappa}\right)^2\right)^\alpha & \text{if} \quad 0 < k \leq \kappa \\ AL \cdot \dfrac{\kappa}{\beta} & \text{if} \quad k \geq \kappa. \end{cases} \tag{3}$$

[3] The superelliptic curve is also sometimes called Lamé curve, after Gabriel Lamé (1795-1870), a French mathematician.

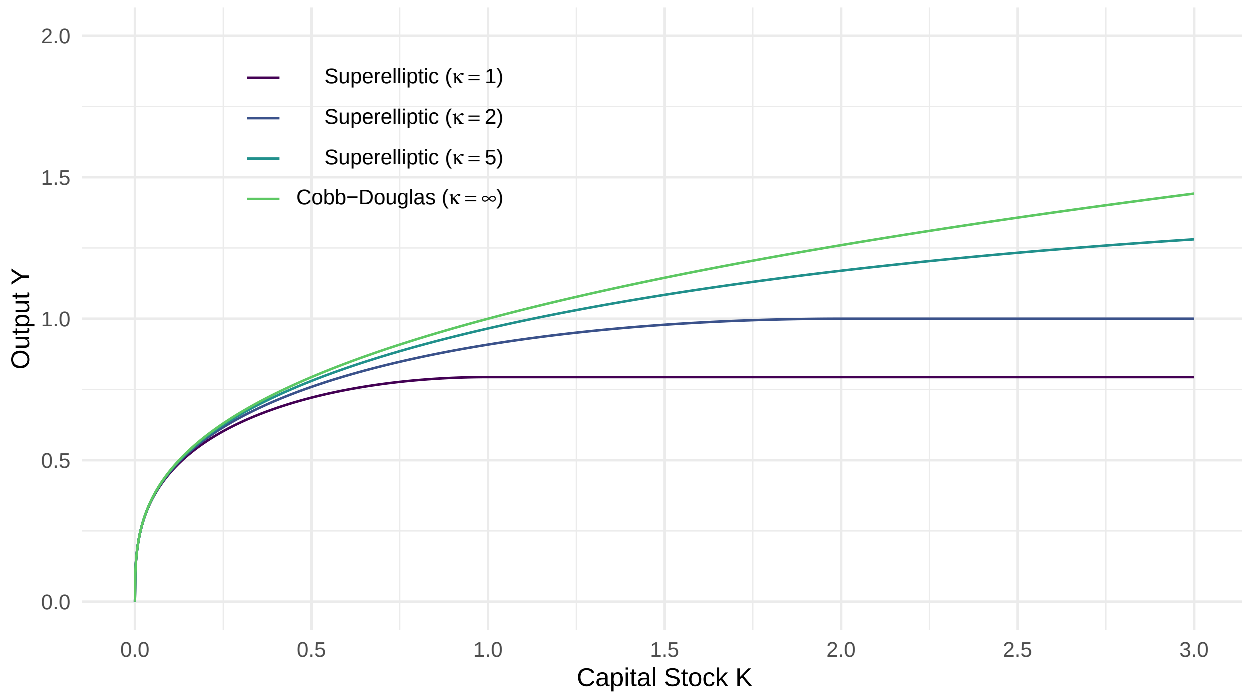

Figure 1: Superelliptic production function with $\kappa = 1, 2, 5, \infty$. $(\alpha = 1/3,\ A = 1,\ L = 1)$

In the concave range, the production function is characterized by a superellipse given by the following equation:

$$\left(\frac{\beta}{\kappa}\frac{Y}{AL}\right)^{1/\alpha} + \left(1 - \frac{1}{\kappa}\frac{K}{AL}\right)^2 = 1.$$

Since $0 < \alpha < 1$, we know that $1/\alpha > 1$. For $\alpha < 1/2$ or $1/\alpha > 2$, the graph of the production function is a hyperellipse: the curve looks superficially like a rectangle with rounded corners. It would be an ellipse if $\alpha = 1/2$ and an hypoellipse if $\alpha > 1/2$. This superellipse is centered on $k = \kappa$, and its maximum is attained for $Y = AL \cdot (\kappa/2)^\alpha$.

For finite κ, when capital intensity is higher than κ, the production function is linear in labor. At this level of capital intensity, adding more capital does not bring any more output: in other words, the marginal product of capital is then equal to zero. When capital intensity is lower than κ, both the quantity of output and labor matter for how much is being produced. When capital intensity is low, relative to κ, that is when $(1/2\kappa) \cdot (K/AL) \ll 1$, we obtain the Cobb-Douglas production function in the limit:

$$k \ll \kappa \quad \Rightarrow \quad Y = F(K, L) = K^\alpha \, (AL)^{1-\alpha}.$$

For higher values of k relative to κ, the production function runs faster into diminishing returns than the Cobb-Douglas, as can be seen from the $(1 - (1/2\kappa) \cdot (K/AL))^\alpha$ term which is decreasing in k. Section 1.2 studies these features more in detail, by computing the Marginal Product of Capital, the elasticity of substitution, and the interest elasticity which are implied by the superelliptic production function.

This production function captures in a straightforward way Hansen (1939)'s hypothesis of a fixed and limited investment demand. In particular, unlike in the benchmark Solow (1956) growth model, there exists no cost of capital which makes firms demand more investment. As Hansen (1939) argued in his 1939 presidential address to the *American Economic Association*, the role of the rate of interest in the determination of investment has perhaps been overstated: "Yet all in all, I venture to assert that the role of the rate of interest as a determinant of investment has occupied a place larger than it deserves in our thinking. If this be granted, we are forced to regard the factors which underlie economic progress as the dominant determinants of investment and employment." Section 1.3 reviews the empirical evidence on investment demand, which confirms that indeed, the cost of capital does not seem to matter for investment.

Two additional observations are in order concerning this superelliptic production function. First, this production function has Harrod-neutral technological change. As a result, this production function does imply balanced growth. Second, the Cobb-Douglas functional form which underlies the neoclassical part of the production function may seem very specific, but just at the Cobb-Douglas production is not needed to Solow (1956)'s growth model, this one is not crucial either. The key assumption is that above some maximum capital to output ratio κ, the economy is satiated with capital. Note that κ is here the maximum book value of capital. The market value of capital, in contrast, can be much greater than the book value, as documented for example in Piketty (2014) and Piketty and Zucman (2014). We shall see later that because of limited investment opportunities, investors may bid assets up because there exists a shortage of stores of value. (Tirole (1985))

1.1.2 Intensive Form

Because of constant returns to scale in production, I now write the production function in its intensive form. I assume Harrod-neutral technological change (consistent with balanced growth), so that I write capital as a function of efficiency units of labor AL. Defining as k the equilibrium capital / efficiency units of labor ratio, also called *capital intensity*:

$$k \equiv \frac{K}{AL},$$

implies an intensive form for the production function expressing output per efficiency units of labor defined as:

$$y \equiv \frac{Y}{AL}.$$

The intensive form of the production function expresses y as a function of k and parameters α and κ:

$$y = f(k) = \begin{cases} k^\alpha \left(1 - \dfrac{1}{2\kappa} k\right)^\alpha & \text{if} \quad 0 < k \leq \kappa \\ \left(\dfrac{\kappa}{2}\right)^\alpha & \text{if} \quad k \geq \kappa. \end{cases}$$

When the threshold κ goes to infinity, we get the Solow production function on the whole domain, so that this function is indeed a generalization of the Cobb-Douglas:

$$\kappa = +\infty \quad \Rightarrow \quad \forall k \geq 0, \quad y = f(k) = k^\alpha.$$

An equivalent formulation of this production function is as follows, where β is defined as a function of κ in equation (2):

$$y = f(k) = \begin{cases} \dfrac{\kappa}{\beta} \left(1 - \left(1 - \dfrac{k}{\kappa}\right)^2\right)^\alpha & \text{if} \quad 0 < k \leq \kappa \\ \dfrac{\kappa}{\beta} & \text{if} \quad k \geq \kappa. \end{cases}$$

This formulation allows to view the superelliptic form of the production function:

$$\left(\frac{\beta}{\kappa} y\right)^{1/\alpha} + \left(1 - \frac{k}{\kappa}\right)^2 = 1.$$

The intensive form of this production function is plotted on Figure 2, with capital intensity k relative to the maximum κ on the x-axis, and assuming that $\alpha = 1/3$. For higher levels of capital accumulation, production runs faster into diminishing returns to capital than Cobb-Douglas, and capital eventually reaches a point where the gross marginal product of capital is equal to zero.

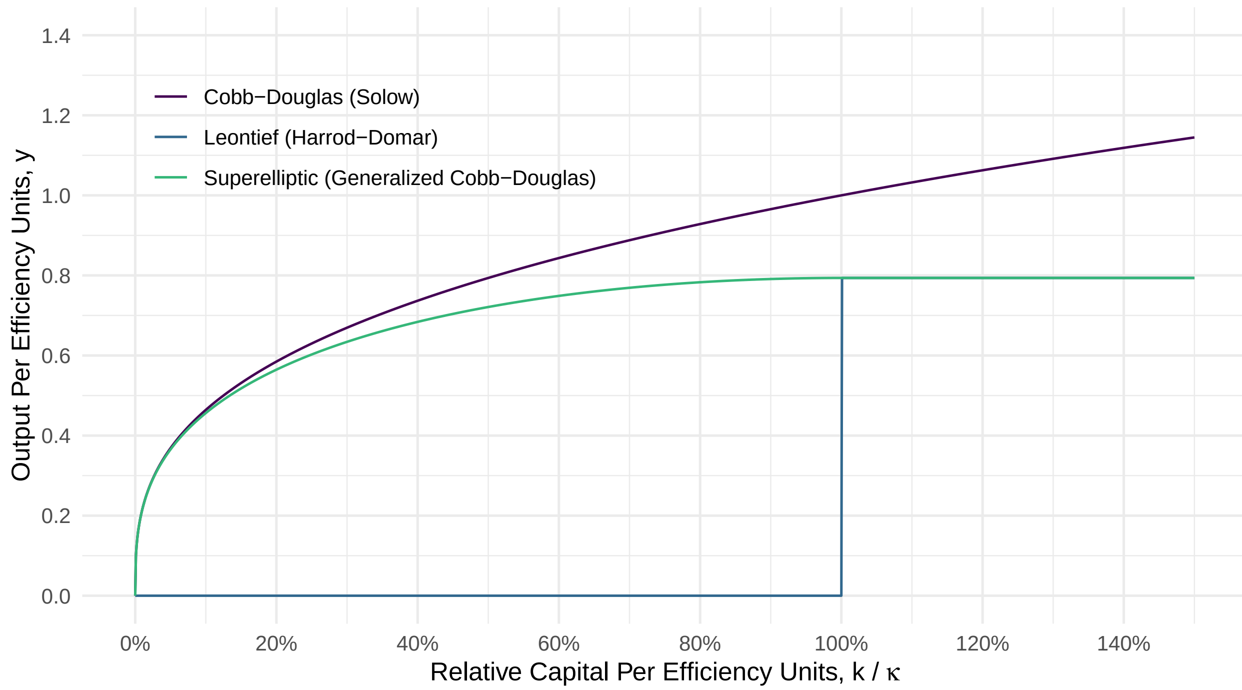

Figure 2: Production Function $y = f(k)$: Cobb-Douglas, Leontief and Superelliptic (Generalized Cobb-Douglas), with $\alpha = 1/3$.

1.2 Properties

I now study the properties from this superelliptic production function. I first compute the Marginal Product of Capital, then go on to compute the elasticity of substitution between capital and labor implied by such a production function, and finally I compute the interest elasticity of investment.

1.2.1 Marginal Product of Capital

Straightforward algebra detailed in Appendix A.1 allows to compute the gross marginal product of capital $\partial Y/\partial K$ for this production function, given by:

$$MPK = \frac{\partial F}{\partial K} = \begin{cases} \alpha \left(\frac{K}{AL}\right)^{\alpha-1} \left(1 - \frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1} \left(1 - \frac{1}{\kappa}\frac{K}{AL}\right) & \text{if} \quad 0 < \frac{K}{AL} \leq \kappa \\ 0 & \text{if} \quad \frac{K}{AL} \geq \kappa \end{cases}$$

Alternatively, the Marginal Product of Capital can be expressed solely as a function of $k \equiv K/AL$, as:

$$MPK = f'(k) = \begin{cases} \alpha k^{\alpha-1} \left(1 - \frac{k}{2\kappa}\right)^{\alpha-1} \left(1 - \frac{k}{\kappa}\right) & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa \end{cases}$$

Again, note the convergence to the Cobb-Douglas case when κ goes to infinity:

$$\kappa = +\infty \quad \Rightarrow \quad \forall k \geq 0, \quad MPK = f'(k) = \alpha k^{\alpha-1}.$$

Figure 3 plots the value of the marginal product of capital, as a function of $k = K/AL$, for the Cobb-Douglas, Leontief, and superelliptic case. I assume that $\kappa = 1$, and $\alpha = 1/3$.

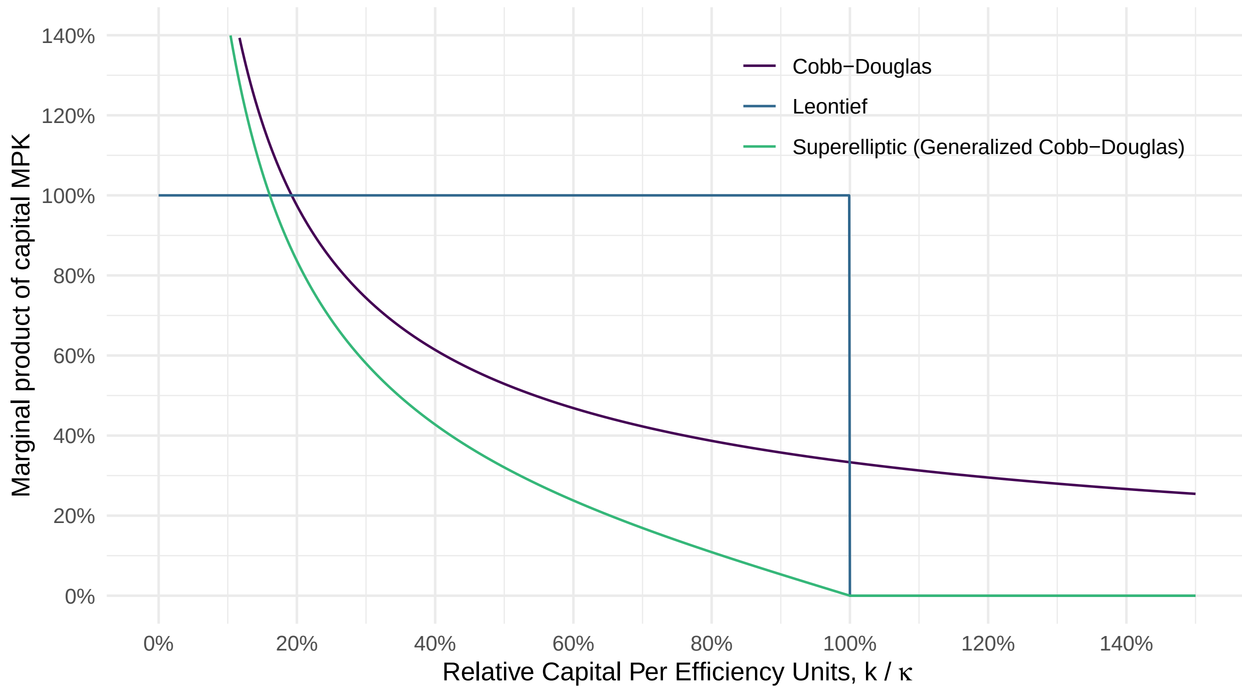

Figure 3: Marginal Product of Capital: Cobb-Douglas, Leontief and Superelliptic ($\kappa = 1$, $\alpha = 1/3$).

The gross marginal product of capital is decreasing with the quantity of capital already installed, as with the Cobb-Douglas production function. However, for any $k \geq \kappa$, the gross marginal product of capital is equal to 0.

1.2.2 Elasticity of Substitution between Capital and Labor

In order to get more economic intuition, I now compute the elasticity of substitution between capital and labor for the Superelliptic production function. I show that the elasticity of substitution between capital and labor is monotonically decreasing from 1 to 0, when the capital intensity varies between 0 and κ. The definition of the elasticity between capital and labor is given by the change in the capital over labor ratio, when the relative price of capital and labor changes:

$$\sigma\left(\frac{K}{L}\right) = -\frac{d\log\left(\frac{K}{L}\right)}{d\log\left(\frac{\partial F/\partial K}{\partial F/\partial L}\right)}.$$

After some algebra detailed in Appendix A.3, we get the elasticity of substitution as a function of $k \equiv K/AL$, which is given by:

$$\sigma(k) = \begin{cases} \dfrac{1}{1 + \dfrac{\kappa}{\kappa - k} - \dfrac{\kappa}{\kappa - \dfrac{1-2\alpha}{2-2\alpha}k}} & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa \end{cases}$$

Using the max operator:

$$\sigma\left(k\right)=\max\left\{\frac{1}{1+\dfrac{\kappa}{\kappa-k}-\dfrac{\kappa}{\kappa-\dfrac{1-2\alpha}{2-2\alpha}k}},0\right\}$$

A key feature of this production function is that it does not belong to the class of Constant Elasticity of Substitution (CES) functions, but that the elasticity of substitution depends on the level of capital that has already been accumulated. Capital runs faster into diminishing returns than in the neoclassical case, where it is assumed that no matter how many machines have already been bought, adding an additional machine without bringing an extra worker always leads to higher output.

Again, note the convergence to the Cobb-Douglas case when κ goes to infinity:

$$\kappa=+\infty \quad\Rightarrow\quad \forall k\geq 0,\quad \sigma(k)=1.$$

Figure 4 plots the elasticity of substitution σ as a function of k, for the Cobb-Douglas, Leontief, and superelliptic case. Again, I assume that $\kappa=1$, and $\alpha=1/3$.

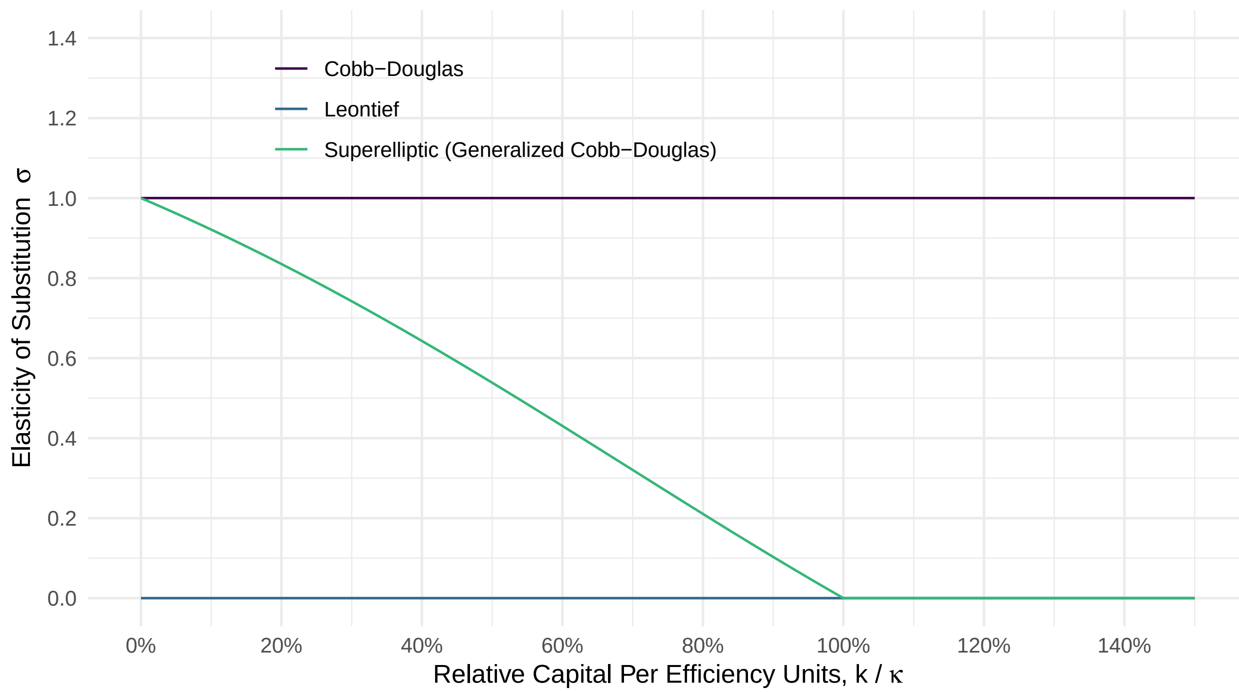

Figure 4: Elasticity of Substitution σ: Cobb-Douglas, Leontief and Superelliptic

A final way to see that the proposed production function is an intermediary case between Solow (1956)'s Cobb-Douglas neoclassical production function, and Harrod (1939) and Domar (1946)'s Leontief production function with fixed proportions, is to compute the elasticity of substitution at the limits of the domain where capital intensity is zero, and capital intensity is κ:

- At low level of capital intensity, we find that the elasticity is that of the Cobb-Douglas production function:

$$\sigma\left(\frac{K}{L}\right)\xrightarrow{K/AL\to 0}1$$

- When the level of capital intensity reaches its maximum κ, the elasticity goes to 0, approaching the Leontief case:
$$\sigma\left(\frac{K}{L}\right)\xrightarrow{K/AL\to\kappa}0$$

When κ becomes large, this production function becomes the Cobb-Douglas production function, so it is simply a generalization allowing for potential capital satiation. In this case, it never is the case that the elasticity substitution reaches 0 eventually.

1.2.3 Interest elasticity of Investment

By definition, the interest elasticity of the capital over output ratio is directly related to the elasticity of substitution between capital and labor, through the formula:

$$-\frac{d\log\left(\frac{K}{L}\right)}{d\log r}=-\frac{d\log\left(\frac{K}{L}\right)}{d\log\left(\frac{r}{w}\right)}=\sigma(k)$$

This implies that the interest elasticity of the capital over output ratio is given by:

$$-\frac{d\log\left(\frac{K}{L}\right)}{d\log r}=\sigma\left(k\right)=\max\left\{\frac{1}{1+\frac{\kappa}{\kappa-k}-\frac{\kappa}{\kappa-\frac{1-2\alpha}{2-2\alpha}k}},0\right\}$$

Since the interest elasticity of investment is equal to the elasticity of substitution between capital and labor $\sigma(k)$, Figure 4 is also the interest elasticity of investment as a function of capital intensity k, for $\alpha=1/3$ and $\kappa=1$.

1.2.4 Neoclassical and Accelerator Model of Investment

The marginal product of capital, the elasticity of substitution and the interest elasticity of investment all demonstrate that with the superelliptic production function (generalized Cobb-Douglas), there exists two very different regimes regarding which model best explains investment demand, which are illustrated on Figure 5:

- **Neoclassical Model of Investment.** When $0\leq k<\kappa$, the marginal product of capital is positive, the elasticity of substitution is positive, and the cost of capital matters to explain investment, as in the neoclassical case.

- **Accelerator Model of Investment.** When $k\geq\kappa$, the marginal product of capital is zero, as is the elasticity of substitution between capital and labor, and investment is completely insensitive to the cost of capital. In this region of the parameter space, investment demand is best described by an accelerator model. The accelerator model is described by Caballero (1999): "The simple accelerator model was based on the view that firms install new capital when they need to produce more. Therefore, firms would invest if output was expected to change, but they would not otherwise undertake net investment. The simple accelerator model did a reasonable job of explaining the data but was regarded as inadequate since it failed to take the costs of investing into account."

1.3 Evidence

The superelliptic production function, whose properties I have just studied, can accommodate a large range of elasticities of substitution, depending on how much capital was already accumulated. Indeed, it runs faster into diminishing returns than the constant elasticity, Cobb-Douglas neoclassical production function. What is the empirical evidence on the elasticity of investment to the cost of capital?

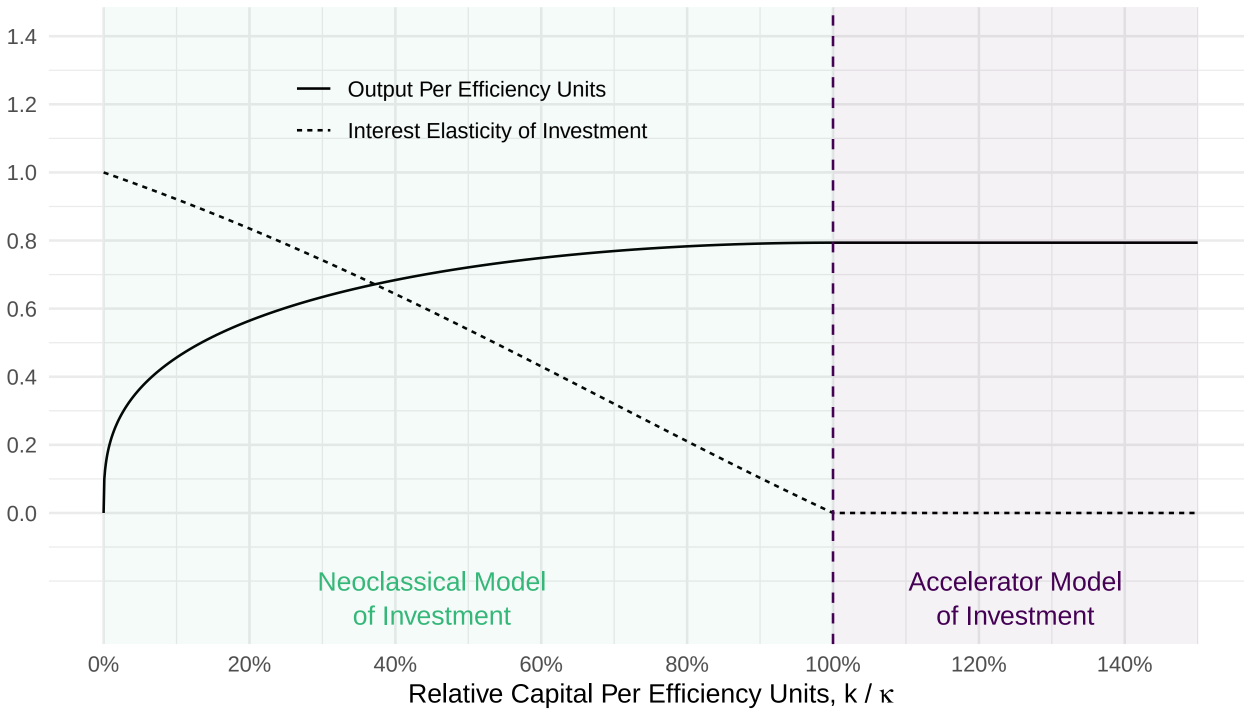

Figure 5: Interest elasticity of investment with a Superelliptic Production Function: Neoclassical Model, Accelerator Model

The consensus view in the literature since the 1970s has been that the neoclassical investment equations fit the data very poorly. This view was summarized in Dornbusch and Fischer's 3rd edition of *Macroeconomics*: "At least on evidence through 1979, it seems that the cost of capital empirically does not much affect investment and that accordingly the simple accelerator model does as well as the neoclassical model at explaining investment" (Dornbusch and Fischer (1979)). Similarly, Shapiro, Blanchard, and Lovell (1986) were summarizing the consensus view in the 1980s: "One of the best established facts in macroeconomics is that business fixed investment and output move strongly together over the business cycle. By contrast, investment and the cost of capital are either uncorrelated or only weakly correlated. These relationships might appear to suggest that business fixed investment can be best explained by an accelerator model of investment, whereby investment responds to changes in the desired capital stock, itself determined by the demand for output. The theory behind the accelerator model is akin to the man-on-the-street view that firms have little incentive to invest when current prospects for selling the output produced by the new capital are relatively poor." An example is that of Olivier Blanchard's discussion of Shapiro, Blanchard, and Lovell (1986): "it is well known that to get the user cost to appear at all in the investment equation, one has to display more than the usual amount of econometric ingenuity, resorting most of the time to choosing a specification that simply forces the effect to be there."

In the 1990s, researchers were struggling to find an effect of the cost of capital on investment. This was considered as a puzzle, which is for example reflected by Caballero's discussion of Cummins et al. (1994): "My expositional complaint is that while the authors succeed in generating an elasticity that I suspect we all like, they are less clear about exactly where it came from. Spread throughout the paper, rather than succinctly listed, are the standard culprits and the proposed remedies. But there is no precise statement as to which of the ingredients in their complex medicine *cured the patient* and as to whether the cure has left the patient with a *life worth living*." The 1990s disappointing results on neoclassical investment equations are well summarized by Oliner, Rudebusch, and Sichel (1995): "To summarize the results, we find that the Euler equations produce extremely poor forecasts of investment for both equipment and nonresidential structures. The time-to-build version of the Euler equation outperforms the basic Euler equation in our tests, but the improvement is modest. All the Euler equations have mean squared forecast errors many times larger than those of the traditional models. Moreover, the Fair-Shiller tests suggest that, as a group, the traditional models for equipment dominate the Euler equations; for nonresidential structures, the Fair- Shiller tests show that neither the Euler equations nor the traditional models have any forecasting ability."

In his chapter on investment in the *Handbook of Macroeconomics*, Ricardo Caballero (1999) summarizes the consensus view after twenty years of more research: "not only have models emphasizing the net return to investing been defeated in forecasting *horse races* by ad hoc models, but, more important, structural variables are frequently found to be economically or statistically insignificant." He added: "The movement from aggregate to microeconomic data, by itself, has not done much to improve affairs. Although microeconomic data has improved precision, coefficients on the cost of capital and q in investment equations have remained embarrassingly small." More recently, Yagan (2015) using the 2003 dividend tax cuts, and a difference in difference approach between C-corporations and S-corporations, has measured that the elasticity of investment to this dividend tax cut was a precisely estimated zero.[4]

From this empirical evidence, I conclude that it is not impossible that capital intensity has indeed reached its maximum point κ, given that the interest elasticity of investment is so low, and that the behavior of investment is best described by an *accelerator model of investment*:

$$-\frac{d\log\left(\frac{K}{L}\right)}{dr} = 0 \quad \Rightarrow \quad k = \kappa.$$

In Geerolf (2013c), I discuss the connection between the share of capital in value added, and the gross marginal product of capital. I show that the data is not inconsistent with a very low gross marginal product of capital, either.

Finally, I want to reassert that this question is not at all a new one. It was posed in the same terms exactly by Hansen (1939) just before World War II, and it was the core at the secular stagnation argument. Hansen (1939) then wrote: "Less agreement can be claimed for the role played by the rate of interest on the volume of investment. Yet few there are who believe that in a period of investment stagnation an abundance of loanable funds at low rates of interest is alone adequate to produce a vigorous flow of real investment. I am increasingly impressed with the analysis made by Wicksell who stressed the prospective rate of profit on new investment as the active, dominant, and controlling factor, and who viewed the rate of interest as a passive factor, lagging behind the profit rate. This view is moreover in accord with competent business judgment." He added: "It is true that it is necessary to look beyond the mere cost of interest charges to the indirect effect of the interest rate structure upon business expectations." Therefore, limited investment demand and the relative inelasticity of investment to the cost of capital are really at the core of the secular stagnation argument.

2 Representative Agent Economy

In this section, I introduce the Superelliptic production function into an otherwise standard growth model with capital accumulation and an exogenous saving rate. I first state the assumptions in section 2.1. I then show that this model implies two different regimes with regards to capital accumulation, investment, and labor utilization: the neoclassical and the secular stagnation regimes. I study the neoclassical regime in section 2.2, and the secular stagnation regime in section 2.3.

2.1 Assumptions

2.1.1 Disposable Income

I consider an economy with a continuum of identical agents with measure 1. Throughout the paper, I abstract away from how income is divided between capital and labor for simplicity (although the distributional effects of this split surely matters to determine the average propensity to save, as I will show in section 3).[5] I assume

[4]Another interpretation he puts forward is that the cost of capital is in fact not sensible to dividend taxes.

[5]Moreover, the marginalist determination of the labor and capital share has a number of problems, as shown in Geerolf (2013c).

that each representative agent owns the same share of the representative firm, so that each agent, in measure 1, gets income before taxes Y, whose sum adds up to total GDP Y, which after taxes gives disposable income Y^D:

$$Y^D = Y - T(Y).$$

2.1.2 Taxes, Transfers

I consider a linear tax system as a linear approximation around the actual tax system. I denote by $T(0)$ the dollar amount which is taxed (if positive) or transferred (if negative), when income is equal to 0. I also assume that locally, each additional dollar of income is taxes at marginal rate τ:

$$T(Y) = T(0) + \tau Y$$

Disposable income Y^D is defined as income after taxes, so that:

$$\begin{aligned} Y^D &= Y - T(Y) \\ Y^D &= (1-\tau)Y - T(0) \end{aligned}$$

I assume fixed government spending G. Importantly in applications, I assume that G does not crowd out private consumption. By this I imply that it is not the government providing free services where people were previously buying them.

2.1.3 Government Saving S_g

Public saving is the excess of net taxes over government spending:

$$\begin{aligned} S_g &= T(Y) - G \\ S_g &= T(0) + \tau Y - G \end{aligned}$$

If government saving is equal to zero, then this implies that $T(Y) = G$, and therefore that aggregate taxes $T(Y)$ are used to pay entirely for government spending G.

2.1.4 Consumption C

As in Solow (1956), I assume that the marginal propensity to consume out of (permanent) disposable income is exogenously given by $1 - s$, so that people save a fraction s of their (permanent) disposable income, and consume $1 - s$ of this income:

$$C = (1-s)\left[Y - T(Y)\right].$$

Replacing taxes by its expression above gives private consumption C:

$$C = -(1-s)T(0) + (1-s)(1-\tau)Y.$$

2.1.5 Private Saving S_p

In turn, private saving S_p is given using that consumers save a fraction s of their disposable income:

$$S_p = s\left[Y - T(Y)\right].$$

Replacing taxes by its expression above gives private saving S_p:

$$S_p = -sT(0) + s(1-\tau)Y.$$

I remain agnostic about the source of these saving. Saving could also arise from life-cycle concerns in an overlapping-generations model a la Diamond (1965). However, the reason why I do not wish to write an overlapping-generations model with life-cycle saving is that empirically, as Saez and Zucman (2016) have shown, much of the wealth does not correspond to pension wealth, but rather mostly to the saving of the rich. Alternatively, consumers could have other reasons to save, such as bequests or prestige (Carroll (2000)).

2.1.6 Total Saving S

From private saving S_p and government saving S_g, I can compute total saving:

$$\begin{aligned} S &= S_p + S_g \\ &= [-sT(0) + s(1-\tau)Y] + [T(0) + \tau Y - G] \\ S &= (1-s)T(0) - G + [1-(1-\tau)(1-s)]\,Y \end{aligned}$$

2.1.7 Accounting Identities

The aggregate demand for goods Z is composed of private consumption C, investment demand I and finally government purchases G such that $Z \equiv C + I + G$. In equilibrium, aggregate demand equals production so that $Y = Z$. As a consequence:

$$Y = C + I + G.$$

This allows us to derive that total saving S, the sum of private saving $S_p = Y - T - C$ and government saving $S_g = T - G$ equals investment:

$$(Y - T - C) + (T - G) = I.$$

As a consequence, total saving, private S plus public S_g, equals investment:

$$S = S_p + S_g = I.$$

As Robinson (1974) explains, "When Lloyd George proposed a campaign to reduce unemployment (which was then at the figure of one million or more) by expenditure on public works, he was answered by the famous *Treasury View* that there is a certain amount of saving at any moment, available to finance investment, and if the government borrows a part, there will be so much the less for industry." The above accounting identity allows to make sense of this statement. For a given amount of private saving S_p, if the government saves less $\Delta S_g < 0$ then firms will be able to invest less, since $\Delta I = \Delta S_g < 0$.

2.1.8 Investment I, and Employment L

As in Solow (1956), I assume that labor supply is inelastic, and that the maximum number of hours is given by:

$$L \leq \bar{L}$$

I also denote by $\bar{Y}$ the maximum attainable level of output when labor is fully employed, and capital intensity is maximum and equal to κ:

$$\bar{Y} = \frac{\kappa}{\beta} A\bar{L} = \left(\frac{\kappa}{2}\right)^{\alpha} A\bar{L} \tag{4}$$

Investment demand, and employment are not necessarily given as in the Solow (1956) growth model. The reason is that there exists some satiation of capital, so that the market for loanable funds might not necessarily be able to clear through the interest rate, but rather through output, and hence, employment L. I shall now study two different cases, depending on the quantity of capital which has already been accumulated, and whether labor is fully employed or not. I will show that there are two cases.

1. **The neoclassical regime ($L = \bar{L}$ and $k \leq \kappa$).** Investment is determined by saving, given the level of technology and capital, assuming that there is full employment of resources ($L = \bar{L}$). This neoclassical regime holds as long as the implied capital intensity stays lower or equal than κ. ($k \leq \kappa$) I will show that in line with previous work on the optimal accumulation of capital (Malinvaud (1953), Phelps (1961), Diamond (1965)), there exists a Golden rule level of capital accumulation, above which the capital stock is "too high". If the capital stock is higher than the Golden rule level of capital accumulation, then there is dynamic inefficiency. If it is lower, then there is dynamic efficiency. These two regimes can be visualized on Figure 6, as a function of the level of capital accumulation.

2. **The secular stagnation regime ($L < \bar{L}$ and $k = \kappa$).** the economy is satiated with enough capital ($k = \kappa$), so that the marginal product of capital is equal to zero - as there is no further possibility of substitution between capital and labor. Investment is then also equal to saving, however instead of investment demand adjusted to the level of saving, we get the exact opposite result: the level of employment, and therefore output adjusts so that saving equals the maximum admissible investment demand. As a result, there is "unemployment" of resources.

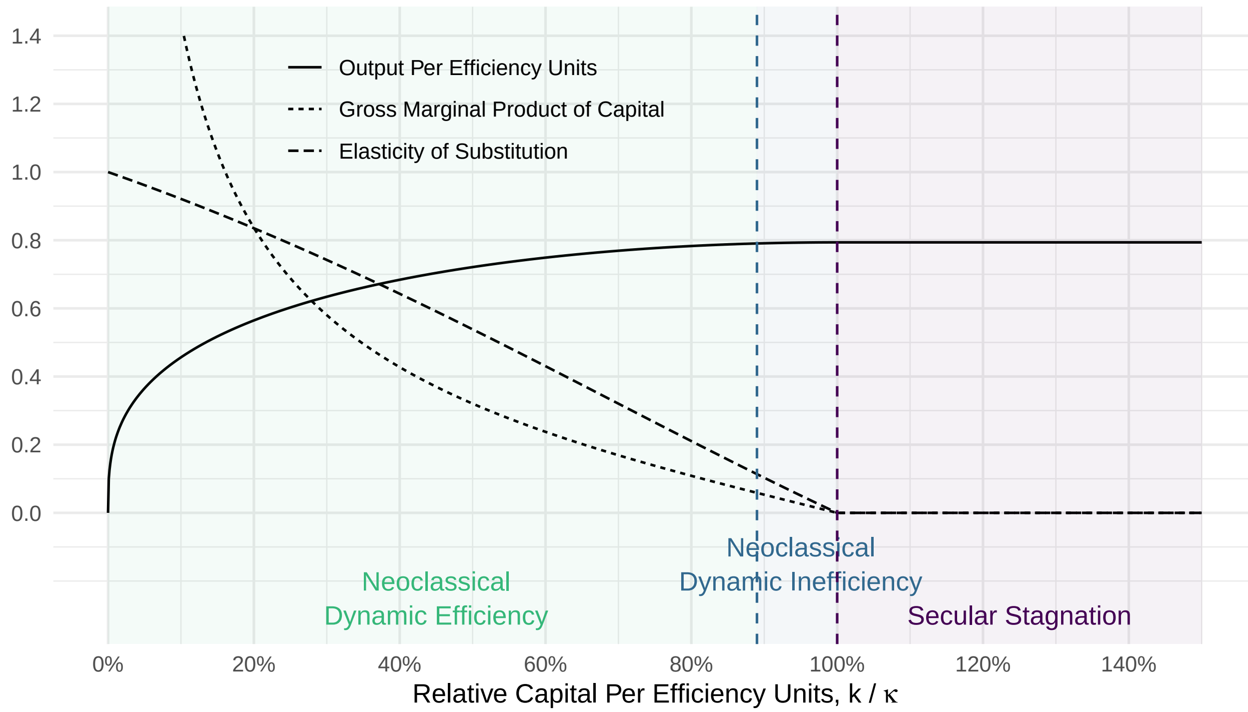

Figure 6: Dynamic Efficiency, Dynamic Inefficiency, Secular Stagnation

Figure 6 illustrates that the secular stagnation regime is a more advanced form of dynamic inefficiency, which is known to occur in the neoclassical regime (and which I have shown in previous work is a feature of advanced economies in Geerolf (2013a), Geerolf (2013c) and Geerolf (2013b)), for example in Malinvaud (1953), Phelps (1961) and Diamond (1965). In these instances however, there is "too much" capital, and the labor market clears at the level of maximum employment. We shall see that the secular stagnation regime has both too much capital given the quantity of labor that is employed, and too little capital compared to how much capital there would be if labor was fully employed. We will also see that it does not have the counterfactual implication that investment should decrease during booms because of crowding out.

I now study these two regimes in turn. I first examine the well-known neoclassical regime in section 2.2. I then move on to the secular stagnation regime, with capital satiation, in section 2.3.

2.2 Neoclassical regime

In the neoclassical regime, labor is fully employed so that $L = \bar{L}$, and the stock of capital implied by the full utilization of labor is such that capital intensity is lower than κ, so that $k \leq \kappa$. I guess and verify that this is the case, and connect the two regimes after having described the two different equilibria.

2.2.1 Steady-state capital, output and consumption

As I already stated, I focus attention on steady-states, as they allow to get at the main results of the paper. However, the results are robust to studying transitional dynamics. As always, there are two ways to compute steady-state output. One is to start from the aggregate demand for goods, another is to start from the total savings equals investment identity.

Capital. Following the tradition in studying the Solow (1956) growth model, I start by equatinng total saving and investment:

$$S_p + S_g = I$$

I assume that steady-state capital intensity consistent with given saving propensities and full employment is such that $k \leq \kappa$, so that this capital intensity corresponds to a level of capital which is actually demanded by firms (no firm would ever pay for a level of capital which brings capital intensity to a level higher than κ). As a consequence, investment in the steady-state must simply equal whatever needs to make up for capital depreciation:

$$I = \delta K.$$

Total saving is:

$$S = (1-s)T(0) - G + [1-(1-\tau)(1-s)]\, Y.$$

Using the form of the production function with full employment $L = \bar{L}$, I get the following implicit equation for the steady-state level of capital:

$$\delta K = [1-(1-\tau)(1-s)] \left(A\bar{L}\right)^{1-\alpha} \left(K - \frac{1}{2\kappa}\frac{K^2}{A\bar{L}}\right)^{\alpha} + (1-s)T(0) - G.$$

Therefore, using that $k = K/A\bar{L}$, the equation for capital intensity k is:

$$\delta k = [1-(1-\tau)(1-s)] \left(k - \frac{k^2}{2\kappa}\right)^{\alpha} + \frac{(1-s)T(0) - G}{A\bar{L}}.$$

Dividing both sides by κ and using the maximum capital over output ratio $\beta = 2^{\alpha}\kappa^{1-\alpha}$ defined in equation (2), as well the maximum level of output defined in equation (4) $\bar{Y} = (\kappa/\beta) \cdot A\bar{L}$ implies:

$$\delta\beta\frac{k}{\kappa} = [1-(1-\tau)(1-s)] \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} + \frac{(1-s)T(0) - G}{\bar{Y}}.$$

Finally, I define $\Lambda_\alpha(a,b,c)$ the solution to:

$$ax = b\left(2x - x^2\right)^{\alpha} + c \quad \Rightarrow \quad x = \Lambda_\alpha(a,b,c).$$

Graphically, $\Lambda_\alpha(-,-,+)$ has the following properties:

$$\frac{\partial \Lambda_\alpha}{\partial a} < 0, \qquad \frac{\partial \Lambda_\alpha}{\partial b} > 0, \qquad \frac{\partial \Lambda_\alpha}{\partial c} > 1.$$

Since $\left(2x - x^2\right)^{\alpha}$ is a concave function of x, this equation defines a unique $x \neq 0$. This implies that:

$$\frac{k}{\kappa} = \Lambda_\alpha\left[\delta\beta, 1-(1-\tau)(1-s), \frac{(1-s)T(0) - G}{\bar{Y}}\right].$$

Output. The level of steady-state output Y is then given by the production function at full employment given by:

$$Y = A\bar{L} \cdot \frac{\kappa}{\beta}\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha}$$

Denoting again by $\bar{Y}$ the full-employment, maximum capital accumulation value of output (as in equation (4)), I get:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha}$$
$$\text{with} \quad \frac{k}{\kappa} = \Lambda_{\alpha}\left[\delta\beta, 1-(1-\tau)(1-s), \frac{(1-s)T(0)-G}{\bar{Y}}\right].$$

Consumption. Finally, steady-state consumption is given by:

$$C = -(1-s)T(0) + (1-s)(1-\tau)Y$$

where Y is given solely as a function of parameters above.

2.2.2 Treasury View, Crowding out

The so-called "Treasury View" is that since there is a certain amount of private saving at any moment, available to finance investment, then if the government borrows a part, there will be less money available for private investment:

$$S = S_p + S_g = I.$$

This view was expressed by Winston Churchill in his discussion of the 1929 Budget: "The orthodox Treasury view... is that when the Government borrow(s) in the money market it becomes a new competitor with industry and engrosses to itself resources which would otherwise have been employed by private enterprise, and in the process raises the rent of money to all who have need of it."

The Teasury View in the Neoclassical Regime. In the neoclassical regime described above, the Treasury view is correct. Indeed, the implicit equation for capital intensity k:

$$\delta\beta\frac{k}{\kappa} = [1-(1-\tau)(1-s)]\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} + \frac{(1-s)T(0)-G}{\bar{Y}},$$

shows that when $T(0)$ is reduced, or that government purchases increase $\Delta G > 0$, then there is a fall in the steady-state capital stock:

$$\Delta G > 0 \quad \Rightarrow \quad \Delta K < 0.$$

This effect is shown on Figure 7. Moreover, this implies that steady-state output is lower. In the neoclassical regime, the Treasury view is correct: more government debt does crowd out private capital accumulation. It should be added that this assumes strong Ricardian non-equivalence: I assume here thatconsumers do not fully offset the increase in debt by saving more (the saving rate was taken here as exogenous, and in particular was independant of the level of government debt), as in Barro (1974).

The fact that more saving, public or private in fact, helps capital accumulation is no doubt at the heart of the success of early capitalists society. According to Keynes (1919), the "virtue of saving" explains how capitalists society came to accumulate a large amount of wealth: "And on the other hand the capitalist classes were allowed to call the best part of the cake theirs and were theoretically free to consume it, on the tacit underlying condition that they consumed very little of it in practice. The duty of *saving* became nine-tenths of virtue and the growth of the cake the object of true religion. There grew round the non-consumption of the cake all those instincts of puritanism which in other ages has withdrawn itself from the world and has neglected the arts of production as well as those of enjoyment. And so the cake increased; but to what end was not clearly contemplated. Individuals would be exhorted not so much to abstain as to defer, and to cultivate the pleasures of security and anticipation. Saving was for old age or for your children; but this was only in theory,—the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you." This argument that government spending crowds out private spending lies behind theories of "expansionary austerity" (Giavazzi and Pagano (1990)), and it is true in a region of the parameter space.

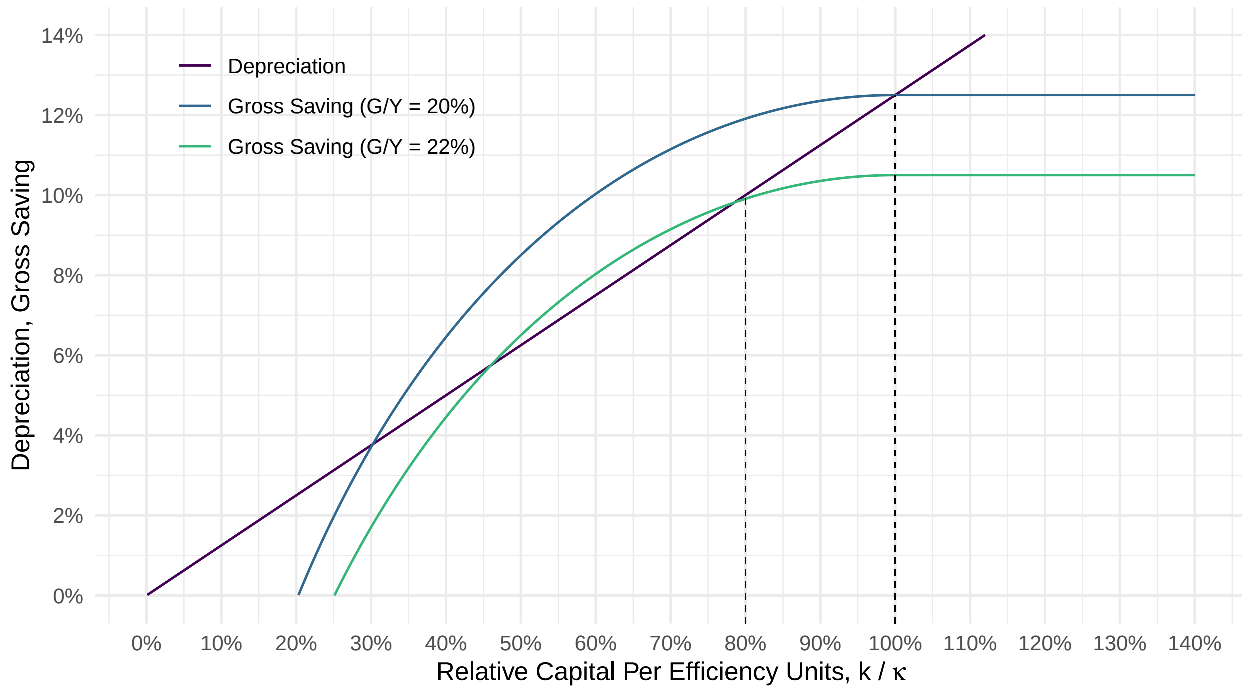

Figure 7: Crowding out Effect of Government Spending ($\alpha = 1/3$, $s = 1/4$, $\tau = 1/4$, $\delta = 1/16$, $\kappa = 2$, $T(0)/\bar{Y} = -15\%$)

In the neoclassical regime, more saving always leads to more investment, and hence more output. However, if capital accumulation is too high, then consumption might in fact be too low, as too many resources are being diverted to replace the depreciating capital stock every period. This result is very well-known in the growth literature. However, since secular stagnation can be viewed as a more advanced form of dynamic inefficiency, I now shall briefly discuss it.

2.2.3 Golden Rule, Dynamic Inefficiency

To express the idea of a Golden rule level of capital accumulation, I focus on the case where there is no government spending and no taxes, that is $G = 0$ and $T(Y) = 0$ (that is both $\tau = 0$ and $T(0) = 0$). Then, consumption is simply given by:

$$C = (1 - s)Y = Y - \delta K.$$

Thus, maximizing consumption implies setting the gross marginal return to capital equal to δ:

$$MPK = \frac{\partial Y}{\partial K} = \delta$$

Therefore, this equation in turn implies a certain level of capital intensity, which solves:

$$\alpha k^{\alpha-1} \left(1 - \frac{k}{2\kappa}\right)^{\alpha-1} \left(1 - \frac{k}{\kappa}\right) = \delta$$

The intuition is very simple: when the gross marginal product of capital is lower than δ, the net marginal product of capital is negative, which implies that it costs more to maintain the marginal unit of capital at the steady-state level than what is obtained from it. Although doing so does increase gross output, it does not increase net output (after depreciation) which matters for welfare and therefore this investment should not be undertaken from a welfarist point of view. For the Superelliptic production function, the marginal product of capital was derived in section 1.2.

For example, if $\delta = 0.06$, and if $\alpha = 1/3$ and $\kappa = 1$, then the Golden rule level of capital solves:

$$\frac{1}{3}k^{-2/3}\left(1-\frac{k}{2}\right)^{-2/3}(1-k) = 0.06 \quad \Rightarrow \quad k \approx 0.89$$

In the neoclassical regime with dynamic inefficiency, the marginal product is “too low”. However, the Treasury view still holds: lower government saving does lead to higher capital accumulation. Thus, the situation is no different in terms of positive economics. In term of normative economics though, this situation does call for Keynesian policies, in the sense of stimulating consumption, either through tax reductions or using other schemes to lower the aggregate saving rate.

2.2.4 Domain of the neoclassical regime

There are two limits worth defining. One is the locus of parameters separating the neoclassical from the secular stagnation regime. A second limit worth defining is that of the locus of parameters separating dynamically efficient from dynamically inefficient economies.

Neoclassical / Secular Stagnation. I first characterize the limit between the neoclassical and the secular stagnation regime. The secular stagnation regime arises when the private saving rate s is so large that even when capital intensity is equal to κ, gross saving is higher than gross investment (corresponding to depreciation) This implies:

$$[1-(1-\tau)(1-s)] + \frac{(1-s)T(0)-G}{\bar{Y}} \geq \delta\beta$$

As an illustrative example, I set $T(0)/\bar{Y} = \tau = 0$, and $G/\bar{Y} = 0$ for the government parameters, then the locus of limit points is given by the equation (using the definition of $\beta = 2^\alpha \kappa^{1-\alpha}$):

$$s \geq \delta\beta = 2^\alpha \delta \kappa^{1-\alpha}.$$

Figure 8 draws this limit assuming that $\alpha = 1/3$ and $\delta = 6\%$. This equation then is $s = \sqrt[3]{2}\cdot 0.06 \cdot \kappa^{2/3} \approx 7.5\% \cdot \kappa^{2/3} + 0.01$.

Dynamic Efficiency / Inefficiency. I next characterize the limit between the dynamic efficiency and dynamic inefficiency regimes. Dynamic inefficiency arises whenever the gross marginal product of capital is lower than the rate of depreciation. This corresponds to:

$$\alpha k^{\alpha-1}\left(1-\frac{k}{2\kappa}\right)^{\alpha-1}\left(1-\frac{k}{\kappa}\right) \leq \delta.$$

Let $\Gamma_\alpha(.)$ be defined as:

$$\Gamma_\alpha(x) \equiv \alpha x^{\alpha-1}\left(1-\frac{x}{2}\right)^{\alpha-1}(1-x). \tag{5}$$

The previous equation can be written as:

$$k \geq \kappa \cdot \Gamma_\alpha^{-1}\left(\delta\kappa^{1-\alpha}\right).$$

Again, I set $G = T(0) = \tau = 0$ to obtain:

$$\delta k \leq s\left(k - \frac{k^2}{2\kappa}\right)^\alpha \quad \Rightarrow \quad s \geq \delta k^{1-\alpha}\left(1-\frac{k}{2\kappa}\right)^{-\alpha}.$$

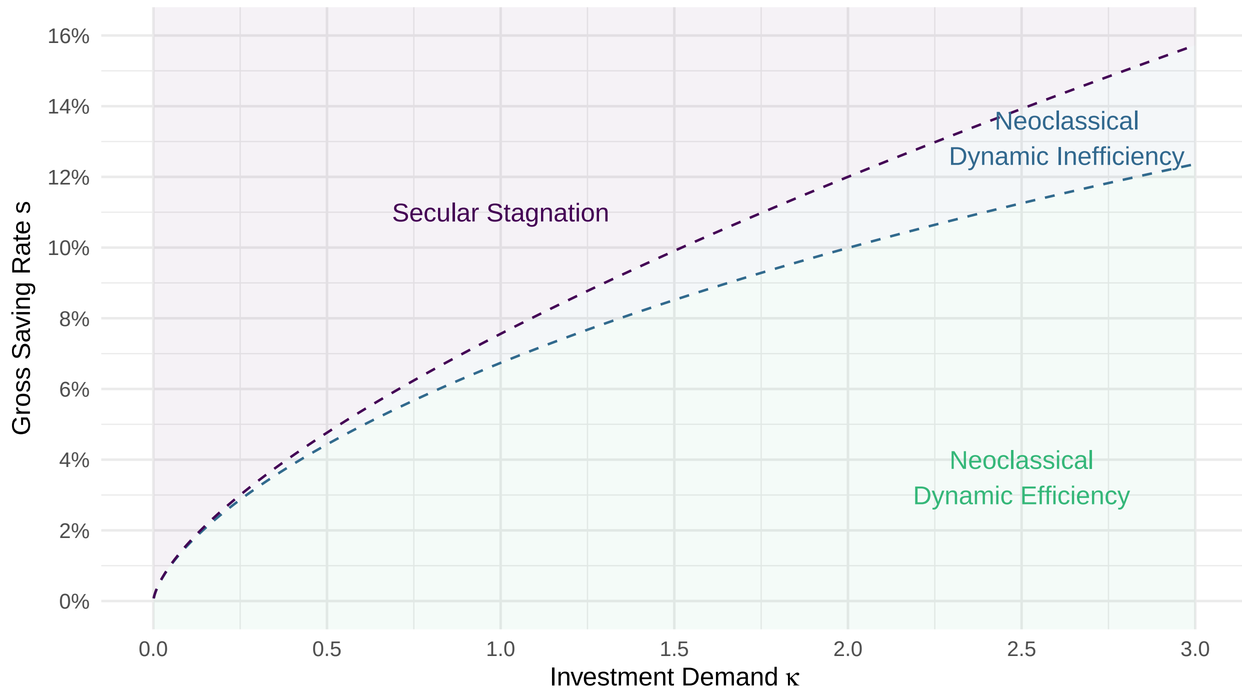

Figure 8: Regimes as a Function of the Saving Propensity s, and investment demand κ, with $\alpha = 1/3$ and $\delta = 6\%$

These two equations define a locus of parameters characterizing the frontier between dynamic efficiency and dynamic inefficiency. Indeed, the first equation defines k as a function of parameters α, κ, δ only, through $k = \kappa \cdot \Gamma_\alpha^{-1}\left(\delta\kappa^{1-\alpha}\right)$:

$$s \geq \delta\kappa^{1-\alpha}\left[\Gamma_\alpha^{-1}\left(\delta\kappa^{1-\alpha}\right)\right]^{1-\alpha}\left[1 - \frac{1}{2}\Gamma_\alpha^{-1}\left(\delta\kappa^{1-\alpha}\right)\right]^{-\alpha}$$

where $\Gamma(.)$ is defined in equation (5). Figure 8 draws this limit assuming that $\alpha = 1/3$ and $\delta = 6\%$. I will now study the secular stagnation regime.

2.3 Secular Stagnation regime

The secular stagnation regime arises when the steady-state capital stock implied by full employment $L = \bar{L}$ and the given propensities to save, and given government policies G, τ, $T(0)$ lead to an implied level of capital intensity that is higher than κ. This is not an equilibrium, because firms would never invest capital whose marginal product is equal to 0. Therefore, instead of "unemployment of capital" as in Harrod (1939), this situation rather leads to the "unemployment of labor" with $L < \bar{L}$. Intuitively, this unemployment lowers output, and therefore private saving (and public, if $\tau > 0$), so that saving equal investment. In the secular stagnation regime, we therefore know that capital intensity is maximum, and labor is underemployed:

$$k = \kappa \quad \text{and} \quad L < \bar{L}.$$

As for the neoclassical regime, I first guess that $k = \kappa$ and then check the conditions on parameters so that indeed $L < \bar{L}$. I show that this gives similar results as those obtained previously, which delimit the neoclassical from the secular stagnation regime.

2.3.1 Steady-state output and employment

Again, I focus attention on steady-states.

Output. I guess that in equilibrium $k = \kappa$, which implies that from capital intensity we can get to a value for the stock of capital in the steady state, as a function of output, using that $\kappa = \frac{K}{AL}$ as well as equation (1):

$$Y = \left(\frac{\kappa}{2}\right)^{\alpha} AL \quad \Rightarrow \quad AL = 2^{\alpha}\kappa^{-\alpha} Y \quad \Rightarrow \quad K = \kappa AL = 2^{\alpha}\kappa^{1-\alpha} Y = \beta Y$$

In the steady-state, the investment needed in order to replace depreciated capital stock is simply δK, and therefore:

$$I = \delta K = \delta\beta Y.$$

There are two ways to compute steady-state output. One is to start from the aggregate demand for goods, another is to start from the total savings equals investment identity. I starting by writing that $S = I$, for symmetry with the neoclassical regime. To derive the Keynesian cross with the aggregate demand for goods, I will be using the output equals demand identity. Equating total saving and investment thus gives:

$$\delta\beta Y = (1-s)T(0) - G + [1 - (1-\tau)(1-s)]\, Y.$$

Therefore, this allows to straightforwardly compute output when $1 - (1-\tau)(1-s) - \delta\beta \neq 0$:

$$Y = \frac{G - (1-s)T(0)}{1 - (1-\tau)(1-s) - \delta\beta}.$$

As a function of maximum output $\bar{Y}$, this is:

$$\frac{Y}{\bar{Y}} = \frac{1}{1 - (1-\tau)(1-s) - \delta\beta} \frac{G - (1-s)T(0)}{\bar{Y}}$$

Employment. When the production function is satiated with capital but there is less than full employment:

$$Y = \frac{\kappa}{\beta} AL.$$

Therefore employment as a fraction of full employment $L/\bar{L}$ is equal to output as a fraction of maximum output defined as (4) $\bar{Y}$, $Y/\bar{Y}$:

$$\frac{L}{\bar{L}} = \frac{1}{1 - (1-\tau)(1-s) - \delta\beta} \frac{G - (1-s)T(0)}{\bar{Y}}$$

2.3.2 Multiplier, Keynesian Cross

The secular stagnation regime of the growth model with a superelliptic function allows to very naturally compute government and tax multipliers, as well as to make sense of the so-called "Keynesian Cross."

Indeed, another way to calculate output is to compute total Aggregate Demand for goods Z as a function of income Y:

$$\begin{aligned} Z &= C + I + G \\ &= (1-s)\,(Y - T(Y)) + \delta\beta Y + G \\ Z &= [G - (1-s)T(0)] + [(1-\tau)(1-s) + \delta\beta]\, Y. \end{aligned}$$

Equilibrium is then obtained as a solution to $Z = Y$, that is equating the aggregate demand for goods to income. The curve representing the demand for goods Z as a function of income Y is represented on Figure 9, for a case where $(1-\tau)(1-s) + \delta\beta > 0$, and the $Z = Y$ curve corresponds to the 45 degree line. Figure 9 corresponds to the Keynesian cross, which obtains here as an outcome in the steady-state of standard model where the production function is saturated with enough capital.

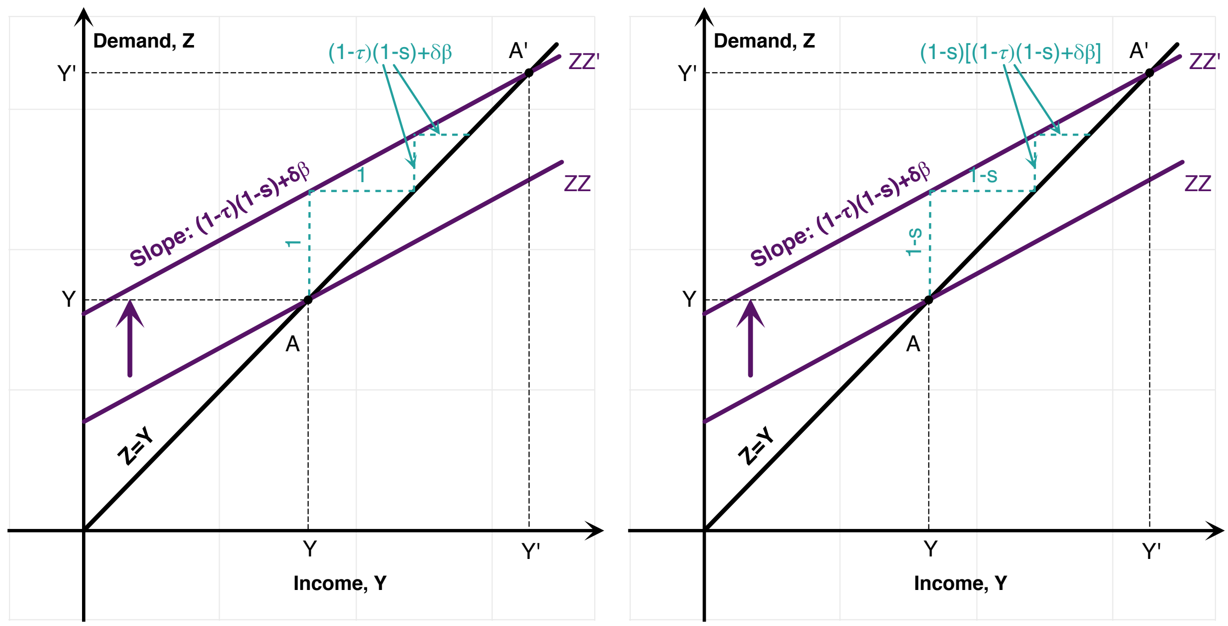

Figure 9: Keynesian Cross: Expenditure-Based $\Delta G = 1$ (Left), Tax-Based $\Delta T = -1$ (Right).

The economic interpretation is that with this assumption, Say (1803)'s law is not verified: supply does not create its own demand. This is because out of one additional dollar of income, only $(1-s)(1-\tau)$ is being consumed, and because of the inelasticity of investment, only $\delta\beta$ can be invested. Therefore, what is demanded in the economy is not equal to the extra income which was generated.

Using that supply must equal the demand for goods in equilibrium $Z = Y$, and solving for Y I get:

$$Y = \frac{G - (1-s)T(0)}{1 - (1-\tau)(1-s) - \delta\beta}.$$

The infinite-horizon government Spending multiplier (assuming no government debt problem):

$$\Delta Y = \frac{1}{1 - (1-\tau)(1-s) - \delta\beta}\Delta G.$$

Therefore, we can conclude that the multiplier is $1/(1-(1-s)(1-\tau)-\delta\beta)$ if $(1-s)(1-\tau)+\delta\beta < 1$. Another way to see the multiplier is to compute successive rounds of spending and add them up. This gives the multiplier:

$$1 + [(1-\tau)(1-s)+\delta\beta] + [(1-\tau)(1-s)+\delta\beta]^2 + ... = \frac{1}{1-(1-\tau)(1-s)-\delta\beta}$$

The tax multiplier is:

$$\Delta Y = -\frac{(1-s)\Delta T(0)}{1-(1-\tau)(1-s)-\delta\beta}$$

Note that here the consequences of higher taxes do not at all come from incentives effects, as the public finance literature usually assumes. This can help explain why micro and macro elasticities are so different: aggregate changes in taxes work through their effect on aggregate saving, and hence on output (Chetty et al. (2011)). There is therefore no reason to expect micro and macro elasticities to be similar, as in the neoclassical regime.

Example 1: Crowding in or crowding out of government expenditures? Depending on the regime, government expenditures can have long-term crowding in or crowding out effects on investment, and therefore output. For example, assuming that $\alpha = 1/3$, $s = 1/4$, $\tau = 1/4$, $\delta = 1/16$, $\kappa = 2$, and that $T(0)/\bar{Y} = -15\%$ in the secular stagnation regime (see appendix B.1.1 for details):

$$\frac{Y}{\bar{Y}} = 36\% + 3.2 \cdot \frac{G}{\bar{Y}}$$

In the neoclassical regime:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha}$$

$$\text{where } \frac{k}{\kappa} \text{ solves:} \quad \frac{7}{16}\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} - \frac{1}{8}\frac{k}{\kappa} - \frac{9}{80} - \frac{G}{\bar{Y}} = 0$$

The crowding in and crowding out effects of government spending are shown on Figure 10. When government spending is low so that the secular stagnation regime prevails, government spending crowds in investment, increases employment and output. When government spending is high so that the regime is neoclassical, government spending crowds out investment, and therefore steady-state output decreases.

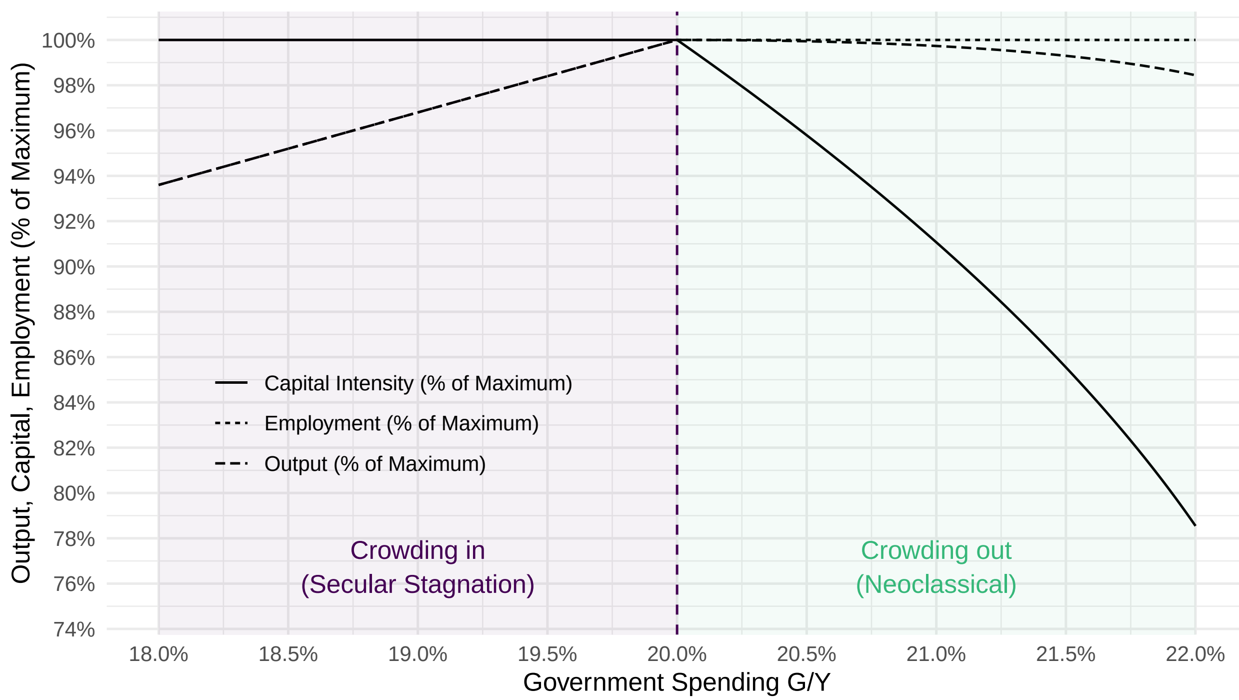

Figure 10: Crowding in, Crowding out. Assumptions: $\alpha = 1/3$, $s = 1/4$, $\tau = 1/4$, $\delta = 1/16$, $\kappa = 2$, $T(0)/\bar{Y} = -15\%$

Example 2: Expansionary or contractionary austerity? Depending on whether saving is plentiful or scarce, "austerity" or increases in taxes can be expansionary or contractionary. For example, assuming that $\alpha = 1/3$, $s = 1/4$, $\tau = 1/4$, $\delta = 1/16$, $\kappa = 2$, and that $G = 20\%$ in the secular stagnation regime (see appendix B.1.1 for details):

$$\frac{Y}{\bar{Y}} = 64\% - 2.4 \cdot \frac{T(0)}{\bar{Y}}$$

In the neoclassical regime:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha}$$

$$\text{where } \frac{k}{\kappa} \text{ solves:} \quad \frac{7}{16}\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} - \frac{1}{8}\frac{k}{\kappa} - \frac{1}{5} + \frac{3}{4}\frac{T(0)}{\bar{Y}} = 0$$

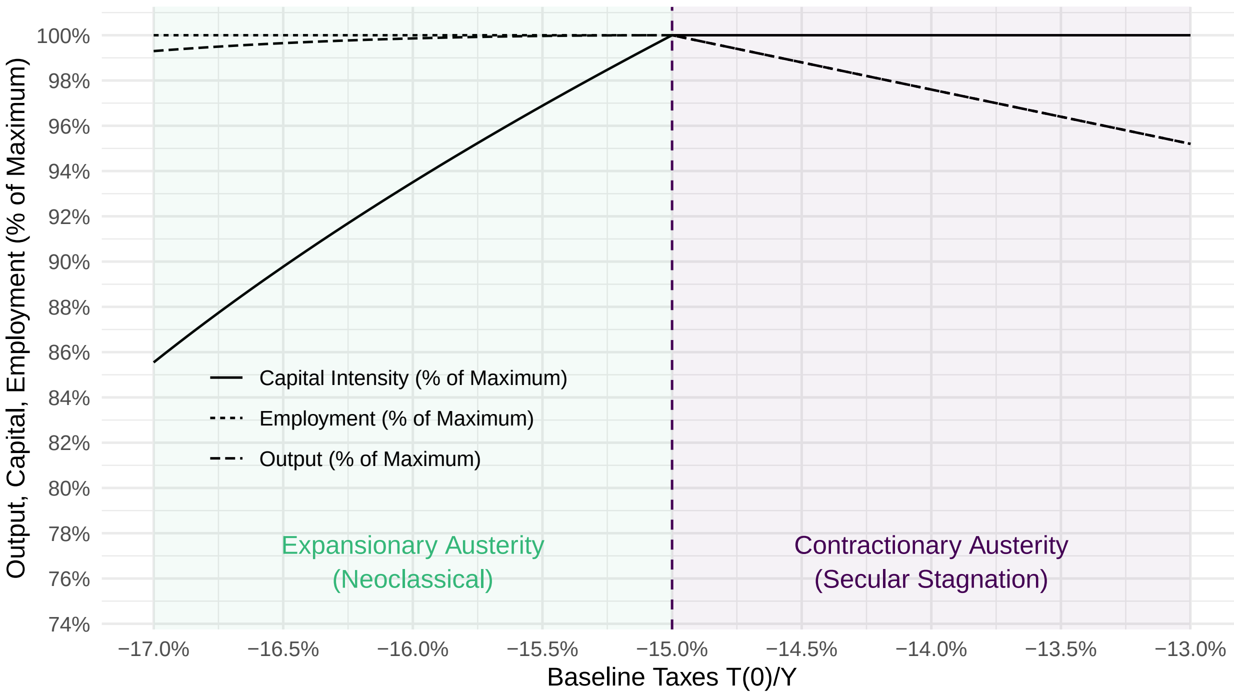

Figure 11: Expansionary or Contractionary Austerity? Assumptions: $\alpha = 1/3$, $s = 1/4$, $\tau = 1/4$, $\delta = 1/16$, $\kappa = 2$, $G/\bar{Y} = 20\%$

2.3.3 Paradox of thrift

Despite flexible prices, this model has a paradox of thrift: efforts to save more are self-defeating. When the saving rate increases in the economy, the total quantity of saving in fact falls, and investment falls as well. There are two ways to see this. The first is to compute the change in total saving, the other is to compute the change in investment.

Change in total saving. Total saving S is:

$$S = (1-s)T(0) - G + [1-(1-\tau)(1-s)]\,Y$$

In turn, outpu Y in the secular stagnation regime is given by:

$$Y = \frac{1}{1-(1-\tau)(1-s)-\delta\beta}\,[G-(1-s)T(0)]\,.$$

Therefore:

$$S = (1-s)T(0) - G + \frac{1-(1-\tau)(1-s)}{1-(1-\tau)(1-s)-\delta\beta}\,[G-(1-s)T(0)]$$

$$S = \frac{\delta\beta}{1-(1-\tau)(1-s)-\delta\beta}\,[G-(1-s)T(0)]$$

In changes:

$$
\begin{aligned}
\Delta S &= \frac{\delta\beta T(0)}{1-(1-\tau)(1-s)-\delta\beta}\Delta s - \frac{\delta\beta(1-\tau)\Delta s}{[1-(1-\tau)(1-s)-\delta\beta]^2}\left[G-(1-s)T(0)\right] \\
\Delta S &= -\delta\beta\frac{(1-\tau)Y-T(0)}{1-(1-\tau)(1-s)-\delta\beta}\Delta s
\end{aligned}
$$

As a result, an increase in the saving rate leads to a decline in total saving ΔS:

$$\Delta s > 0 \quad \Rightarrow \quad \Delta S < 0$$

Change in investment. Another way to see the paradox of thrift is to directly calculate the change in investment. The change in investment is a linear function of the change in output $\Delta I = \delta\beta\Delta Y$. The change in output due to a change in the saving rate Δs is:

$$
\begin{aligned}
\Delta Y &= -\frac{(1-\tau)}{1-(1-\tau)(1-s)-\delta\beta}Y\Delta s + \frac{T(0)\Delta s}{1-(1-\tau)(1-s)-\delta\beta} \\
\Delta Y &= -\frac{(1-\tau)Y-T(0)}{1-(1-\tau)(1-s)-\delta\beta}\Delta s
\end{aligned}
$$

The change in investment is equal to the change in total saving:

$$
\begin{aligned}
\Delta I &= \delta\beta\Delta Y \\
\Delta I &= -\frac{\delta\beta\left[(1-\tau)Y-T(0)\right]}{1-(1-\tau)(1-s)-\delta\beta}Y\Delta s
\end{aligned}
$$

Example (the paradox of thrift): Does a higher saving rate lead to more investment? Depending on the regime, an increase in the saving rate $\Delta s > 0$ can either increase total saving, or decrease it; therefore investment can increase or decrease. For example, assuming that $\alpha = 1/3$, $\tau = 1/4$, $\delta = 1/16$, $\kappa = 2$, $G/\bar{Y} = 20\%$ and that $T(0)/\bar{Y} = -15\%$ in the secular stagnation regime (see appendix B.1.3 for details):

$$\frac{Y}{\bar{Y}} = \frac{2}{5}\cdot\frac{7-3s}{1+6s}$$

Denoting by $\bar{I}$ the maximum level for investment, corresponding to the maximum level of output $\bar{Y}$, investment is given by:

$$\frac{I}{\bar{I}} = \frac{2}{5}\cdot\frac{7-3s}{1+6s}$$

In the secular stagnation regime, investment is therefore a monotonically decreasing function of the saving rate s.

In contast, in the neoclassical regime, we get the Solow result that more saving leads to more investment, and therefore more output in the steady-state:

$$
\begin{aligned}
&\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} \\
&\text{where } \frac{k}{\kappa} \text{ solves:} \quad \left(\frac{1}{4}+\frac{3}{4}s\right)\left(2\frac{k}{\kappa}-\frac{k^2}{\kappa^2}\right)^{\alpha} - \frac{1}{8}\frac{k}{\kappa} - \frac{1}{5} - \frac{3}{20}(1-s) = 0
\end{aligned}
$$

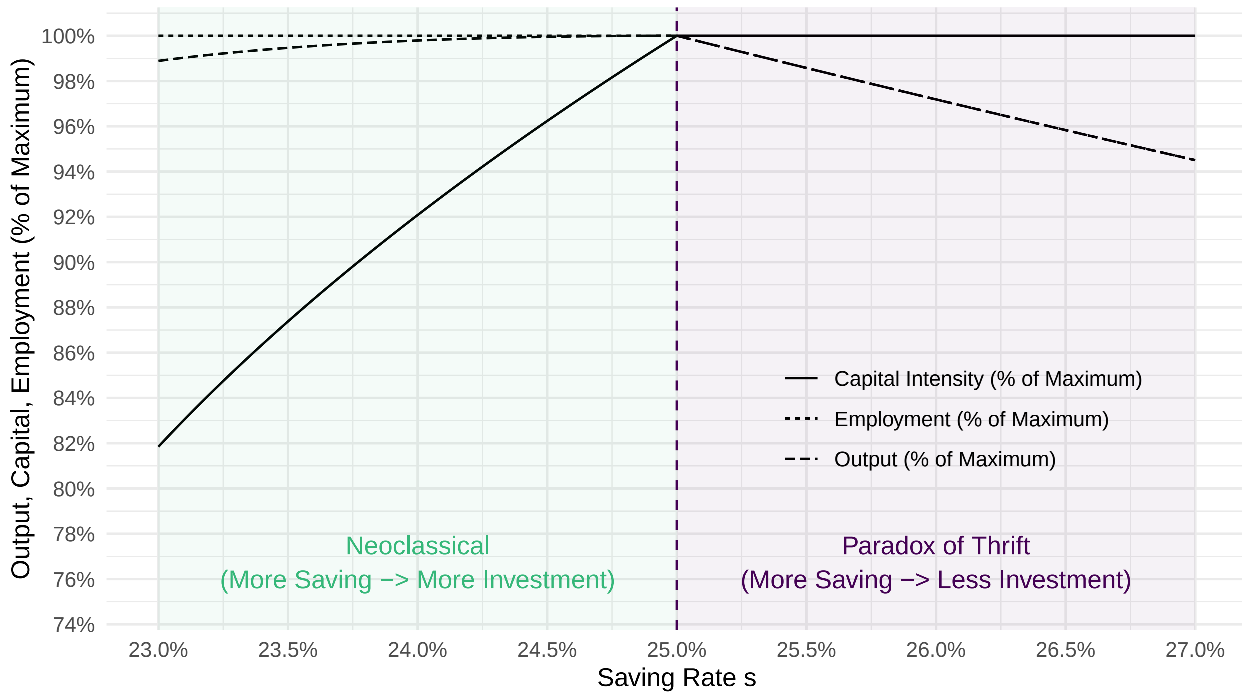

Figure 12: Paradox of thrift. Assumptions: $\alpha = 1/3$, $\tau = 1/4$, $\delta = 1/16$, $\kappa = 2$, $G/\bar{Y} = 20\%$ and $T(0)/\bar{Y} = -15\%$

2.3.4 Treasury View, Crowding in

I now reexamine the Treasury View in the secular stagnation regime. Using the expression for investment above, we see that investment increases when government spending is increased:

$$\Delta I = \delta\beta\Delta Y = \frac{\delta\beta}{1-(1-\tau)(1-s)-\delta\beta}\Delta G > 0.$$

According to the Treasury view, the corresponding extra borrowing from the public sector, should crowd out private investment: if the amont of saving is fixed, then when the government borrows, there should be less funds available for investment. I first examine why this intuition is incorrect in the secular stagnation regime, and I then relate this explanation of crowding in to another explanation which has been proposed for the absence of crowding out effects, namely Ricardian equivalence.

Economic Intuition. It is worth examining the economic intuition for why the Treasury view is incorrect in the secular stagnation regime of this model. There are in fact two reasons why the Treasury View is then incorrect:

- First, a stimulus $\Delta G > 0$ increases output, and therefore disposable income and private saving. Thus, it is incorrect to reason as if there was a certain amount of saving.
- Second, a stimulus $\Delta G > 0$ might not in fact increase the government deficit $\Delta(G-T)$ by an equal amount, as more economic activity generates more tax receipts $\Delta T > 0$ which offsets the negative impact on the budget. If the government purchase multiplier is sufficiently high, the stimulus might in fact even be self-financing.

The increase in output caused by an increase in government purchases ΔG is:

$$\Delta Y = \frac{1}{1-(1-\tau)(1-s)-\delta\beta}\Delta G.$$

Private saving is:

$$S_p = s\left[(1-\tau)Y - T(0)\right].$$

We therefore have:

$$\begin{aligned}
\Delta S_p &= s(1-\tau)\Delta Y \\
\Delta S_p &= \frac{s(1-\tau)}{1-(1-\tau)(1-s)-\delta\beta}\Delta G.
\end{aligned}$$

The impact on government saving is:

$$\begin{aligned}
\Delta S_g &= \tau\Delta Y - \Delta G \\
&= \frac{\tau}{1-(1-\tau)(1-s)-\delta\beta}\Delta G - \Delta G \\
\Delta S_g &= \frac{\delta\beta - s(1-\tau)}{1-(1-\tau)(1-s)-\delta\beta}\Delta G
\end{aligned}$$

Total saving therefore increases:

$$\Delta S = \Delta S_p + \Delta S_g = \frac{\delta\beta}{1-(1-\tau)(1-s)-\delta\beta}\Delta G > 0.$$

This implies that investment increases too, which invalidates the Treasury view: expenditure on public works induces more private investment, not less:

$$\Delta I = \Delta S_p + \Delta S_g > 0$$

An alternative to "Ricardian" Equivalence. The fact that budget deficits do not lead to investment crowding out has been observed before at numerous occasions. For example, Barro (1989) notes that despite large budget deficits in the Reagan years, the U.S. economy had seen large capital accumulation and high growth.[6] However, his explanation was different: according to him, the reason why crowding out did not take place was what is now called "Ricardian Equivalence"[7]: private saving increased in anticipation of higher taxes in the future to repay booming government debt. Secular stagnation therefore provides an alternative explanation for the same phenomenon, one that is consistent with the fact that tax cuts did boost consumption during this period (Poterba and Summers (1987)) and that not only did aggregate saving and investment not fall, but that they in fact rose, something that Ricardian equivalence does not explain.

2.3.5 Comovements

It is usually believed that only aggregate supply shocks can lead to a positive comovement of output, consumption, and investment. In the secular stagnation regime, negative aggregate demand shocks drive consumption, investment, output, and employment down simultaneously, whether they come from shocks to government spending or taxes. This is consistent with Romer and Romer (2010)'s findings after a 1% of GDP tax cut in the United States, as shown on Figures 18, 19, 20 and 21.

Similar results have been found by Cloyne (2013) in the United Kingdom, Hayo and Uhl (2014) in Germany, and Geerolf and Grjebine (2019) using property taxes to isolate aggregate demand effects. House, Proebsting,

[6] The exact quote in Barro (1989) is: "In recent years there has been a lot of discussion about U.S. budget deficits. Many economists and other observers have viewed these deficits as harmful to the U.S. and world economies. The supposed harmful effects include high real interest rates, low saving, low rates of economic growth, large current-account deficits in the United States and other countries with large budget deficits, and either a high or low dollar (depending apparently on the time period). This crisis scenario has been hard to maintain along with the robust performance of the U.S. economy since late 1982. This performance features high average growth rates of real GNP, declining unemployment, much lower inflation, a sharp decrease in nominal interest rates and some decline in expected real interest rates, high values of real investment expenditures, and (until October 1987) a dramatic boom in the stock market. Persistent budget deficits have increased economists' interest in theories and evidence about fiscal policy. At the same time, the conflict between standard predictions and actual outcomes in the U.S. economy has, I think, increased economists' willingness to consider approaches that depart from the standard paradigm."

[7] I use quotation marks, because David Ricardo, in fact explicitly refuted this idea which now bears his name.

and Tesar (2019) have found large tax multipliers using austerity plans enacted after the 2007-2009 financial crisis. This recent literature has been reviewed by Ramey (2019), who observes that "Narrative methods (which use historical documents to find exogenous changes) for tax rate changes typically yield multiplier estimates that are surprisingly large and surprisingly uniform across a number of countries. The bulk of the empirical estimates vary between –2 and –3." Although the magnitude of the multiplier is subject of much controversy, the comovement patterns are robust.

2.3.6 Permanent Effects of Aggregate Demand Shocks

A central assumption of both classical models and Keynesian models that macroeconomic policy is all about dealing with fluctuations in output and unemployment – fluctuations around a given mean. According to both types of theories, this mean is itself determined by aggregate supply forces, outside of the reach of the market – and the goal of monetary and fiscal policy should be to have less volatility, not change this given mean. According to well-accepted Keynesian theory, aggregate demand only affects GDP in the short run, but permanent influences to output and employment must be due to aggregate supply shocks (Nelson and Plosser (1982)). For instance, Blanchard and Quah (1989) have used the transitory nature of aggregate demand shocks as a restriction helping identify supply from demand in a Vector Auto Regression. This so-called neoclassical synthesis was already almost universally accepted in 1953, at least according to Paul Samuelson's in the 3rd edition of his *Economics* textbook. (Samuelson (1953)). "In recent years 90 percent of American economists have stopped being 'Keynesian economists' or 'anti-Keynesian economists'. Instead they have worked towards a synthesis of whatever is valuable in older economics and in modern theories of income determination. The result might be called neo-classical synthesis and is accepted in its broad outlines by all but about 5 per cent of extreme left wing and right wing writers."[8]

However, the evidence weighs against that hypothesis. Indeed, a large literature has investigated the permanent effects of aggregate demand shocks (Fatás and Summers (2018)), which are typically attributed to hysteresis effects (as in Blanchard and Summers (1986)).[9] An example is shown on Figure 14, comparing the experience of the U.S. and Europe after the financial crisis: Europe applied much more severe austerity policies than the United States, and the level of output still is depressed relative to that of the United States, even 10 years after. As Summers (2013) noted: "I wonder if a set of older and much more radical ideas that I have to say were pretty firmly rejected in 14.462[10], Stan, a set of older ideas that went under the phrase secular stagnation, are not profoundly important in understanding Japan's experience in the 1990s, and may not be without relevance to America's experience today." The superelliptic production function studied in this paper, and presented in section 1, can explain how aggregate demand shocks can have permanent effects. Since the effects of aggregate demand on output do not come from "frictions", such as sticky prices or sticky information, but from technology, these effects can be very long lasting.

As a consequence of aggregate demand shocks being long lasting in this model, remedies can be long lasting as well. This is not the case in mainsteam New-Keynesian models, where public deficits should be reduced during booms and increased during busts for stabilization purposes. As Reis (2018) writes: "Most macroeconomists support countercyclical fiscal policy, where public deficits rise in recessions, both in order to smooth tax rates over time and to provide some stimulus to aggregate demand. Looking at fiscal policy across the OECD countries over the last 30 years, it is hard to see too much of this advice being taken. Rather, policy is best described as deficits almost all the time, which does not match normative macroeconomics." In contrast, my results suggest that policymakers are in fact doing the right thing by continuously rolling over the debt, and

[8] He added: "Repeatedly in the book I have set forth what I call a 'grand neoclassical synthesis'. This is a synthesis of (1) the valid core of modern income determination with (2) the classical economic principles." Finally, in this book he defined the neoclassical synthesis as the following: "by means of appropriately reinforcing monetary and fiscal policies, our mixed-enterprise system can avoid the excesses of boom and slump and can look forward to healthy progressive growth."

[9] Fatás and Summers (2018) write: "But there is also a very different interpretation of the persistence of GDP, if we are willing to deviate from the tradition of separating long-term dynamics and business cycles. It is possible that cyclical conditions leave permanent scars on output, what is typically referred to as hysteresis. It was originally applied to models of the labor market as in Blanchard and Summers (1986) where cyclical unemployment turned into structural one. But the logic extends even more naturally when we start thinking of long-term growth as endogenous and we allow for the possibility that economic cycles interrupt temporarily these long-term dynamics."

[10] 14.462 refers to the course number for the most advanced graduate course in macroeconomics at the Massachusetts of Technology.

taking up deficits all the time, and that they probably should even do more of it, particularly in Germany and Japan, whose current account surpluses are a sign that they are a drag on global demand.

Finally, multipliers are not greater in times of extreme slack or at the zero lower bound, as argued by Ramey (2019): "There is no robust evidence of higher multipliers during recessions or times of slack, for either spending or taxes. In fact, all studies of state dependence for tax multipliers find higher multipliers during expansions." This finding does not appear consistent with the New Keynesian model, but it is consistent with the present model.

3 Heterogeneous Agents Model

I now develop a model with two types of workers and consumers. This is necessary in order to understand Zidar (2019)'s evidence on tax cuts being more effective to boost employment and growth on low income earners rather than on high income earners. This is also necessary in order to understand how monetary policy can have real effects, despite prices being completely flexible. I first introduce the assumptions in section 3.1. I then study the neoclassical regime in section 3.2, and the secular stagnation regime in section 3.3. I focus on the effects of redistributive policies on growth and capital accumulation. However, technology-driven increases in inequality have similar effects.

3.1 Assumptions

I again consider a continuum of identical agents with measure 1. However, instead of a representative worker, there are two types of workers: high income and low income. Before looking at the equilibrium of this growth model, I first compute income, taxes, and consumption in the case of hetereogeneity.

3.1.1 Disposable Income

Again, I shall abstract from how income is divided between capital and labor for simplicity, and simply assume that capital income is distributed in proportion to labor income in this economy. There are two types of workers:

- Low income workers earn income Y_L, and their share is λ. After taxes equal to $T_L(Y_L)$, I get disposable income Y_L^D:
$$Y_L^D \equiv Y_L - T_L(Y_L)$$
- High income workers earn income Y_H, and their share is $1-\lambda$. Moreover, I assume that Y_H is γ times Y_L, so that:
$$Y_H = \gamma Y_L.$$
Disposable income Y_H^D is given by:
$$Y_H^D \equiv Y_H - T_H(Y_H)$$

Aggregate income is given by:

$$Y = \lambda Y_L + (1-\lambda) Y_H$$
$$Y = [\lambda + (1-\lambda)\gamma]\, Y_L$$

Therefore individual incomes as a function of total income Y are:

$$Y_L = \frac{1}{\lambda + (1-\lambda)\gamma} Y \qquad Y_H = \frac{\gamma}{\lambda + (1-\lambda)\gamma} Y$$

3.1.2 Simple and income-weighted averages: definitions

For any quantity f, I define $\langle f \rangle$ as the simple average of f:

$$\langle f \rangle \equiv \lambda f_L + (1-\lambda) f_H., \tag{6}$$

and $\langle f \rangle_y$ as the income weighted average of f:

$$\langle f \rangle_y \equiv \frac{\lambda f_L + (1-\lambda)\gamma f_H}{\lambda + (1-\lambda)\gamma}. \tag{7}$$

3.1.3 Taxes, Transfers

Again, I consider a linear tax system for each group, with different marginal and average tax rates. For the low income, I assume that:

$$T_L(Y) = T_L(0) + \tau_L Y_L$$

Therefore, disposable income Y^D is:

$$Y_L^D = Y_L - T_L(Y_L)$$
$$Y_L^D = (1-\tau_L) Y_L - T_L(0)$$

Symmetrically, high income earners pay taxes:

$$T_H(Y) = T_H(0) + \tau_H Y_H$$

Their disposable income Y^D is:

$$Y_H^D = (1-\tau_H) Y_H - T_H(0)$$

Total aggregate taxes T are thus given by:

$$T = \lambda T_L(Y) + (1-\lambda) T_H(Y)$$
$$T = [\lambda T_L(0) + (1-\lambda) T_H(0)] + \left[\frac{\lambda \tau_L + (1-\lambda)\gamma \tau_H}{\lambda + (1-\lambda)\gamma}\right] Y$$

Using the simple average $\langle . \rangle$ and income-weighted average $\langle . \rangle_y$ concepts defined in equations (6) and (7):

$$T(Y) = \langle T(0) \rangle + \langle \tau \rangle_y \cdot Y.$$

3.1.4 Government Saving S_g

Public saving is the excess of net taxes over government spending:

$$S_g = T(Y) - G$$
$$S_g = \langle T(0) \rangle + \langle \tau \rangle_y \cdot Y - G$$

3.1.5 Consumption C

There is much evidence that the propensity to spend out of *permanent income* is different across income groups (Straub (2019), Saez and Zucman (2016)): the rich save more than the poor, on average. Note that with secular stagnation, what matters is the propensity to consume out of permanent income, and not the propensity to consume out of transitory income. Thus, the mechanism is very different from that in Heterogeneous Agent New Keynesian (HANK) models. In particular, models of precautionary saving do not generate heterogeneous propensities to consume out of permanent income. Therefore, I shall assume that there exists two level of the saving rate for each one of the income groups:

- The poor save a fraction s_L of their permanent income:

$$C_L = (1 - s_L)\left[Y_L - T_L(Y_L)\right].$$

Therefore:

$$C_L = -(1 - s_L)T_L(0) + (1 - s_L)(1 - \tau_L)Y_L.$$

- The rich save a fraction s_H of their permanent income:

$$C_H = (1 - s_H)\left[Y_H - T_H(Y_H)\right].$$

Therefore:

$$C_H = -(1 - s_H)T_H(0) + (1 - s_H)(1 - \tau_H)Y_H.$$

Aggregate consumption C is:

$$C = \lambda C_L + (1 - \lambda)C_H$$
$$C = -\left[\lambda(1 - s_L)T_L(0) + (1 - \lambda)(1 - s_H)T_H(0)\right] + \left[\frac{\lambda(1 - s_L)(1 - \tau_L) + (1 - \lambda)\gamma(1 - s_H)(1 - \tau_H)}{\lambda + (1 - \lambda)\gamma}\right] Y$$

Using the simple average $\langle . \rangle$ and income-weighted average $\langle . \rangle_y$ concepts defined in equations (6) and (7):

$$C = -\langle (1 - s)T(0) \rangle + \langle (1 - s)(1 - \tau) \rangle_y \cdot Y.$$

3.1.6 Private Saving S_p

A similar calculation as for consumption shows that private saving S_p is given by:

$$S_p = -\langle sT(0) \rangle + \langle s(1 - \tau) \rangle_y \cdot Y.$$

3.1.7 Total Saving S

From private saving S_p and government saving S_g, I can compute total saving:

$$\begin{aligned} S &= S_p + S_g \\ &= -\langle sT(0) \rangle + \langle s(1 - \tau) \rangle_y \cdot Y + \langle T(0) \rangle + \langle \tau \rangle_y \cdot Y - G \\ S &= \langle (1 - s)T(0) \rangle - G + \langle 1 - (1 - s)(1 - \tau) \rangle_y \cdot Y \end{aligned}$$

3.2 Neoclassical regime

3.2.1 Steady-state output

In the neoclassical regime, inequality is a good thing because it fosters capital accumulation and growth. Redistribution from the poor to the rich is desirable, as it increases overall saving and therefore, investment. Public debt is detrimental to growth, especially to the extent that individuals are not Ricardian, as it crowds out capital accumulation. As in the representative agent case, investment in the steady-state is:

$$I = \delta K.$$

From the previous section, total saving $S = S_p + S_g$ is:

$$S = \langle (1 - s)T(0) \rangle - G + \langle 1 - (1 - s)(1 - \tau) \rangle_y \cdot Y.$$

Similarly as in the representative agent case (see section 2.2), it is easy to show that:

$$\delta K = \langle 1 - (1 - s)(1 - \tau) \rangle_y \left(A\bar{L}\right)^{1-\alpha} \left(K - \frac{1}{2\kappa}\frac{K^2}{A\bar{L}}\right)^{\alpha} + \langle (1 - s)T(0) \rangle - G.$$

Therefore, in intensive form:

$$\delta\beta\frac{k}{\kappa} = \langle 1-(1-s)(1-\tau)\rangle_y \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^\alpha + \frac{\langle (1-s)T(0)\rangle - G}{\bar{Y}}.$$

To conclude, output is given by:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^\alpha$$

$$\text{where } \frac{k}{\kappa} \text{ solves:} \quad \delta\beta\frac{k}{\kappa} = \langle 1-(1-s)(1-\tau)\rangle_y \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^\alpha + \frac{\langle (1-s)T(0)\rangle - G}{\bar{Y}}.$$

3.2.2 Redistribution hurts growth

In the neoclassical regime, redistribution hurts growth. I consider ex-ante revenue neutral redistribution, one which preserves total tax receipts equal to $\langle T(0)\rangle$. I denote the difference between the tax that high income pay $T_H(0)$ and the tax that low income pay $T_L(0)$ by Δ. Therefore, $T_H(0)$ and $T_L(0)$ solve:

$$T_H(0) - T_L(0) = \Delta$$
$$(1-\lambda)T_H(0) + \lambda T_L(0) = \langle T(0)\rangle$$

The solution to this system is:

$$\begin{cases} T_L(0) = \langle T(0)\rangle - (1-\lambda)\Delta \\ T_H(0) = \langle T(0)\rangle + \lambda\Delta \end{cases}$$

This allows to compute $\langle (1-s)T(0)\rangle$ (see Appendix B.2):

$$\langle (1-s)T(0)\rangle = \langle 1-s\rangle \cdot \langle T(0)\rangle - \lambda(1-\lambda)\left(s_H - s_L\right)\Delta$$

To conclude, output is given by:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^\alpha$$

$$\text{where } \frac{k}{\kappa} \text{ solves:} \quad \delta\beta\frac{k}{\kappa} = \langle 1-(1-s)(1-\tau)\rangle_y \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^\alpha + \frac{\langle 1-s\rangle \cdot \langle T(0)\rangle - \lambda(1-\lambda)\left(s_H - s_L\right)\Delta - G}{\bar{Y}}.$$

As a consequence, given differences in propensities to save, when $\Delta > 0$, there is less capital accumulation in the steady-state, and therefore lower output. In the neoclassical model, offsetting income inequality hurts growth, because it leads to lower saving, and as a consequence lower capital accumulation.

The fact that more saving leads to more investment is the reason why inequalities in income are positive to growth, according to neoclassical theory. Keynes (1931) describes this view in *Essays in Persuation*: "Since the end of the nineteenth century significant progress towards the removal of very great disparities of wealth and income has been achieved through the instrument of direct taxation - income tax and surtax and death duties - especially in Great Britain. Many people would wish to see this process carried much further, but they are deterred by two considerations; partly by the fear of making skilful evasions too much worth while and also of diminishing unduly the motive towards risk-taking, but mainly, I think, by the belief that the growth of capital depends upon the strength of the motive towards individual saving and that for a large proportion of this growth we are dependent on the savings of the rich out of their superfluity."

3.2.3 Domain of the neoclassical regime

Again, we can now delimit the neoclassical and the secular stagnation regime. The secular stagnation regime arises when the average private saving rate is so large that even when capital intensity is equal to κ, gross saving is higher than gross investment (corresponding to depreciation). This implies:

$$\langle 1-(1-s)(1-\tau)\rangle_y + \frac{\langle (1-s)T(0)\rangle - G}{\bar{Y}} \geq \delta\beta.$$

With the notations defined before, this corresponds to redistribution Δ being bounded above:

$$\langle 1-(1-s)(1-\tau)\rangle_y + \frac{\langle 1-s\rangle \cdot \langle T(0)\rangle - \lambda(1-\lambda)\,(s_H - s_L)\,\Delta - G}{\bar{Y}} \geq \delta\beta.$$

$$\frac{\Delta}{\bar{Y}} \leq \frac{1}{\lambda(1-\lambda)\,(s_H - s_L)} \left(\langle 1-(1-s)(1-\tau)\rangle_y - \delta\beta + \frac{\langle 1-s\rangle \cdot \langle T(0)\rangle - G}{\bar{Y}} \right)$$

3.3 Secular Stagnation regime

3.3.1 Steady-state output

Again, from $S = I$, and writing that in the secular stagnation regime $I = \delta\beta Y$:

$$\langle (1-s)T(0)\rangle - G + \langle 1-(1-s)(1-\tau)\rangle_y \cdot Y = \delta\beta Y$$

Therefore:

$$\frac{Y}{\bar{Y}} = \frac{1}{1 - \langle (1-s)(1-\tau)\rangle_y - \delta\beta} \frac{G - \langle (1-s)T(0)\rangle}{\bar{Y}}.$$

This implies that the government spending multiplier, for example, is given by a very similar expression to the representative agent case, except that $(1-s)(1-\tau)$, the propensity to consume out of pre-tax income of the representative agent, is replaced by $\langle (1-s)(1-\tau)\rangle_y$, the income-weighted propensity to consume out of pre-tax income of heterogeneous agents:

$$\Delta Y = \frac{1}{1 - \langle (1-s)(1-\tau)\rangle_y - \delta\beta} \Delta G.$$

3.3.2 Redistribution

Again, as in the neoclassical case above, I consider ex-ante revenue neutral redistribution, one which preserves total tax receipts equal to $\langle T(0)\rangle$. I denote the difference between the tax that high income pay $T_H(0)$ and the tax that low income pay $T_L(0)$ by Δ. Again, this allows to compute $\langle (1-s)T(0)\rangle$ (see Appendix B.2), which implies that secular stagnation output is:

$$\frac{Y}{\bar{Y}} = \frac{1}{1 - \langle (1-s)(1-\tau)\rangle_y - \delta\beta} \frac{G - \langle 1-s\rangle \cdot \langle T(0)\rangle + \lambda(1-\lambda)\,(s_H - s_L)\,\Delta}{\bar{Y}}.$$

This expression shows that in the secular stagnation regime, output is all the greater when the difference between the baseline taxes Δ that high income and low income earners pay is high. The higher the difference in saving rate between high and low income $s_H - s_L$, the more redistribution has powerful effects.

Therefore, this model provides a microfoundation for Keynes (1931)'s intuition: "For we have seen that, up to the point where full employment prevails, the growth of capital depends not at all on a low propensity to consume but is, on the contrary, held back by it; and only in conditions of full employment is a low propensity to consume conducive to the growth of capital. Moreover, experience suggests that in existing conditions saving by institutions and through sinking funds is more than adequate, and that measures for the redistribution of incomes in a way likely to raise the propensity to consume may prove positively favourable to the growth of capital."

Given that $T = \langle T(0)\rangle + \langle\tau\rangle_y Y$, government saving increase, there is a reduction in the deficit, in public debt. Therefore, even though the policies are budget neutral ex-ante, they are not budget neutral ex-post, as they boost output which increases revenues through automatic stabilizers:

$$\begin{aligned}\Delta\left(T-G\right) &= \Delta T \\ &= \langle \Delta T(0)\rangle + \langle\tau\rangle_y \Delta Y \\ \Delta\left(T-G\right) &= \langle\tau\rangle_y \frac{\lambda(1-\lambda)\left(s_H - s_L\right)\Delta}{1-\langle(1-s)(1-\tau)\rangle_y - \delta\beta} > 0\end{aligned}$$

Example: Effects of redistributive policies on output. Depending on the regime, redistributive policies can either decrease or raise output. For example, assuming that:

$$\begin{aligned}&\tau_L = \tau_H = \frac{1}{4}, \quad s_L = 0, \quad s_H = \frac{1}{2}, \quad \alpha = \frac{1}{3}, \quad \delta = \frac{1}{16}, \\ &\kappa = 2, \quad \frac{G(0)}{\bar{Y}} = 20\%, \quad -\frac{\langle T(0)\rangle}{\bar{Y}} = 10\% \quad \gamma = 9, \quad \lambda = \frac{9}{10},\end{aligned}$$

then if $\Delta/\bar{Y} \leq 38.8\%$, the secular stagnation expression applies:

$$\frac{Y}{\bar{Y}} = \frac{944}{1000} + \frac{144}{1000}\cdot\frac{\Delta}{\bar{Y}}$$

If $\Delta/\bar{Y} \geq 38.8\%$, then we are in the neoclassical regime:

$$\begin{aligned}&\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^\alpha \\ &\text{where } \frac{k}{\kappa} \text{ solves:} \quad \frac{1}{8}\frac{k}{\kappa} = \frac{7}{16}\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^\alpha - \frac{59}{200} - \frac{9}{200}\frac{\Delta}{\bar{Y}}.\end{aligned}$$

3.3.3 Public Debt, Bubbles, Credit Cycles

Secular stagnation is a more severe form of dynamic inefficiency. Therefore, results which are well-known in overlapping-generations model of dynamic inefficiency apply to secular stagnation too. In particular, as in Tirole (1985), rational bubbles can appear when public debt does not provide enough stores of value. Since returns to capital are low, physical investment has low returns, so that investors are incentivized to "speculate", or to try to coordinate on high asset prices around a particular asset class. This can explain why wealth to income ratios have risen substantially, predominantly from real estate, but also from stock prices in the United States, which corresponds to a fluctuation in the price of capital. The accumulation component of wealth, on the other hand, has in fact declined, as I previously showed.

To the extent that sellers of assets tend to have higher marginal propensities to consume than buyers of assets, rational bubbles redistribute purchasing power from buyers of assets to sellers of assets, which tends to boost aggregate demand, as we have just shown. This idea has also been expressed by Summers (2013): "Let me say a little bit more about why I'm led to think in those terms. If you go back and you study the economy prior to the crisis, there is something a little bit odd. Many people believe that monetary policy was too easy. Everybody agrees that there was a vast amount of imprudent lending going on. Almost everybody believes that wealth, as it was experienced by households, was in excess of its reality: too much easy money, too much borrowing, too much wealth. Was there a great boom? Capacity utilization wasn't under any great pressure. Unemployment wasn't at any remarkably low level. Inflation was entirely quiescent. So, somehow, even a great bubble wasn't enough to produce any excess in aggregate demand."

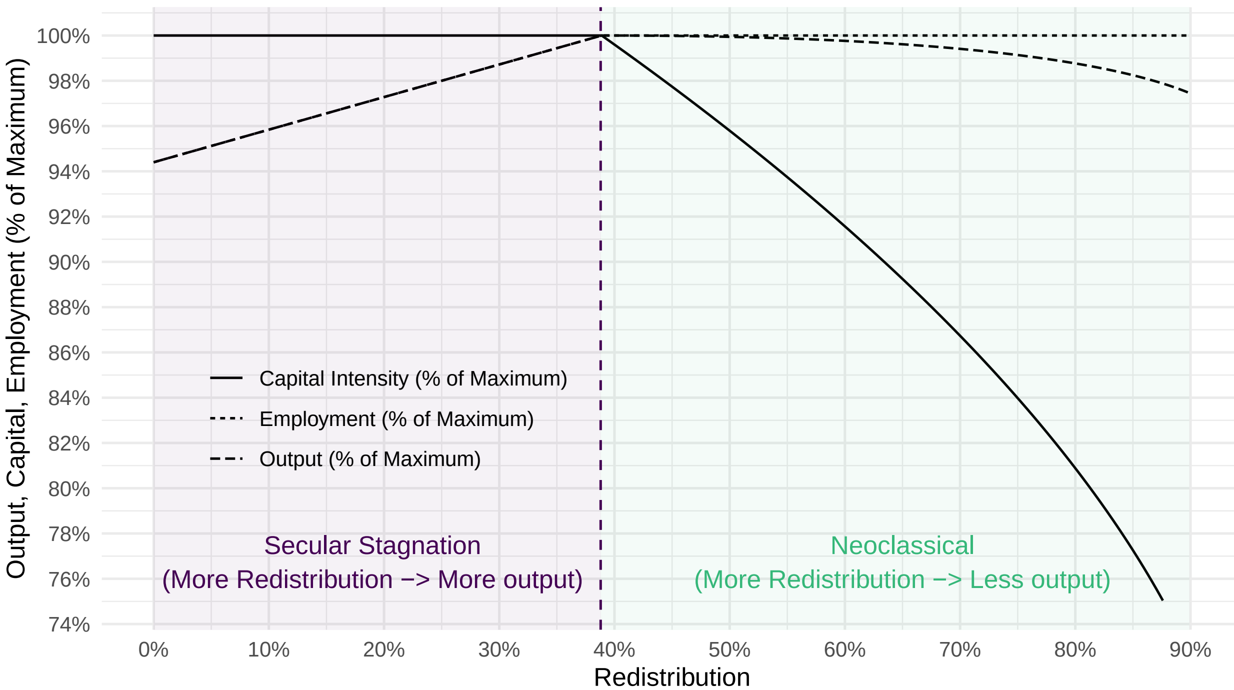

Figure 13: Effects of Redistributive Policies. Assumptions: $\gamma = 9$, $\lambda = 0.9$, $\alpha = 1/3$, $\tau_L = \tau_H = 1/4$, $\delta = 1/16$, $\kappa = 2$, $G/\bar{Y} = 20\%$, $s_L = 0$, $s_H = 1/2$ and $T(0)/\bar{Y} = -10\%$

Public debt are superior over privately created liabilities, which produce a similar burden on young generations, in that public debt is far more stable and predictable, and the redistributive consequences of public debt are more controlled. On the other hand, returns from a bubble can really be interpreted as coming from pure luck.

3.4 On Monetary Policy

One argument in favor of the sticky-price view of Keynesian economics is often that monetary policy has real effects, as is shown on Figure 22 reproducing Romer and Romer (2004). If prices were not sticky, then prices would double following a doubling of the money supply. Moreover, the interest rate is a key channel through which monetary policy is supposed to affect output in the Keynesian IS/LM model. For example, commenting about the zero elasticity of investment with respect to the cost of capital, Shapiro, Blanchard, and Lovell (1986) write: "Neo-Keynesians should not, however, find comfort in this embarrassment of neoclassical theory. The textbook IS-LM model is also inconsistent with the empirical finding that output shocks rather than cost-of-capital shocks determine investment. If the interest rate does not affect investment, then the IS curve is vertical, and there is no role for the Keynesian transmission mechanism from money to output."

In the secular stagnation regime of a growth model with satiation of capital, monetary policy works very differently than in the IS-LM model, but it is operative nonethless. This is because redistributing income from low to high marginal propensity to consume (for example creditors to lenders) boosts consumption, and in particular of durable goods such as housing. This is in fact what Romer and Romer (2004) find, as shown on Figures 23 for housing construction and 24 for house prices. I now review the reasons why monetary policy tends to redistribute income in a closed economy, and I then consider the case of an open economy.

3.4.1 Closed Economy

In a closed economy, the first reason why monetary policy redistributes income is that many contracts are nominal, and therefore price inflation redistributes between away from creditors and towards debitors. A second, even more direct, reason is that monetary policy has effects on interest rates at which borrowers borrow from creditors (particularly those with a mortgage). Monetary policy has redistributive effects across different types of people for many reasons. This idea goes back to Fischer, whose ideas are exposed in Tobin (1980): “Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and debtors. But if the spending propensity were systematically greater for debtors, even by a small amount, the Pigou effect would be swamped by this Fisher effect. There are indeed reasons for expecting, or at least for suspecting, just that. The population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from wealth or from current income or from any liquid resources they can command.” These effects have been studied more formally in Auclert (2019).

There is growing empirical evidence that the redistribution channel of monetary policy is in fact, very important. Cloyne, Ferreira, and Surico (2019), for example, have shown that after a temporary cut in interest rates, households with mortgage debt increase their spending significantly, homeowners without debt do not adjust expenditure at all and renters increase spending but by less than mortgagors. Wong (2016) has shown very similar results, working through the refinancing channel of monetary policy.

Compared with this literature on the redistributive effects of monetary policy, my model therefore shows that monetary policy has real effects even if prices are perfectly flexible and the Phillips curve is absent (see Geerolf (2018)). As a consequence, the well-documented effects of monetary policy on output is no evidence that prices are sticky.

3.4.2 Open Economy

In an open economy, potential devaluation allows the redenomination of nominal assets and liabilities in a new currency. Again, and especially to the extent that debts are denominated in the domestic currency, devaluations allow to substantially ease the burden of debtors, and can trigger a substantial recovery from a regime change in monetary policy. Moreover, for exporters devaluation allows to get much more revenue when denominated in local currency (since the foreign currency is now more valuable). A recent example has been given by Hausman, Rhode, and Wieland (2019) who have shown that an important source of the 1933 recovery was the effect of dollar devaluation on farm prices, incomes, and consumption. Devaluation immediately raised traded crop prices, raising farmers' income, and substantially contributing to the recovery.

Conclusion

Before World War II, Hansen (1939) warned of impending “secular stagnation.” With the closing of the frontier, the U.S. economy would not be capable of creating a sufficient number of new outlets for investment: “We are thus rapidly entering a world in which we must fall back upon a more rapid advance of technology than in the past if we are to find private investment opportunities adequate to maintain full employment.” (Hansen (1939)) This theory has recently been reignited by Larry Summers' Speech in 2013 at the IMF conference in honor of Stanley Fischer (Summers (2013)), where Larry Summers wondered whether aggregate demand problems were only about fluctuations, or whether it could also potentially be an explanation for lackluster growth. For example, he questioned whether the U.S. economy prior to the 2007-2009 should be thought of as “overheating” or whether in fact, it also was then suffering from deficient aggregate demand, temporarily boosted by a housing bubble. As Summers (2017) has emphasized, the Keynesian aspiration was not to merely reduce the amplitude of cyclical fluctuations, but also to increase overall growth. For instance, President Kennedy's 1963 Economic Report of the President argued that fiscal stimulus would boost long-run potential output: “among the costs of prolonged slack is slow growth. An economy that fails to use its productive potential fully feels no need to increase it rapidly. The incentive to invest is bent beneath the weight of excess capacity. Lack of employment opportunities slows the growth of the labor force.”

This paper has formalized Hansen (1939) / Summers (2013) secular stagnation hypothesis. It proves to be an alternative to Hicks (1937)‘s IS-LM sticky price model. According to this interpretation, labor is underemployed because more labor would create saving that would not be met with enough investment demand. This happens because with the proposed production function, marginal returns to capital are equal to zero, past some level of capital intensity. Although J.M. Keynes' early work such as Keynes (1919) contains many allusions to the possibility of excess saving, and of an excess of saving over investment, this is not the route which was pursued in the General Theory. According to Axel Leijonhufvud, "It does not even seem to have occurred to Keynes that investment might be exceedingly interest inelastic, as later Keynesians would have it. Instead, he was concerned to convince the reader that it is reasonable to assume that *a moderate change in the prospective yield of capital-assets or in the rate of interest will not involve an indefinitely great change in the rate of investment*." In this paper, I have formalized this hypothesis, simply adding a small twist to the neoclassical production function. In turns out that most Keynesian paradoxes seem to fit well with a view of a production function exhibiting satiation of capital.

This paper suggests a way too favorable view of expansionary fiscal policy. With a production function exhibiting satiation, aggregate demand stimulating policies only have positive effects assuming that we find ourselves in a secular stagnation regime. In Geerolf (2018), I have investigated some potential trade-offs arising from the external balance constraint, and argued that the Phillips curve of the neoclassical synthesis might in fact really be a real exchange rate Phillips curve. The most desirable course of action would be coordinated fiscal stimulus policies across countries, to alleviate the external balance problem while avoiding tariffs or capital controls, which are detrimental to the international division of labor, and useful foreign direct investment. At the same time, both Japan and Germany have so far appeared unwilling to move away from an export led model of growth to serve internal demand. To the contrary, Germany has undertaken important pension reform in the 2001, to make pensions less generous, and encourage a move from a pay-as-you-go to a funded system. Similarly, it has twice increased the very regressive Value Added Tax, from 16% to 19% in 2007, which has also depressed consumption as well as compressed imports from its neighbors. How to best boost aggregate demand without further increasing the trade deficit, while maintaining a commitment to openness of capital and goods, remains an important question for future research and policy.

References

Auclert, Adrien. 2019. "Monetary Policy and the Redistribution Channel." *American Economic Review* 109 (6): 2333–67. https://doi.org/10.1257/aer.20160137.

Barro, Robert J. 1974. "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82 (6): 1095–1117. http://www.jstor.org/stable/1830663.

———. 1989. "The Ricardian Approach to Budget Deficits." *Journal of Economic Perspectives* 3 (2): 37–54. https://doi.org/10.1257/jep.3.2.37.

Benigno, Gianluca, and Luca Fornaro. 2018. "Stagnation Traps." *The Review of Economic Studies* 85 (3): 1425–70. https://doi.org/10.1093/restud/rdx063.

Blanchard, Olivier Jean, and Danny Quah. 1989. "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *The American Economic Review* 79 (4): 655–73. http://www.jstor.org/stable/1827924.

Blanchard, Olivier, and Lawrence H. Summers. 1986. "Hysteresis and the European Unemployment Problem." *NBER Macroeconomics Annual 1986, Volume 1*, January, 15–90. https://www.nber.org/chapters/c4245.

Caballero, Ricardo J. 1999. "Chapter 12 Aggregate Investment." In *Handbook of Macroeconomics*, 1:813–62. Elsevier. https://doi.org/10.1016/S1574-0048(99)10020-X.

Carroll, Christopher D. 2000. "Why Do the Rich Save so Much ?" In *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, edited by Joel B. Slemrod. Vol. D. Harvard Univ Pr.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins." *American Economic Review* 101 (3): 471–75. https://doi.org/10.1257/aer.101.3.471.

Cloyne, James. 2013. "Discretionary Tax Changes and the Macroeconomy: New Narrative Evidence from the United Kingdom." *American Economic Review* 103 (4): 1507–28. https://doi.org/10.1257/aer.103.4.1507.

Cloyne, James, Clodomiro Ferreira, and Paolo Surico. 2019. "Monetary Policy When Households Have Debt: New Evidence on the Transmission Mechanism." *The Review of Economic Studies*. https://doi.org/10.1093/restud/rdy074.

Crocker, Uriel H., and S. M. Macvane. 1887. "General Overproduction." *The Quarterly Journal of Economics* 1 (3): 362–66. https://doi.org/10.2307/1882763.

Cummins, Jason G., Kevin A. Hassett, R. Glenn Hubbard, Robert E. Hall, and Ricardo J. Caballero. 1994. "A Reconsideration of Investment Behavior Using Tax Reforms as Natural Experiments." *Brookings Papers on Economic Activity* 1994 (2): 1–74. https://doi.org/10.2307/2534654.

Diamond, Peter A. 1965. "National Debt in a Neoclassical Growth Model." *The American Economic Review* 55 (5): 1126–50. http://www.jstor.org/stable/1809231.

Domar, Evsey D. 1946. "Capital Expansion, Rate of Growth, and Employment." *Econometrica* 14 (2): 137–47. https://doi.org/10.2307/1905364.

Dornbusch, Rudiger, and Stanley Fischer. 1979. *Macroeconomics*. Third Edition.

Eccles, Marriner Stoddard. 1951. *Beckoning Frontiers: Public and Personal Recollections. Marriner S. Eccles. Edited by Sidney Hyman.* A.A. Knopf.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins. 2019. "A Model of Secular Stagnation: Theory and Quantitative Evaluation." *American Economic Journal: Macroeconomics* 11 (1): 1–48. https://doi.org/10.1257/mac.20170367.

Engels, Friedrich. 1878. *Anti-Dühring*. https://data.bnf.fr/fr/11939335/friedrich_engels_anti-duhring/.

Fatás, Antonio, and Lawrence H. Summers. 2018. "The Permanent Effects of Fiscal Consolidations." *Journal of International Economics* 112 (May): 238–50. https://doi.org/10.1016/j.jinteco.2017.11.007.

Fazzari, Steven M., R. Glenn Hubbard, Bruce C. Petersen, Alan S. Blinder, and James M. Poterba. 1988. "Financing Constraints and Corporate Investment." *Brookings Papers on Economic Activity* 1988 (1): 141–206. https://doi.org/10.2307/2534426.

Geerolf, François. 2013a. *Bubbles and Asset Supply.* Ph.D. Thesis Sciences Po. https://fgeerolf.com/thesis.pdf.

———. 2013b. "Dynamic Inefficiency and Capital Taxation." UCLA Working Paper. https://fgeerolf.com/land.pdf.

———. 2013c. "Reassessing Dynamic Efficiency." UCLA Working Paper. https://fgeerolf.com/r-g.pdf.

———. 2018. "The Phillips Curve: A Relation Between Real Exchange Rate Growth and Unemployment." UCLA Working Paper. https://fgeerolf.com/phillips.pdf.

Geerolf, François, and Thomas Grjebine. 2019. "The Macroeconomic Effects of Lump-Sum Taxes." Working Paper. https://fgeerolf.com/tax-multipliers.pdf.

Giavazzi, Francesco, and Marco Pagano. 1990. "Can Severe Fiscal Contractions Be Expansionary? Tales of Two Small European Countries." *NBER Macroeconomics Annual* 5 (January): 75–111. https://doi.org/10.1086/654131.

Hansen, Alvin H. 1939. "Economic Progress and Declining Population Growth." *The American Economic Review* 29 (1): 1–15. http://www.jstor.org/stable/1806983.

Harrod, R. F. 1939. "An Essay in Dynamic Theory." *The Economic Journal* 49 (193): 14–33. https://doi.org/10.2307/2225181.

Hausman, Joshua K., Paul W. Rhode, and Johannes F. Wieland. 2019. "Recovery from the Great Depression: The Farm Channel in Spring 1933." *American Economic Review* 109 (2): 427–72. https://doi.org/10.1257/aer.20170237.

Hayo, Bernd, and Matthias Uhl. 2014. "The Macroeconomic Effects of Legislated Tax Changes in Germany." *Oxford Economic Papers* 66 (2): 397–418. https://doi.org/10.1093/oep/gpt017.

Hicks, J. R. 1937. "Mr. Keynes and the "Classics"; A Suggested Interpretation." *Econometrica* 5 (2): 147–59. https://doi.org/10.2307/1907242.

Hobson, J. A. 1902. *Imperialism: A Study.* Cosimo, Inc.

House, Christopher L., Christian Proebsting, and Linda L. Tesar. 2019. "Austerity in the Aftermath of the Great Recession." *Journal of Monetary Economics*, May. https://doi.org/10.1016/j.jmoneco.2019.05.004.

Keynes, John Maynard. 1919. *The Economic Consequences of the Peace.*

———. 1931. *Essays in Persuasion.* Springer.

———. 1936. *The General Theory of Employment, Interest, and Money.*

Malinvaud, Edmond. 1953. "Capital Accumulation and Efficient Allocation of Resources." *Econometrica* 21 (2): 233–68. https://doi.org/10.2307/1905538.

Malthus, Thomas Robert. 1820. *Principles of Political Economy.* https://doi.org/10.1093/library/s5-XVII.3.238.

Mandeville, Bernard. 1732. *The Fable of the Bees.* Penguin UK.

Marx, Karl. 1867. "Capital, Vol. 1: A Critique of Political Economy." *New York: Vintage.*

Michau, Jean-Baptiste. 2018. "Secular Stagnation: Theory and Remedies." *Journal of Economic Theory* 176 (July): 552–618. https://doi.org/10.1016/j.jet.2018.04.011.

Nakamura, Emi, and Jón Steinsson. 2008. "Five Facts About Prices: A Reevaluation of Menu Cost Models." *The Quarterly Journal of Economics* 123 (4): 1415–64. https://doi.org/10.1162/qjec.2008.123.4.1415.

Nelson, Charles R., and Charles R. Plosser. 1982. "Trends and Random Walks in Macroeconmic Time Series: Some Evidence and Implications." *Journal of Monetary Economics* 10 (2): 139–62. https://doi.org/10.1016/0304-3932(82)90012-5.

Oliner, Stephen, Glenn Rudebusch, and Daniel Sichel. 1995. "New and Old Models of Business Investment: A Comparison of Forecasting Performance." *Journal of Money, Credit and Banking* 27 (3): 806–26. https://doi.org/10.2307/2077752.

Phelps, Edmund. 1961. "The Golden Rule of Accumulation: A Fable for Growthmen." *The American Economic Review* 51 (4): 638–43. http://www.jstor.org/stable/1812790.

Piketty, Thomas. 2014. *Capital in the Twenty-First Century*. Harvard University Press.

Piketty, Thomas, and Gabriel Zucman. 2014. "Capital Is Back: Wealth-Income Ratios in Rich Countries 17002010." *The Quarterly Journal of Economics* 129 (3): 1255–1310. https://doi.org/10.1093/qje/qju018.

Poterba, James M., and Lawrence H. Summers. 1987. "Finite Lifetimes and the Effects of Budget Deficits on National Saving." *Journal of Monetary Economics* 20 (2): 369–91. https://doi.org/10.1016/0304-3932(87)90021-3.

Ramey, Valerie A. 2019. "Ten Years After the Financial Crisis: What Have We Learned from the Renaissance in Fiscal Research?" *Journal of Economic Perspectives* 33 (2): 89–114. https://doi.org/10.1257/jep.33.2.89.

Reis, Ricardo. 2018. "Is Something Really Wrong with Macroeconomics?" *Oxford Review of Economic Policy* 34 (1-2): 132–55. https://doi.org/10.1093/oxrep/grx053.

Robinson, Joan. 1974. "What Has Become of the Keynesian Revolution?" *Challenge* 16 (6): 6–11. https://fgeerolf.com/phillips/bib/Robinson1974.pdf.

Romer, Christina D., and David H. Romer. 2004. "A New Measure of Monetary Shocks: Derivation and Implications." *American Economic Review* 94 (4): 1055–84. https://doi.org/10.1257/0002828042002651.

———. 2010. "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks." *American Economic Review* 100 (3): 763–801. https://doi.org/10.1257/aer.100.3.763.

Saez, Emmanuel, and Gabriel Zucman. 2016. "Wealth Inequality in the United States Since 1913: Evidence from Capitalized Income Tax Data." *The Quarterly Journal of Economics* 131 (2): 519–78. https://doi.org/10.1093/qje/qjw004.

Samuelson, Paul A. 1953. *Economics*. Third Edition.

Say, Jean-Baptiste. 1803. *Traité d'économie Politique*.

Shapiro, Matthew D., Olivier J. Blanchard, and Michael C. Lovell. 1986. "Investment, Output, and the Cost of Capital." *Brookings Papers on Economic Activity* 1986 (1): 111–64. https://doi.org/10.2307/2534415.

Smith, Adam. 1776. "An Inquiry into the Nature and Causes of the Wealth of Nations," 1131.

Solow, Robert M. 1956. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics* 70 (1): 65–94. https://doi.org/10.2307/1884513.

Straub, Ludwig. 2019. "Consumption, Savings, and the Distribution of Permanent Income." Working Paper.

Summers, Lawrence H. 1991. "Should Keynesian Economics Dispense with the Phillips Curve?" In *Issues in Contemporary Economics*, 3–20. International Economic Association Series. Palgrave Macmillan, London.

———. 2013. "Conference Address." IMF Fourtheenth Annual Research Conference in Honor of Stanley Fischer. https://www.youtube.com/watch?v=KYpVzBbQIX0.

———. 2017. "Crises in Economic Thought, Secular Stagnation, and Future Economic Research." *NBER Macroeconomics Annual* 31 (1): 557–77. https://doi.org/10.1086/690245.

Sweezy, A. R. 1940. "Population Growth and Investment Opportunity." *The Quarterly Journal of Economics* 55 (1): 64–79. https://doi.org/10.2307/1881666.

Tirole, Jean. 1985. "Asset Bubbles and Overlapping Generations." *Econometrica* 53 (5): 1071–1100. https://doi.org/10.2307/1911012.

Tobin, James. 1980. *Asset Accumulation and Economic Activity: Reflections on Contemporary Macroeconomic Theory.* University of Chicago Press.

Wong, Arlene. 2016. "Population Aging and the Transmission of Monetary Policy to Consumption." 2016 Meeting Paper 716. Society for Economic Dynamics. https://econpapers.repec.org/paper/redsed016/716.htm.

Yagan, Danny. 2015. "Capital Tax Reform and the Real Economy: The Effects of the 2003 Dividend Tax Cut." *American Economic Review* 105 (12): 3531–63. https://doi.org/10.1257/aer.20130098.

Zidar, Owen. 2019. "Tax Cuts for Whom? Heterogeneous Effects of Income Tax Changes on Growth and Employment." *Journal of Political Economy* 127 (3): 1437–72. https://doi.org/10.1086/701424.

A Proofs

A.1 Marginal Product of Capital

In the case where $0 < K/AL \leq \kappa$, the marginal product of capital $\partial F/\partial K$ is:

$$\begin{aligned}
\frac{\partial F}{\partial K} &= \alpha K^{\alpha-1}\left(AL\right)^{1-\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha} - \frac{\alpha}{2\kappa AL}K^{\alpha}\left(AL\right)^{1-\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1} \\
&= \alpha K^{\alpha-1}\left(AL\right)^{1-\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left[\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)-\frac{1}{2\kappa}\frac{K}{AL}\right] \\
\frac{\partial F}{\partial K} &= \alpha\left(\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{\kappa}\frac{K}{AL}\right)
\end{aligned}$$

You can note already that at the maximum level of capital, the marginal product of capital is equal to zero, as then:

$$MPK = 1-\frac{1}{\kappa}\frac{K}{AL} = 0$$

Whenever $K/AL > \kappa$ the marginal product of capital is also equal to zero: there is satiation of capital in the production function.

To conclude, the gross Marginal Product of Capital is equal to:

$$MPK = \frac{\partial F}{\partial K} = \begin{cases} \alpha\left(\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{\kappa}\frac{K}{AL}\right) & \text{if} \quad 0 < \frac{K}{AL} \leq \kappa \\ 0 & \text{if} \quad \frac{K}{AL} > \kappa \end{cases}$$

The Marginal Product of Capital can be expressed solely as a function of $k \equiv K/AL$, as:

$$MPK = \frac{\partial F}{\partial K} = \begin{cases} \alpha k^{\alpha-1}\left(1-\frac{k}{2\kappa}\right)^{\alpha-1}\left(1-\frac{k}{\kappa}\right) & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa. \end{cases}$$

Note that I could have obtained this expression by differentiating the intensive form of the production function directly. The intensive form is:

$$y = f(k) = \begin{cases} k^{\alpha}\left(1-\frac{1}{2\kappa}k\right)^{\alpha} & \text{if} \quad 0 < k \leq \kappa \\ \left(\frac{\kappa}{2}\right)^{\alpha} & \text{if} \quad k \geq \kappa \end{cases}$$

Therefore, when the k is below the threshold value κ, I can compute the marginal product of capital given by $f'(k)$:

$$\begin{aligned}
f'(k) &= \alpha k^{\alpha-1}\left(1-\frac{k}{2\kappa}\right)^{\alpha} - \frac{\alpha}{2\kappa}k^{\alpha}\left(1-\frac{k}{2\kappa}\right)^{\alpha-1} \\
&= \alpha k^{\alpha-1}\left(1-\frac{k}{2\kappa}\right)^{\alpha-1}\left[\left(1-\frac{k}{2\kappa}\right)-\frac{k}{2\kappa}\right] \\
f'(k) &= \alpha k^{\alpha-1}\left(1-\frac{k}{2\kappa}\right)^{\alpha-1}\left(1-\frac{k}{\kappa}\right)
\end{aligned}$$

Therefore:

$$f'(k) = \begin{cases} \alpha k^{\alpha-1}\left(1-\frac{k}{2\kappa}\right)^{\alpha-1}\left(1-\frac{k}{\kappa}\right) & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa. \end{cases}$$

Finally, note that an even more direct way to derive the same expression is simply to write the intensive production function as:

$$y = f(k) = \begin{cases} \left(k - \dfrac{1}{2\kappa}k^2\right)^\alpha & \text{if} \quad 0 < k \leq \kappa \\ \left(\dfrac{\kappa}{2}\right)^\alpha & \text{if} \quad k \geq \kappa \end{cases}$$

This gives immediately:

$$f'(k) = \begin{cases} \alpha \left(k - \dfrac{1}{2\kappa}k^2\right)^{\alpha-1} \left(1 - \dfrac{k}{\kappa}\right) & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa. \end{cases}$$

Therefore:

$$f'(k) = \begin{cases} \alpha k^{\alpha-1} \left(1 - \dfrac{k}{2\kappa}\right)^{\alpha-1} \left(1 - \dfrac{k}{\kappa}\right) & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa. \end{cases}$$

A.2 Marginalist Capital Share

Assuming that capital gets its marginal product as return, the share of capital in value added can be determined using the Marginal Product of Capital determined above:

$$\frac{\partial F}{\partial K} = \begin{cases} \alpha \left(\dfrac{K}{AL}\right)^{\alpha-1} \left(1 - \dfrac{1}{2\kappa}\dfrac{K}{AL}\right)^{\alpha-1} \left(1 - \dfrac{1}{\kappa}\dfrac{K}{AL}\right) & \text{if} \quad 0 < \dfrac{K}{AL} \leq \kappa \\ 0 & \text{if} \quad \dfrac{K}{AL} > \kappa \end{cases}$$

In the case where $0 \leq K/AL \leq \kappa$, I can write:

$$\frac{\frac{\partial F}{\partial K}K}{Y} = \frac{\alpha \left(\dfrac{K}{AL}\right)^{\alpha-1} \left(1 - \dfrac{1}{2\kappa}\dfrac{K}{AL}\right)^{\alpha-1} \left(1 - \dfrac{1}{\kappa}\dfrac{K}{AL}\right) K}{K^\alpha (AL)^{1-\alpha} \left(1 - \dfrac{1}{2\kappa}\dfrac{K}{AL}\right)^\alpha}$$

$$\frac{\frac{\partial F}{\partial K}K}{Y} = \alpha \frac{1 - \dfrac{1}{\kappa}\dfrac{K}{AL}}{1 - \dfrac{1}{2\kappa}\dfrac{K}{AL}}$$

Expressing this using capital intensity $k = K/AL$, I get:

$$\frac{\frac{\partial F}{\partial K}K}{Y} = \alpha \frac{1 - \dfrac{k}{\kappa}}{1 - \dfrac{k}{2\kappa}}$$

A.3 Elasticity of Substitution σ

By definition, the elasticity of substitution between capital and labor is the percentage change in the capital over labor ratio, when ratio of the price of capital over the wage rate (which is equal to the marginal rate of transformation) changes by 1%:

$$\sigma\left(\frac{K}{L}\right) = -\frac{d\log\left(\dfrac{K}{L}\right)}{d\log\left(\dfrac{\partial F/\partial K}{\partial F/\partial L}\right)} = -\frac{d\log\left(\dfrac{K}{AL}\right)}{d\log\left(\dfrac{\partial F/\partial K}{\partial F/\partial AL}\right)}.$$

We note that this elasticity is the same regardless of whether it is computed with labor L, or efficiency units of labor AL. The latter will prove more convenient. To compute the marginal rate of transformation $(\partial F/\partial K)/(\partial F/\partial AL)$, I first compute the marginal product of capital $\partial F/\partial K$ with:

$$Y = F(K,L) = \begin{cases} K^{\alpha}(AL)^{1-\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha} & \text{if} \quad 0<\frac{K}{AL}\leq\kappa \\ \left(\frac{\kappa}{2}\right)^{\alpha}AL & \text{if} \quad \frac{K}{AL}>\kappa \end{cases}$$

I have already shown that the gross Marginal Product of Capital is equal to:

$$MPK = \frac{\partial F}{\partial K} = \begin{cases} \alpha\left(\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{\kappa}\frac{K}{AL}\right) & \text{if} \quad 0<\frac{K}{AL}\leq\kappa \\ 0 & \text{if} \quad \frac{K}{AL}>\kappa \end{cases}$$

In case $0 < K/AL \leq \kappa$, the marginal product of efficiency units of labor $\partial F/\partial AL$ is:

$$\begin{aligned}
\frac{\partial F}{\partial AL} &= (1-\alpha)K^{\alpha}(AL)^{-\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha} + K^{\alpha}(AL)^{1-\alpha}\frac{\alpha K}{2\kappa A^2L^2}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1} \\
&= K^{\alpha}(AL)^{-\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left[(1-\alpha)\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)+\frac{\alpha}{2\kappa}\frac{K}{AL}\right] \\
\frac{\partial F}{\partial AL} &= \left(\frac{K}{AL}\right)^{\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left[1-\alpha-(1-2\alpha)\frac{1}{2\kappa}\frac{K}{AL}\right].
\end{aligned}$$

The marginal rate of transformation is:

$$\begin{aligned}
\frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial AL}} &= \frac{\alpha\left(\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left(1-\frac{1}{\kappa}\frac{K}{AL}\right)}{\left(\frac{K}{AL}\right)^{\alpha}\left(1-\frac{1}{2\kappa}\frac{K}{AL}\right)^{\alpha-1}\left[1-\alpha-(1-2\alpha)\frac{1}{2\kappa}\frac{K}{AL}\right]} \\
&= \frac{\alpha\left(1-\frac{1}{\kappa}\frac{K}{AL}\right)}{\frac{K}{AL}\left[1-\alpha-(1-2\alpha)\frac{1}{2\kappa}\frac{K}{AL}\right]} \\
\frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial AL}} &= \frac{2\alpha\left(\kappa-\frac{K}{AL}\right)}{\frac{K}{AL}\left[(2-2\alpha)\kappa-(1-2\alpha)\frac{K}{AL}\right]}
\end{aligned}$$

Expressing everything as a function of:

$$k \equiv \frac{K}{AL}.$$

allows to get:

$$\frac{\partial F/\partial K}{\partial F/\partial AL} = \frac{2\alpha(\kappa-k)}{k\left[(2-2\alpha)\kappa-(1-2\alpha)k\right]}$$

I can now compute the inverse of the elasticity of substitution:

$$\begin{aligned}
\frac{1}{\sigma} &= -\frac{d\log\left(\dfrac{\partial F/\partial K}{\partial F/\partial AL}\right)}{d\log k} \\
&= -\left(-\frac{d\log k}{d\log k} + \frac{d\log(\kappa - k)}{d\log k} - \frac{d\log\left[(2-2\alpha)\kappa - (1-2\alpha)k\right]}{d\log k}\right) \\
&= -\left(-1 - \frac{k}{\kappa - k} + \frac{(1-2\alpha)k}{(2-2\alpha)\kappa - (1-2\alpha)k}\right) \\
&= \frac{\kappa}{\kappa - k} - \frac{k}{\dfrac{2-2\alpha}{1-2\alpha}\kappa - k} \\
\frac{1}{\sigma} &= 1 + \frac{\kappa}{\kappa - k} - \frac{\kappa}{\kappa - \dfrac{1-2\alpha}{2-2\alpha}k}
\end{aligned}$$

There, I get the elasticity of substitution as a function of k, which is given by:

$$\sigma(k) = \begin{cases} \dfrac{1}{1 + \dfrac{\kappa}{\kappa - k} - \dfrac{\kappa}{\kappa - \dfrac{1-2\alpha}{2-2\alpha}k}} & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa \end{cases}$$

For example, with $\kappa = 1$, and $\alpha = 1/3$, I arrive at the following elasticity of substitution:

$$\sigma(k) = \begin{cases} \dfrac{(4-k)(1-k)}{(1-k)^2 + 3} & \text{if} \quad 0 < k \leq \kappa \\ 0 & \text{if} \quad k \geq \kappa \end{cases}$$

A.4 Neoclassical regime: Example

Assuming that $\alpha = 1/3$, and using

$$k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}} \left(1 - \frac{k^*}{2\kappa}\right)^{\frac{\alpha}{1-\alpha}} \qquad \Rightarrow \qquad k^* = \left(\frac{s}{\delta}\right)^{3/2} \left(1 - \frac{k^*}{2\kappa}\right)^{1/2}$$

Therefore:

$$(k^*)^2 + \frac{1}{2\kappa}\left(\frac{s}{\delta}\right)^3 k^* - \left(\frac{s}{\delta}\right)^3 = 0.$$

Finally:

$$k^* = -\frac{1}{4\kappa}\left(\frac{s}{\delta}\right)^3 + \left(\frac{s}{\delta}\right)^{3/2}\sqrt{1 + \frac{1}{16\kappa^2}\left(\frac{s}{\delta}\right)^3}$$

B Numerical Examples

B.1 Representative Agent

Depending on parameters, steady-state output can take two values. If:

$$\delta\beta \leq 1 - (1-\tau)(1-s) + \frac{(1-s)T(0) - G}{\bar{Y}}$$

Then we get the secular stagnation expression:

$$\frac{Y}{\bar{Y}} = \frac{1}{1-(1-\tau)(1-s)-\delta\beta}\frac{G-(1-s)T(0)}{\bar{Y}}.$$

If on the other hand this condition is not satisfied then:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha}$$
$$\text{where } \frac{k}{\kappa} \text{ solves:} \quad \delta\beta\frac{k}{\kappa} = [1-(1-\tau)(1-s)]\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} + \frac{(1-s)T(0)-G}{\bar{Y}}$$

B.1.1 Government Spending

Given the stated parameter values:

$$\alpha = \frac{1}{3}, \quad s = \frac{1}{4}, \quad \tau = \frac{1}{4}, \quad \delta = \frac{1}{16} \quad \kappa = 2, \quad \frac{T(0)}{\bar{Y}} = -15\%,$$

we have:

$$\delta\beta = \delta \cdot 2^{\alpha}\kappa^{1-\alpha} = \frac{1}{8}, \qquad 1-(1-\tau)(1-s) = \frac{7}{16}, \qquad (1-s)\frac{T(0)}{\bar{Y}} = -\frac{3}{4}\cdot\frac{3}{20} = -\frac{9}{80}.$$

Two regimes. The condition for secular stagnation is:

$$\delta\beta \leq 1-(1-\tau)(1-s) + \frac{(1-s)T(0)-G}{\bar{Y}}$$

This is obtained for low enough government spending:

$$\begin{aligned}
\frac{G}{\bar{Y}} &\leq 1-(1-\tau)(1-s)-\delta\beta+(1-s)\frac{T(0)}{\bar{Y}} \\
&\leq \frac{7}{16} - \frac{1}{8} - \frac{9}{80} \\
&\leq \frac{7\cdot 5 - 10 - 9}{80} \\
\frac{G}{\bar{Y}} &\leq 20\%
\end{aligned}$$

Secular stagnation regime. If $G/\bar{Y} \leq 0.2$, then the secular stagnation expression applies:

$$\begin{aligned}
\frac{Y}{\bar{Y}} &= \frac{1}{1-(1-\tau)(1-s)-\delta\beta}\frac{G-(1-s)T(0)}{\bar{Y}} \\
&= \frac{1}{\frac{7}{16}-\frac{1}{8}}\left(\frac{G}{\bar{Y}} + \frac{9}{80}\right) \\
&= \frac{16}{5}\left(\frac{G}{\bar{Y}} + \frac{9}{80}\right) \\
\frac{Y}{\bar{Y}} &= 36\% + 3.2\cdot\frac{G}{\bar{Y}}
\end{aligned}$$

Taxes are:

$$\frac{T(Y)}{\bar{Y}} = \frac{T(0)}{\bar{Y}} + \tau\frac{Y}{\bar{Y}}$$

Therefore, full output are given by:

$$\frac{T(\bar{Y})}{\bar{Y}} = \frac{T(0)}{\bar{Y}} + \tau = -15\% + 25\% = 10\%.$$

Neoclassical regime. In the neoclassical regime:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha}$$
$$\text{where } \frac{k}{\kappa} \text{ solves:} \quad \frac{7}{16}\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} - \frac{1}{8}\frac{k}{\kappa} - \frac{9}{80} - \frac{G}{\bar{Y}} = 0$$

B.1.2 Austerity

Given the stated parameter values:

$$\alpha = \frac{1}{3}, \quad s = \frac{1}{4}, \quad \tau = \frac{1}{4}, \quad \delta = \frac{1}{16} \quad \kappa = 2, \quad \frac{G(0)}{\bar{Y}} = 20\%,$$

We have:

$$\delta\beta = \delta \cdot 2^{\alpha}\kappa^{1-\alpha} = \frac{1}{8}, \qquad 1 - (1-\tau)(1-s) = \frac{7}{16}, \qquad \frac{G(0)}{\bar{Y}} = \frac{1}{5} = 20\%$$

Two regimes. The condition for secular stagnation is:

$$\delta\beta \leq 1 - (1-\tau)(1-s) + \frac{(1-s)T(0) - G}{\bar{Y}}$$

This is obtained for high enough taxes (which decrease aggregate demand):

$$\begin{aligned}
\frac{T(0)}{\bar{Y}} &\geq \frac{1}{1-s}\left((1-\tau)(1-s) + \delta\beta - 1 + \frac{G}{\bar{Y}}\right) \\
&\geq \frac{4}{3}\cdot\left(-\frac{7}{16} + \frac{1}{8} + \frac{1}{5}\right) \\
&\geq \frac{4}{3}\cdot\frac{-7\cdot 5 + 10 + 16}{80} \\
\frac{T(0)}{\bar{Y}} &\geq 15\%
\end{aligned}$$

Secular stagnation regime. If $T(0)/\bar{Y} \geq 0.15$, then the secular stagnation expression applies:

$$\begin{aligned}
\frac{Y}{\bar{Y}} &= \frac{1}{1-(1-\tau)(1-s)-\delta\beta}\frac{G-(1-s)T(0)}{\bar{Y}} \\
&= \frac{1}{\frac{7}{16} - \frac{1}{8}}\left(\frac{1}{5} - \frac{3}{4}\frac{T(0)}{\bar{Y}}\right) \\
&= \frac{16}{5}\left(\frac{1}{5} - \frac{3}{4}\frac{T(0)}{\bar{Y}}\right) \\
\frac{Y}{\bar{Y}} &= -2.4\frac{T(0)}{\bar{Y}} + 64\%
\end{aligned}$$

Neoclassical regime. If $T(0)/\bar{Y} \leq 0.15$, then we are in the neoclassical regime:

$$\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha}$$
$$\text{where } \frac{k}{\kappa} \text{ solves:} \quad \frac{7}{16}\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2}\right)^{\alpha} - \frac{1}{8}\frac{k}{\kappa} - \frac{1}{5} + \frac{3}{4}\frac{T(0)}{\bar{Y}} = 0$$

B.1.3 Paradox of thrift

Given the stated parameter values:

$$\alpha = \frac{1}{3}, \quad s = \frac{1}{4}, \quad \tau = \frac{1}{4}, \quad \delta = \frac{1}{16}, \quad \kappa = 2, \quad \frac{G(0)}{\bar{Y}} = 20\%, \quad -\frac{T(0)}{\bar{Y}} = 15\%,$$

We have:

$$\delta\beta = \delta \cdot 2^{\alpha}\kappa^{1-\alpha} = \frac{1}{8}, \qquad 1-(1-\tau)(1-s) = \frac{7}{16}, \qquad \frac{G(0)}{\bar{Y}} = \frac{1}{5} = 20\%, \qquad -\frac{T(0)}{\bar{Y}} = \frac{3}{20} = 15\%$$

Secular stagnation regime. Output is given by:

$$\begin{aligned}
\frac{Y}{\bar{Y}} &= \frac{1}{1-(1-\tau)(1-s)-\delta\beta}\frac{G-(1-s)T(0)}{\bar{Y}} \\
&= \frac{1}{1-\frac{3}{4}(1-s)-\frac{1}{8}}\left(\frac{1}{5}+(1-s)\frac{3}{20}\right) \\
&= \frac{8}{1+6s}\left(\frac{7}{20}-\frac{3}{20}s\right) \\
\frac{Y}{\bar{Y}} &= \frac{2}{5}\cdot\frac{7-3s}{1+6s}
\end{aligned}$$

Neoclassical regime. In the neoclassical regime:

$$\begin{aligned}
&\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa}-\frac{k^2}{\kappa^2}\right)^{\alpha} \\
&\text{where } \frac{k}{\kappa} \text{ solves:} \quad \left(\frac{1}{4}+\frac{3}{4}s\right)\left(2\frac{k}{\kappa}-\frac{k^2}{\kappa^2}\right)^{\alpha} - \frac{1}{8}\frac{k}{\kappa} - \frac{1}{5} - \frac{3}{20}(1-s) = 0
\end{aligned}$$

B.2 Heterogeneous Agents

B.2.1 Increase in inequality

I consider ex-ante revenue neutral redistribution, one which preserves total tax receipts equal to $\langle T(0)\rangle$. I denote the difference between the tax that high income pay $T_H(0)$ and the tax that low income pay $T_L(0)$ by Δ. Therefore, $T_H(0)$ and $T_L(0)$ solve:

$$\begin{aligned}
T_H(0) - T_L(0) &= \Delta \\
(1-\lambda)T_H(0) + \lambda T_L(0) &= \langle T(0)\rangle
\end{aligned}$$

The solution to this system is:

$$\begin{cases} T_L(0) = \langle T(0)\rangle - (1-\lambda)\Delta \\ T_H(0) = \langle T(0)\rangle + \lambda\Delta \end{cases}$$

This allows to compute $\langle(1-s)T(0)\rangle$:

$$\begin{aligned}
\langle(1-s)T(0)\rangle &= \lambda(1-s_L)T_L(0) + (1-\lambda)(1-s_H)T_H(0) \\
&= \lambda(1-s_L)\left[\langle T(0)\rangle - (1-\lambda)\Delta\right] + (1-\lambda)(1-s_H)\left[\langle T(0)\rangle + \lambda\Delta\right] \\
\langle(1-s)T(0)\rangle &= \langle 1-s\rangle \cdot \langle T(0)\rangle - \lambda(1-\lambda)\left(s_H - s_L\right)\Delta
\end{aligned}$$

Secular stagnation output is:

$$\frac{Y}{\bar{Y}} = \frac{1}{1-\langle(1-s)(1-\tau)\rangle_y - \delta\beta}\frac{G-\langle(1-s)T(0)\rangle}{\bar{Y}}.$$

Therefore:

$$\frac{Y}{\bar{Y}} = \frac{1}{1 - \langle (1-s)(1-\tau) \rangle_y - \delta\beta} \frac{G - \langle 1-s \rangle \cdot \langle T(0) \rangle + \lambda(1-\lambda)\left(s_H - s_L\right)\Delta}{\bar{Y}}.$$

This expression shows that in the secular stagnation regime, output is all the greater when the difference between the baseline taxes that high income and low income earners pay is high. The higher the difference in saving rate between high and low income $s_H - s_L$, the more redistribution has powerful effects.

B.2.2 Redistribution from high income to low income

Given the stated parameter values:

$$\begin{aligned} &\tau_L = \tau_H = \frac{1}{4}, \quad s_L = 0, \quad s_H = \frac{1}{2}, \quad \alpha = \frac{1}{3}, \quad \delta = \frac{1}{16}, \\ &\kappa = 2, \quad \frac{G(0)}{\bar{Y}} = 20\%, \quad -\frac{\langle T(0) \rangle}{\bar{Y}} = 10\% \quad \gamma = 9, \quad \lambda = \frac{9}{10}, \end{aligned}$$

We have:

$$\lambda(1-\lambda)\left(s_H - s_L\right) = \frac{9}{200}$$

The unweighted marginal propensity to consume is:

$$\langle 1-s \rangle = \frac{1}{10} \cdot \frac{1}{2} + \frac{9}{10} = \frac{19}{20}.$$

Therefore:

$$-\frac{\langle 1-s \rangle \langle T(0) \rangle}{\bar{Y}} = \frac{19}{20}\frac{1}{10} = \frac{19}{200} = 9.5\%.$$

Note that even though the income-weighted propensities to consume are equal to the representative agent case, the corresponding fiscal impulse is lower than in the redistributive case where it was 15%.

The weighted marginal propensity to consume is:

$$\langle 1-s \rangle_y = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} = \frac{3}{4}.$$

Therefore:

$$1 - \langle (1-s)(1-\tau) \rangle_y = 1 - (1-\tau) \cdot \langle 1-s \rangle_y = \frac{7}{16}$$

Moreover:

$$\delta\beta = \delta \cdot 2^{\alpha}\kappa^{1-\alpha} = \frac{1}{8}, \qquad \tau = \frac{1}{4}, \qquad \frac{G(0)}{\bar{Y}} = \frac{1}{5} = 20\%.$$

Two regimes. The condition for secular stagnation is:

$$\langle 1 - (1-s)(1-\tau) \rangle_y + \frac{\langle 1-s \rangle \cdot \langle T(0) \rangle - \lambda(1-\lambda)\left(s_H - s_L\right)\Delta - G}{\bar{Y}} \geq \delta\beta.$$

This is equivalent to:

$$\frac{\Delta}{\bar{Y}} \leq \frac{1}{\lambda(1-\lambda)\left(s_H - s_L\right)} \left(\langle 1 - (1-s)(1-\tau) \rangle_y - \delta\beta + \frac{\langle 1-s \rangle \cdot \langle T(0) \rangle - G}{\bar{Y}} \right)$$

This is obtained for low enough redistribution (since redistribution increases aggregate demand):

$$
\begin{aligned}
\frac{\Delta}{\bar{Y}} &\leq \frac{1}{\lambda(1-\lambda)\left(s_H - s_L\right)} \left(\langle 1-(1-s)(1-\tau)\rangle_y - \delta\beta + \frac{\langle 1-s\rangle \cdot \langle T(0)\rangle - G}{\bar{Y}} \right) \\
&\leq \frac{1}{\frac{1}{10}\cdot\frac{9}{10}\cdot\frac{1}{2}} \cdot \left(\frac{7}{16} - \frac{1}{8} - \frac{1}{5} - \frac{19}{200} \right) \\
&\leq \frac{200}{9} \cdot \frac{7\cdot 25 - 50 - 80 - 38}{400} \\
\frac{\Delta}{\bar{Y}} &\leq \frac{7}{18} = 38.8\%
\end{aligned}
$$

Secular stagnation regime. If $\Delta/\bar{Y} \leq 38.8\%$, then the secular stagnation expression applies:

$$
\begin{aligned}
\frac{Y}{\bar{Y}} &== \frac{1}{1 - \langle (1-s)(1-\tau)\rangle_y - \delta\beta} \frac{G - \langle 1-s\rangle \cdot \langle T(0)\rangle + \lambda(1-\lambda)\left(s_H - s_L\right)\Delta}{\bar{Y}} \\
&= \frac{1}{\frac{7}{16} - \frac{1}{8}} \left(\frac{1}{5} + \frac{19}{200} + \frac{1}{10}\cdot\frac{9}{10}\cdot\frac{1}{2}\frac{\Delta}{\bar{Y}} \right) \\
&= \frac{16}{5}\left(\frac{59}{200} + \frac{9}{200}\frac{\Delta}{\bar{Y}} \right) \\
\frac{Y}{\bar{Y}} &= \frac{944}{1000} + \frac{144}{1000} \cdot \frac{\Delta}{\bar{Y}}
\end{aligned}
$$

Neoclassical regime. If $\Delta/\bar{Y} \geq 38.8\%$, then we are in the neoclassical regime:

$$
\begin{aligned}
&\frac{Y}{\bar{Y}} = \left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2} \right)^{\alpha} \\
&\text{where } \frac{k}{\kappa} \text{ solves:} \quad \frac{1}{8}\frac{k}{\kappa} = \frac{7}{16}\left(2\frac{k}{\kappa} - \frac{k^2}{\kappa^2} \right)^{\alpha} - \frac{59}{200} - \frac{9}{200}\frac{\Delta}{\bar{Y}}.
\end{aligned}
$$

C Figures

C.1 Permanent Effects of Aggregate Demand Shocks

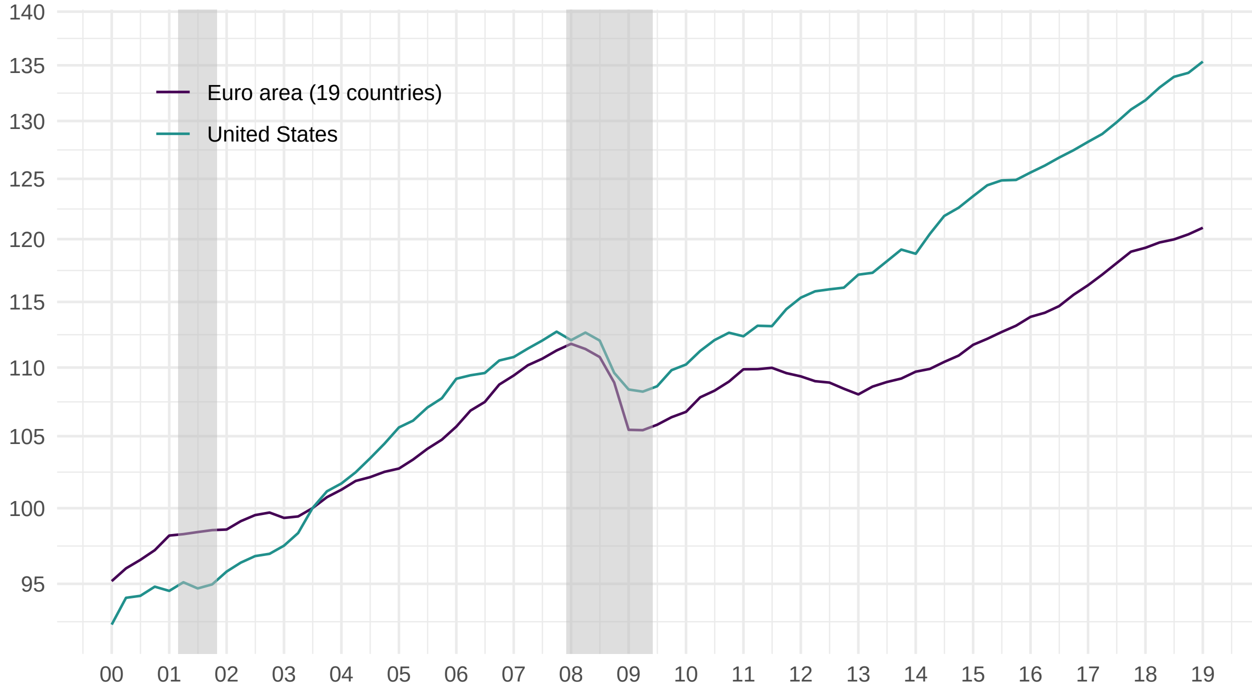

Figure 14: Real GDP in the Euro Area (19 countries) and the U.S.

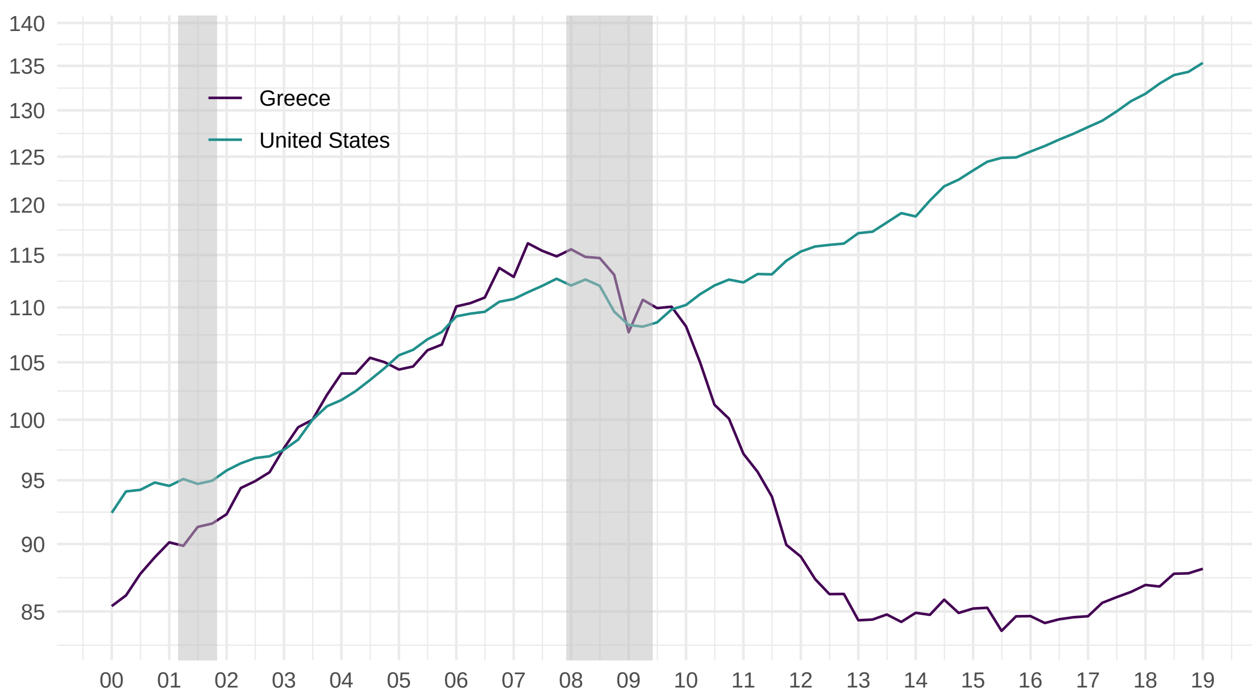

Figure 15: Real GDP in Greece and the U.S.

C.2 U.S. Growth Slowdown

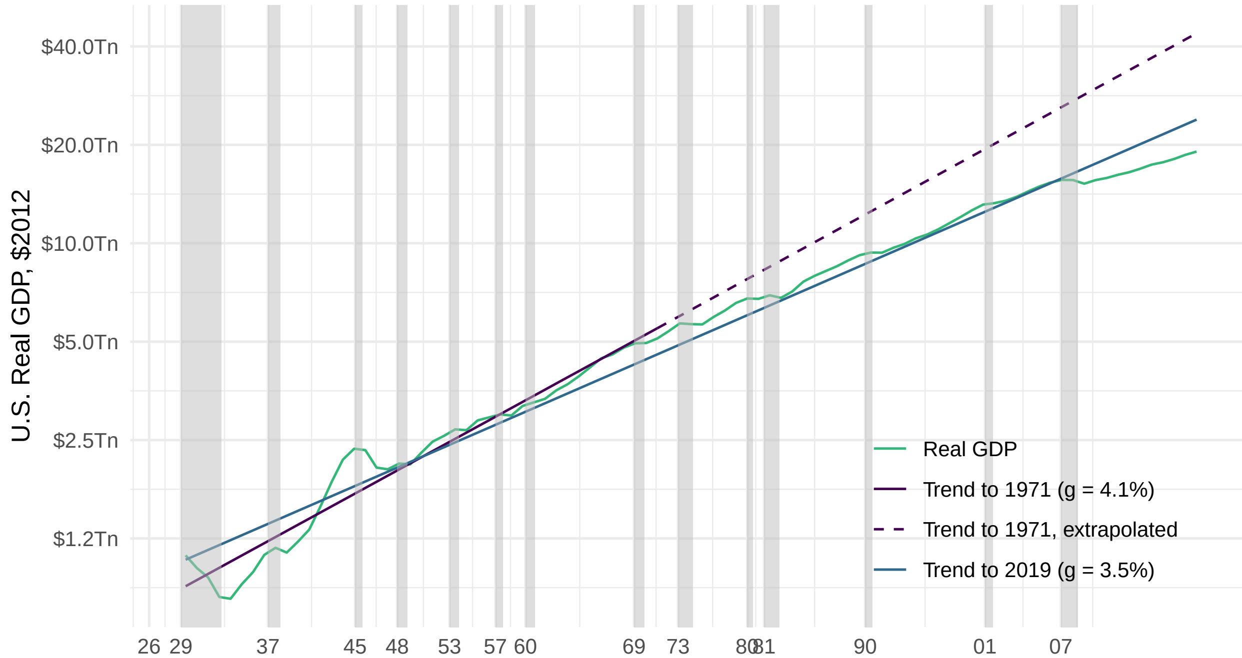

Figure 16: U.S. Real GDP Trends (1929-2019) - Log Scale.

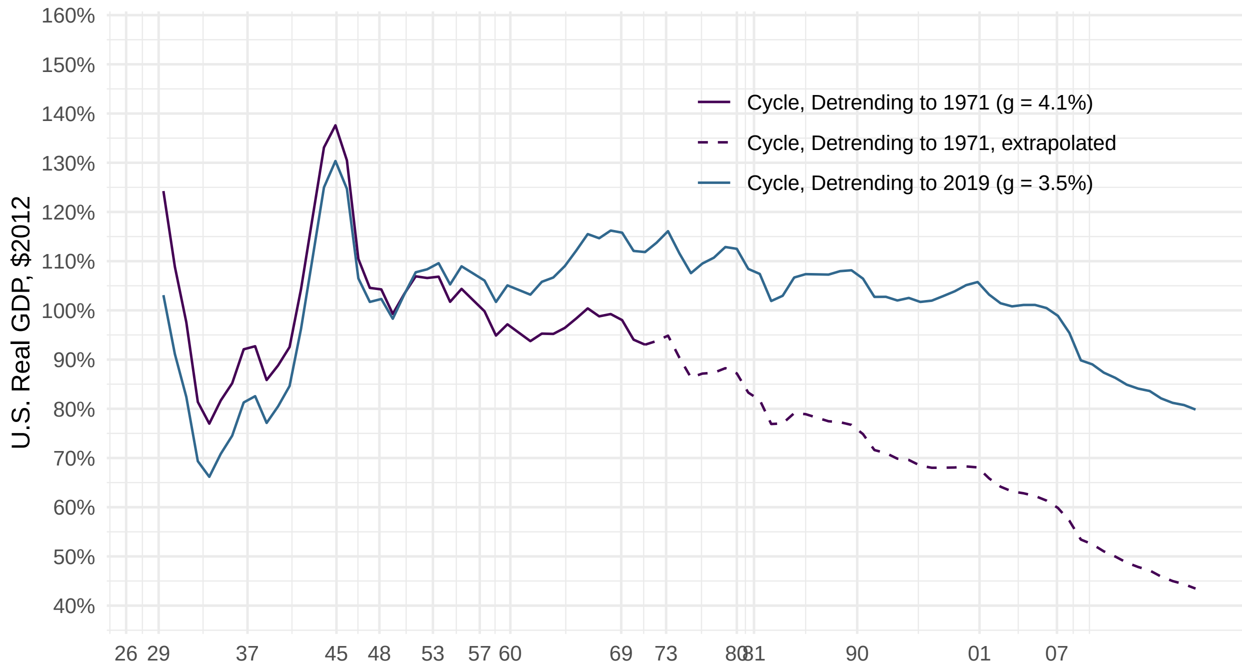

Figure 17: U.S. Real GDP Cycles (1929-2019).

C.3 Long-Term Effects of Tax Increases (Romer and Romer (2010))

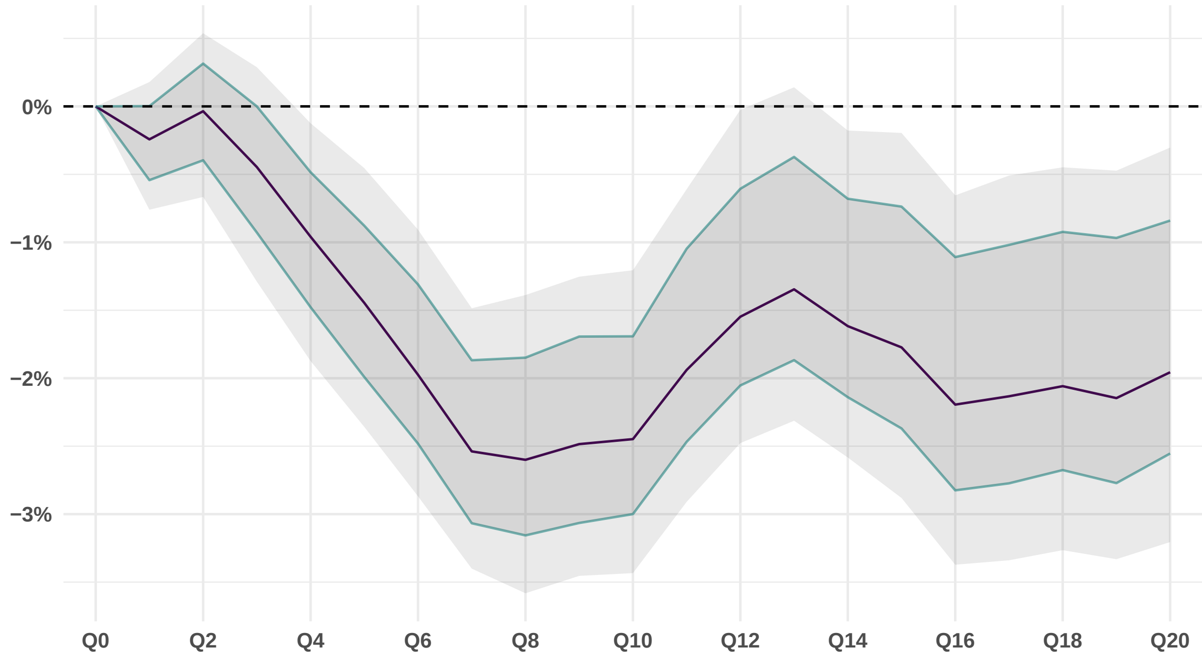

Figure 18: GDP (Real) after a 1% of GDP increase in taxes (Romer and Romer (2010))

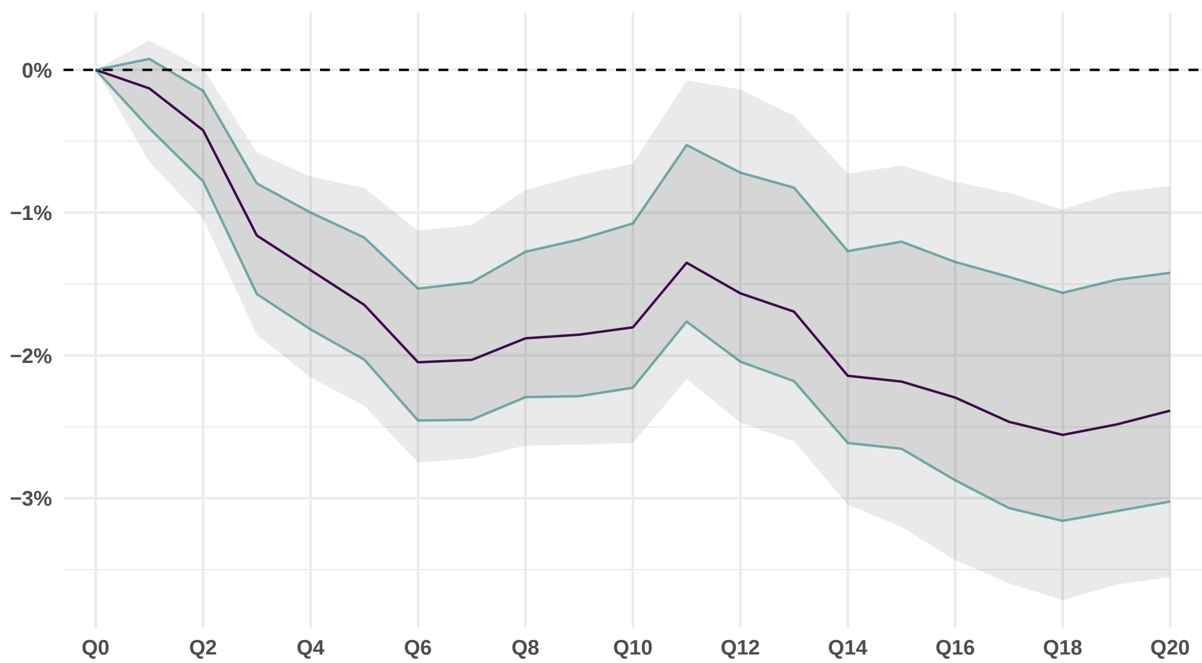

Figure 19: Personal Consumption Expenditures (Real) after a 1% of GDP increase in taxes (Romer and Romer (2010))

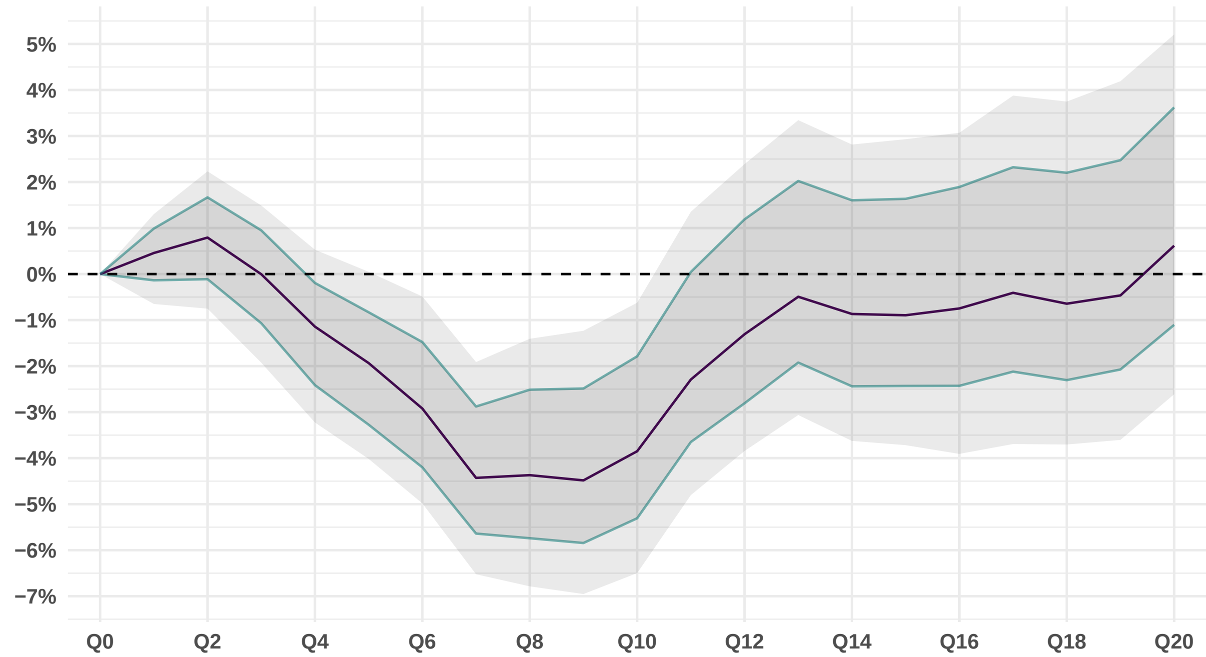

Figure 20: Investment (Real) after a 1% of GDP increase in taxes (Romer and Romer (2010))

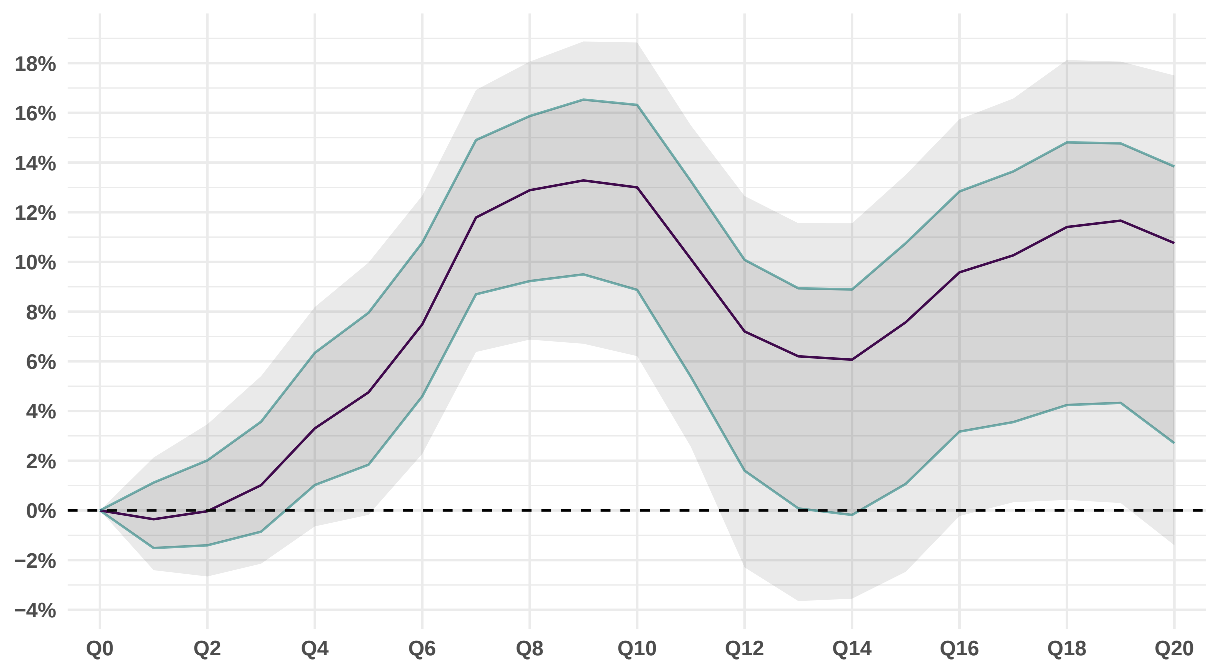

Figure 21: Unemployment Rate after a 1% of GDP increase in taxes (Romer and Romer (2010))

C.4 Real Effects of Monetary Policy Shocks (Romer and Romer (2004))

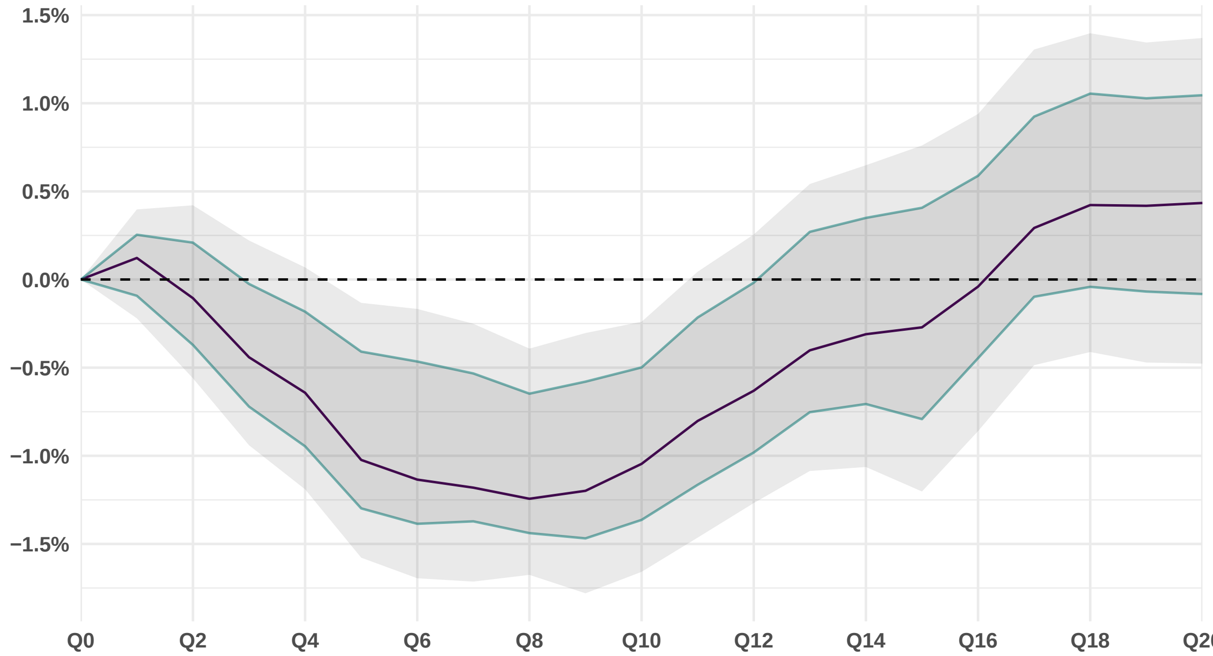

Figure 22: GDP (Real) after a 1% increase in the Fed Funds Rate (Romer and Romer (2004))

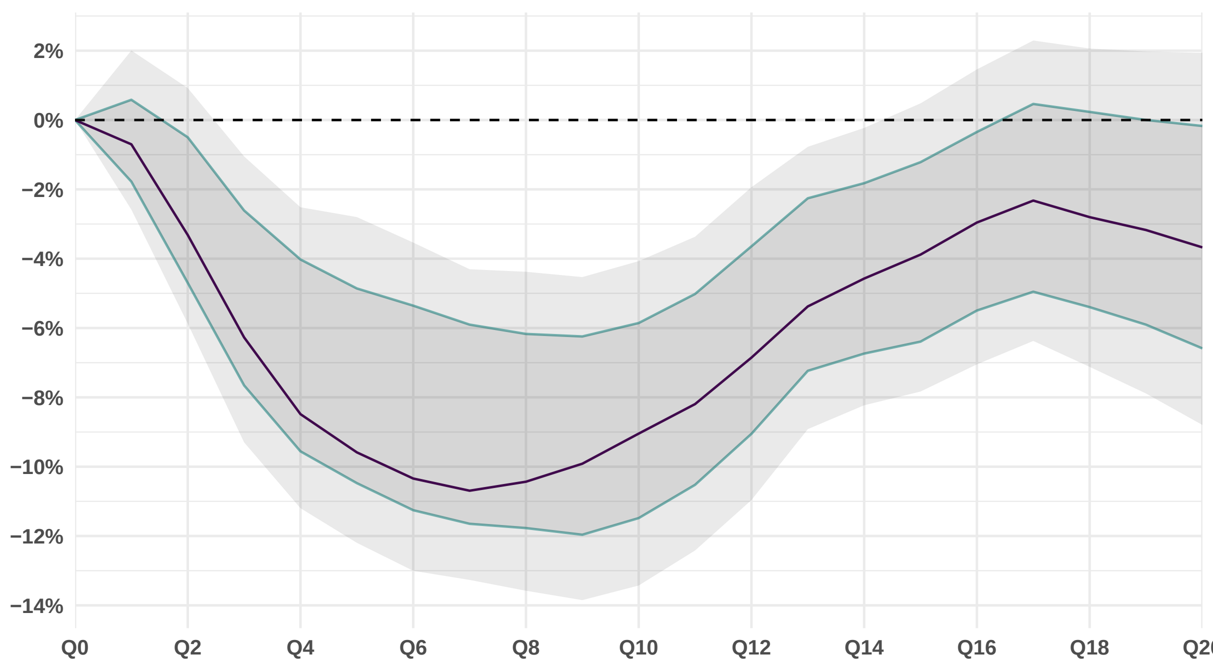

Figure 23: Production of Total Construction

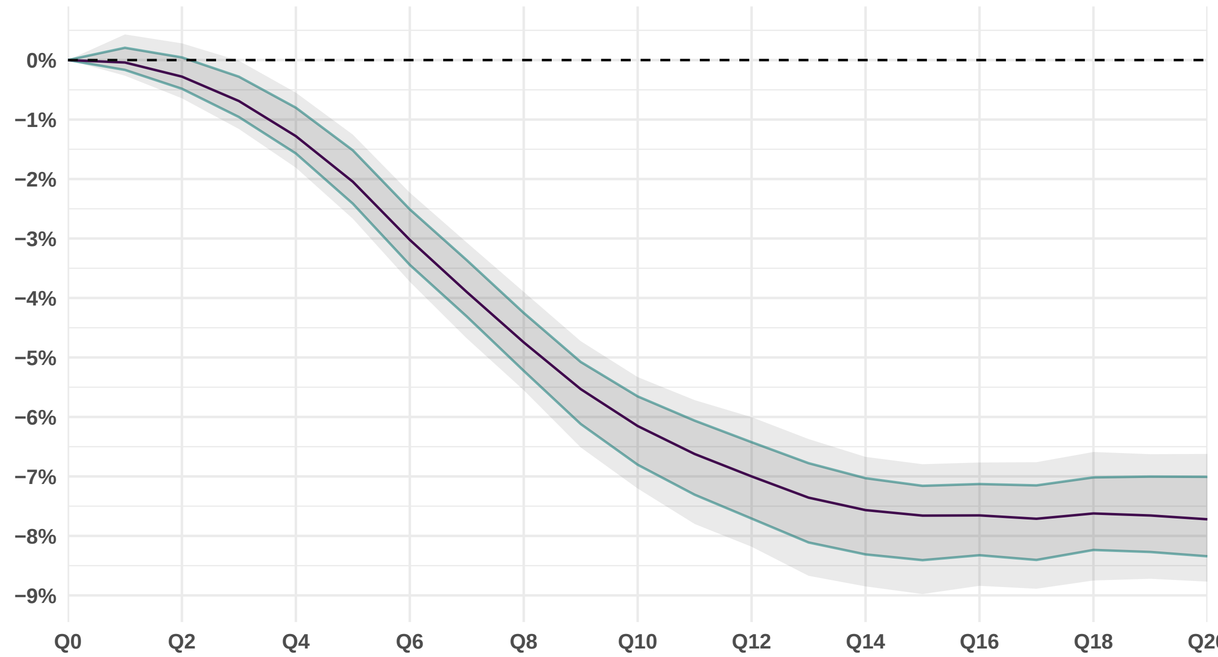

Figure 24: House Prices

C.5 Saving and Wealth Inequality

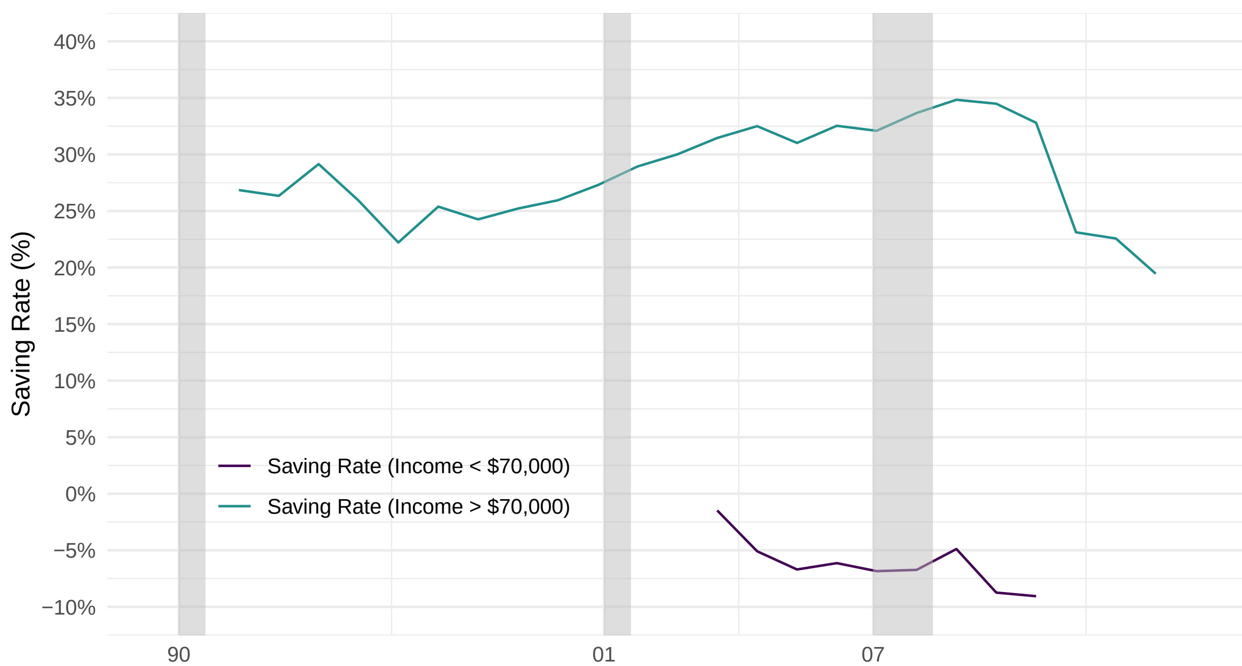

Figure 25: Saving by Income Class. Source: CEX

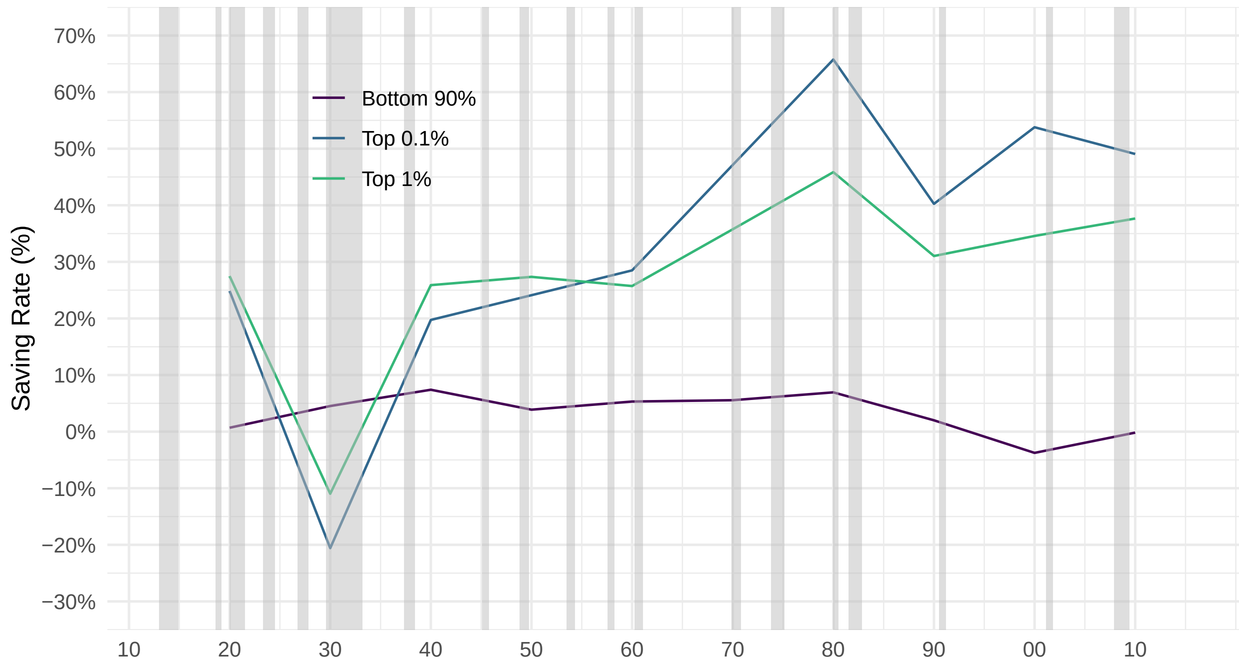

Figure 26: Saving by Wealth Class. Source: Saez, Zucman (2016)

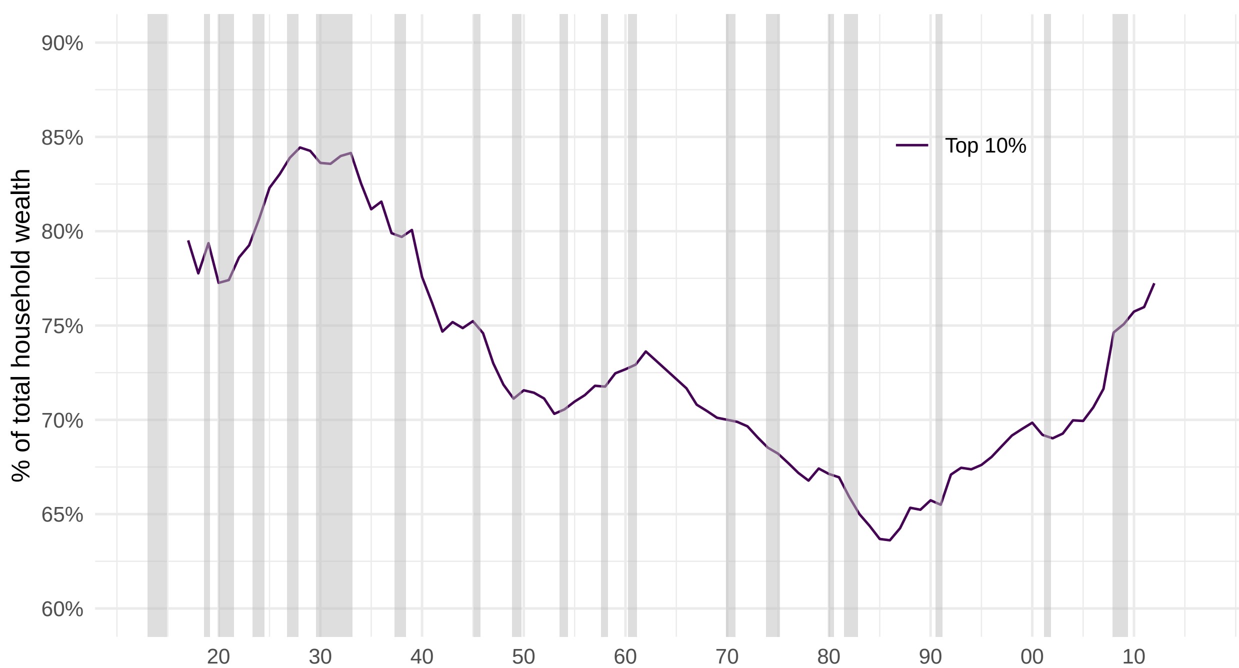

Figure 27: Top 10% wealth share. Source: Saez, Zucman (2016)

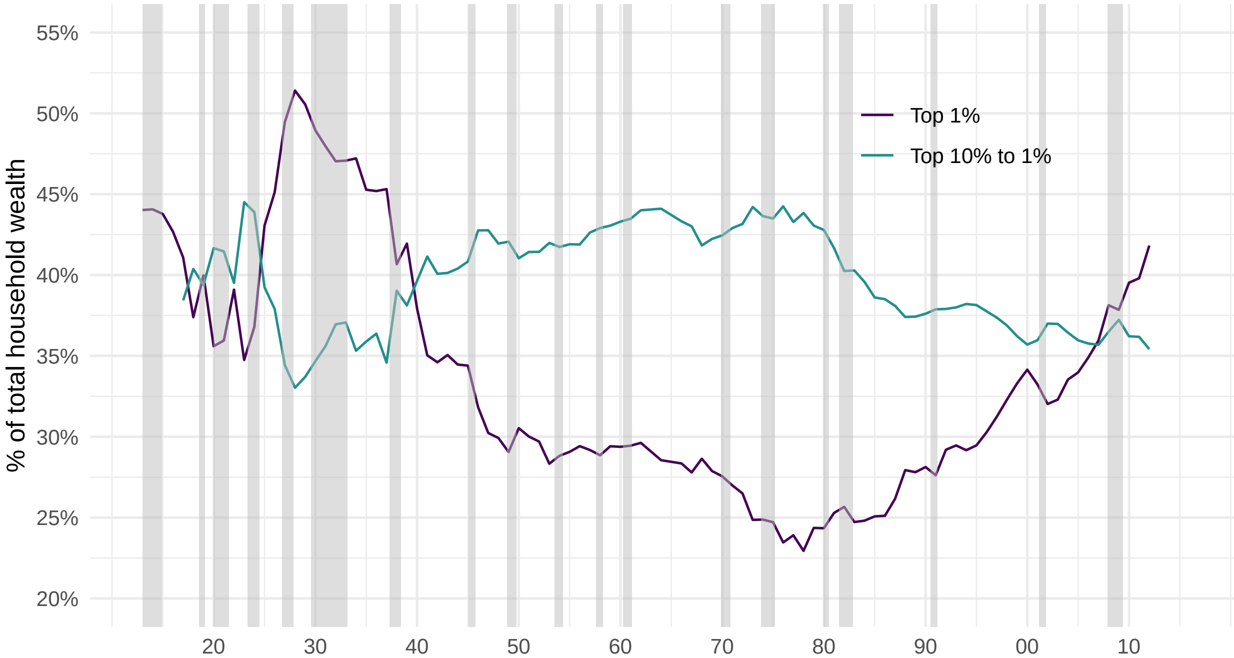

Figure 28: Top 1-10% and Top 1% wealth share. Source: Saez, Zucman (2016)

D Tables

D.1 Consumption

Table 1: Consumption for the 9th and 10th decile, Consumer Expenditure Survey (2017)

	9th 10%	High 10%
Total average annual expenditures	$ 87,432	$136,873
Food	$ 10,328	$ 14,692
Alcoholic beverages	$ 785	$ 1,378
Housing	$ 26,719	$ 40,547
Apparel and services	$ 2,526	$ 4,493
Transportation	$ 14,495	$ 17,724
Healthcare	$ 6,772	$ 8,577
Entertainment	$ 4,604	$ 7,165
Personal care products and services	$ 1,085	$ 1,643
Reading	$ 157	$ 300
Education	$ 2,097	$ 5,104
Tobacco products and smoking supplies	$ 386	$ 219
Miscellaneous expenditures	$ 1,462	$ 2,031
Cash contributions	$ 2,739	$ 7,711
Personal insurance and pensions	$ 13,278	$ 25,290
Income after taxes	$108,743	$205,391

E (NOT FOR PUBLICATION) Excess saving in the history of economic thought

In this appendix, I aim to show that the idea of excess saving, and the paradox of thrift, has a long tradition in the history of economic thought.

E.1 Old Testament: Book of Proverbs

Perhaps one of the oldest reference to potential excess saving can be found in the Book of Proverbs 11:24:

> There is that scattereth, and yet increaseth; and there is that withholdeth more than is meet, but it tendeth to poverty.

E.2 Mandeville (1732)

Similarly, Bernard Mandeville in *The Fable of the Bees: or, Private Vices, Public Benefits*, published in 1714, expresses the idea of the paradox of thrift. Collective efforts to save more can be self-defeating:

> As this prudent economy, which some people call Saving, is in private families the most certain method to increase an estate, so some imagine that, whether a country be barren or fruitful, the same method if generally pursued (which they think practicable) will have the same effect upon a whole nation, and that, for example, the English might be much richer than they are, if they would be as frugal as some of their neighbours. This, I think, is an error.

As Keynes (1936) explains in chapter 23 "Note on Mercantilism, the usury laws, stamped money and theories of under-consumption" of the *General Theory*, the "fear of goods" was a very important aspect of mercantilist thinking, against which classical authors were writing:

> It is impossible to study the notions to which the mercantilists were led by their actual experiences, without perceiving that there has been a chronic tendency throughout human history for the propensity to save to be stronger than the inducement to invest. The weakness of the inducement to invest has been at all times the key to the economic problem. To-day the explanation of the weakness of this inducement may chiefly lie in the extent of existing accumulations; whereas, formerly, risks and hazards of all kinds may have played a larger part. But the result is the same. The desire of, the individual to augment his personal wealth by abstaining from consumption has usually been stronger than the inducement to the entrepreneur to augment the national wealth by employing labour on the construction of durable assets.

E.3 Smith (1776)

On the contrary, classical thinkers were very much against this view that there might be a problem of excess saving. In Chapter 3 "Of the accumulation of capital, or of productive and unproductive labour." of the Wealth of Nations, Adam Smith writes:

> Capitals are increased by parsimony, and diminished by prodigality and misconduct.
>
> What is annually saved, is as regularly consumed as what is annually spent, and nearly in the same time too : but it is consumed by a different set of people. That portion of his revenue which a rich man annually spends, is, in most cases, consumed by idle guests and menial servants, who leave nothing behind them in return for their consumption. That portion which he annually saves, as, for the sake of the profit, it is immediately employed as a capital, is consumed in the same manner, and nearly in the same time too, but by a different set of people: by labourers, manufacturers, and artificers, who reproduce, with a profit, the value of their annual consumption. His revenue, we shall suppose, is paid him in money. Had he spent the whole, the food, clothing, and lodging, which the whole could have purchased, would have been distributed among the former set of people. By saving a part of it, as that part is, for the sake of the profit, immediately employed as a capital, either by himself or by some other person, the food, clothing, and lodging,

> which may be purchased with it, are necessarily reserved for the latter. The consumption is the same, but the consumers are different.

Keynes (1936) comments:

> The extraordinary achievement of the classical theory was to overcome the beliefs of the 'natural man' and, at the same time, to be wrong. (...) I remember Bonar Law's mingled rage and perplexity in face of the economists, because they were denying what was obvious. He was deeply troubled for an explanation. One recurs to the analogy between the sway of the classical school of economic theory and that of certain religions. For it is a far greater exercise of the potency of an idea to exorcise the obvious than to introduce into men's common notions the recondite and the remote.

E.4 Malthus (1820)

Malthus (1820)

> Adam Smith has stated that capitals are increased by parsimony, that every frugal man is a public benefactor, and that the increase of wealth depends upon the balance of produce above consumption. That these propositions are true to a great extent is perfectly unquestionable... But it is quite obvious that they are not true to an indefinite extent, and that the principles of saving, pushed to excess, would destroy the motive to production. If every person were satisfied with the simplest food, the poorest clothing, and the meanest houses, it is certain that no other sort of food, clothing, and lodging would be in existence.

Thomas Malthus was wondering why unemployment could occur, and he was inspired by events surrounding the post-Napoleonic wars period, during which time industrial depression in Britain was causing serious unemployment of labor and capital:

> While it is quite certain that an adequate passion for consumption may fully keep up the proper proportion between supply and demand, whatever may be the powers of production, it appears to be quite as certain that a passion for accumulation must inevitably lead to a supply of commodities beyond what the structure and habits of such a society will permit to be consumed.

E.5 Marx (1867); Capital Vol. III Part V

Marx (1867) also has elements of thinking about excess saving:

> The ultimate reason for all real crises always remains the poverty and restricted consumption of the masses as opposed to the drive of capitalist production to develop the productive forces as though only the absolute consuming power of society constituted their limit.

In a market economy overproduction refers only to what can be profitably sold. Marx explained:

> The English, for example, are forced to lend their capital to other countries in order to create a market for their commodities.
>
> Overproduction, the credit system, etc., are means by which capitalist production seeks to break through its own barriers and to produce over and above its own limits... Hence crises arise, which simultaneously drive it onward and beyond [its own limits] and force it to put on seven-league boots, in order to reach a development of the productive forces which could only be achieved very slowly within its own limits.

E.6 Engels (1878) - Engels, Anti-Dühring

Engels (1878):

> The underconsumption of the masses, the restriction of the consumption of the masses to what is necessary for their maintenance and reproduction, is not a new phenomenon. It has existed as

> long as there have been exploiting and exploited classes. The underconsumption of the masses is a necessary condition of all forms of society based on exploitation, consequently also of the capitalist form; but it is the capitalist form of production which first gives rise to crises. The underconsumption of the masses is therefore also a prerequisite condition for crises, and plays in them a role which has long been recognised. But it tells us just as little why crises exist today as why they did not exist before.

E.7 Crocker and Macvane (1887)

Crocker and Macvane (1887):

> It has happened that, in recent years, this principle, so generally accepted by the learned, has apparently been as generally contradicted by the every-day experience of practical men. During the last twelve or fifteen years, business men have, almost without exception, complained that, so far at least as the particular business of each was concerned, there has been an actual overproduction; and, unless a majority of these men have been mistaken as to the proportion of demand to production in their own specialties, general overproduction must have been, in spite of the theories of the economists, an actual existing fact. In this conflict of theory with apparent fact, it becomes important carefully to test the theory, in order to see whether it is based on sound reasoning.

E.8 Hansen (1939)

Hansen (1939):

> This view was inspired by a century in which the forces of economic progress were powerful and strong, in which investment outlets were numerous and alluring. Spiethoff saw clearly that technological progress, the development of new industries, the discovery of new resources, the opening of new territory were the basic causes of the boom, which in turn was the progenitor of depression.
>
> The expanding economy of the last century called forth a prodigious growth of capital formation. So much was this the case, that this era in history has by common consent been called the capitalistic period. No one disputes the thesis that without this vast accumulation of capital we should never have witnessed the great rise in the standard of living achieved since the beginning of the Industrial Revolution.
>
> Consumption may be strengthened by the relief from taxes which drain off a stream of income which otherwise would flow into consumption channels. Public investment may usefully be made in human and natural resources and in consumers'capital goods of a collective character designed to serve the physical, recreational and cultural needs of the community as a whole.

Very importantly:

> Thus we may postulate a consensus on the thesis that in the absence of a positive program designed to stimulate consumption, full employment of the productive resources is essentially a function of the vigor of investment activity. Less agreement can be claimed for the role played by the rate of interest on the volume of investment. Yet few there are who believe that in a period of investment stagnation an abundance of loanable funds at low rates of interest is alone adequate to produce a vigorous flow of real investment. I am increasingly impressed with the analysis made by Wicksell who stressed the prospective rate of profit on new investment as the active, dominant, and controlling factor, and who viewed the rate of interest as a passive factor, lagging behind the profit rate. This view is moreover in accord with competent business judgment." It is true that it is necessary to look beyond the mere cost of interest charges to the indirect effect of the interest rate structureupon business expectations. Yet all in all, I venture to assert that the role of the rate of interest as a determinant of investment has occupied a place larger than it deserves in our thinking. If this be granted, we are forced to regard the factors which underlie economic progress as the dominant determinants of investment and employment.

Deepening versus Widening of capital:

> A growth in real investment may take the form either of a deepening of capital or of a widening of capital, as Hawtrey has aptly put it. The deepening process means that more capital is used per unit of output, while the widening process means that capital formation grows pari-passu with the increase in the output of final goods. If the ratio of real capital to real income remains constant, there is no deepening of capital; but if this ratio is constant and real income rises, then there is a widening of capital.
>
> According to Douglas, the growth of real capital formation in England from 1875 to 1909 proceeded at an average rate of two per cent per annum; and the rate of growth of capital formation in the United States from 1890 to 1922 was four per cent per annum. The former is less than the probable rate of increase of output in England, while the latter is somewhat in excess of the annual rise of production in the United States. Thus, during the last fifty years or more, capital formation for each economy as a whole has apparently consisted mainly of a widening of capital. Surprising as it may seem, as far as we may judge from such data as are available, there has been little, if any, deepening of capital. The capital stock has increased approximately in proportion to real income. This is also the conclusion of Gustav Cassel; while Keynes thinks that real capital formation in England may have very slightly exceeded the rise in real income in the period from 1860 to the World War. If this be true, it follows that, in terms of the time element in production, which is the very essence of the capital concept, our system of production is little more capitalistic now than fifty or seventy-five years ago.

A big component of investment demand results from building residential structures:

> Now the rate of population growth must necessarily play an important role in determining the character of the output; in other words, the composition of the flow of final goods. Thus a rapidly growing population will demand a much larger per capita volume of new residential building construction than will a stationary population. A stationary population with its larger proportion of old people may perhaps demand more personal services; and the composition of consumer demand will have an important influence on the quantity of capital required. The demand for housing calls for large capital outlays, while the demand for personal services can be met without making large investment expenditures. It is therefore not unlikely that a shift from a rapidly growing population to a stationary or declining one may so alter the composition of the final flow of consumption goods that the ratio of capital to output as a whole will tend to decline.
>
> An interesting problem for statistical research would be to determine the proportion of investment in the nineteenth century which could be attributed (a) to population growth, (b) to the opening up of new territory and the discovery of new resources, and (c) to technical innovations.
>
> These figures, while only suggestive, point unmistakably to the conclusion that the opening of new territory and the growth of population were together responsible for a very large fraction - possibly somewhere near one-half-of the total volume of new capital formation in the nineteenth century. These outlets for new investment are rapidly being closed.
>
> We are thus rapidly entering a world in which we must fall back upon a more rapid advance of technology than in the past if we are to find private investment opportunities adequate to maintain full employment. Should we accept the advice of those who would declare a moratorium on invention and technical progress, this one remaining avenue for private investment would also be closed. There can be no greater error in the analysis of the economic trends of our times than that which finds in the advance of technology, broadly conceived,a major cause of unemployment. It is true that we cannot discount the problem of technological unemployment, a problem which may be intensified by the apparently growing importance of capital-saving inventions. But, on the other side, we cannot afford to neglect that type of innovation which creates new industries and which thereby opens new outlets for real investment. The problem of our generation is, above all, the problem of inadequate private investment outlets. What we need is not a slowing down in the progress of science and technology, but rather an acceleration of that rate.

Of first-rate importance is the development of new industries. There is certainly no basis for the assumption that these are a thing of the past. But there is equally no basis for the assumption that we can take for granted the rapid emergence of new industries as rich in investment opportunities as the railroad, or more recently the automobile, together with all the related developments, including the construction of public roads, to which it gave rise.

Can any tax system, designed to increase the propensity to consume by means of a drastic change in income distribution, be devised which will not progressively encroach on private investment?

Totalitarian states have the great advantage that they can rigorously check the advance of costs, including wage rates, while engaging in an expansionist program of public investment. Democratic countries cannot in modern times escape from the influence exerted by organized groups upon the operation of the price system.

E.9 Sweezy (1940)

After a century of neglect the problem of long-run investment opportunity is once more in the center of economic controversy. To the classical economists it was a matter of very real concern. With the continuing expansion and prosperity of the nineteenth century, however, apprehensions about an eventual exhaustion of profitable investment opportunities faded into the background. Economists, in effect, stopped asking whether there might be an outer limit to the process of capital accumulation.

All of these factors tend to diminish investment opportunity in a period when the potential supply of investment-seeking funds is greater than ever.

E.10 Eccles (1951)

According to Marriner S. Eccles, who was the Federal Reserve Chairman from 1934 to 1948, the reason why so much debt was taken on in the period leading up to the Great Depression was too much concentration of wealth:

As mass production has to be accompanied by mass consumption; mass consumption, in turn, implies a distribution of wealth — not of existing wealth, but of wealth as it is currently produced — to provide men with buying power equal to the amount of goods and services offered by the nation's economic machinery. Instead of achieving that kind of distribution, a giant suction pump had by 1929-30 drawn into a few hands an increasing portion of currently produced wealth. This served them as capital accumulations. But by taking purchasing power out of the hands of mass consumers, the savers denied to themselves the kind of effective demand for their products that would justify a reinvestment of their capital accumulations in new plants. In consequence, as in a poker game where the chips were concentrated in fewer and fewer hands, the other fellows could stay in the game only by borrowing. When their credit ran out, the game stopped.

That is what happened to us in the twenties. We sustained high levels of employment in that period with the aid of an exceptional expansion of debt outside of the banking system. This debt was provided by the large growth of business savings as well as savings by individuals, particularly in the upper-income groups where taxes were relatively low. Private debt outside of the banking system increased about fifty per cent. This debt, which was at high interest rates, largely took the form of mortgage debt on housing, office, and hotel structures, consumer installment debt, brokers' loans and foreign debt.

The stimulation to spending by debt-creation of this sort was short lived and could not be counted on to sustain high levels of employment for long periods of time. Had there been a better distribution of the current income from the national product — in other words, had there been less savings by business and the higher-income groups and more income in the lower groups — we should have had far greater stability in our economy. Had the six billion dollars, for instance, that were loaned by corporations and wealthy individuals for stock-market speculation been distributed

> to the public as lower prices or higher wages and with less profits to the corporations and the well-to-do, it would have prevented or greatly moderated the economic collapse that began at the end of 1929.

E.11 Robinson (1974)

- Joan Robinson, "What has become of the Keynesian Revolution?" pdf

  > At the time when the General Theory was being written, Keynes, projecting the situation of the slump into the future. threw out the suggestion that the need for accumulation could be overcome in thirty years of invcstment at the full-employment level, provided that wars were avoided and population ceased to grow. (He was taking an insular view. The Third World had not yet corne to mind.)
  >
  > Alvin Hansen took this up and turned it into a horror story. With the closing of the frontier in North America, there would not be sufficient outlets for the saving that capitalism generates and chronic stagnation would set in.

E.12 Summers (1991)

Before leaving academic for the policy world, Larry Summers wrote forcefully against the neoclassical synthesis, and the sticky price approach to Keynesian economics:

> Indeed, while American Keynesians condemn the 'three Eisenhower recessions', and the recessions of 1975 and 1982, as the result of excessively contractionary policies, there is no peacetime period when any consensus regarded policy as too expansionary.

E.13 Summers (2013)

He recently revived Hansen (1939)'s idea of secular stagnation:

> It is a central pillar of both classical models and Keynesian models that stabilization policy is all about fluctuations – fluctuations around a given mean – and that the achievable goal and therefore the proper objective of macroeconomic policy is to have less volatility. I wonder if a set of older and much more radical ideas that I have to say were pretty firmly rejected in 14.462, Stan, a set of older ideas that went under the phrase secular stagnation, are not profoundly important in understanding Japan's experience in the 1990s, and may not be without relevance to America's experience today.
>
> Let me say a little bit more about why I'm led to think in those terms. If you go back and you study the economy prior to the crisis, there is something a little bit odd. Many people believe that monetary policy was too easy. Everybody agrees that there was a vast amount of imprudent lending going on. Almost everybody believes that wealth, as it was experienced by households, was in excess of its reality: too much easy money, too much borrowing, too much wealth. Was there a great boom? Capacity utilization wasn't under any great pressure. Unemployment wasn't at any remarkably low level. Inflation was entirely quiescent. So, somehow, even a great bubble wasn't enough to produce any excess in aggregate demand.