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October 23, 2019
Time Limit: 1 hour 15 minutes
Student ID Number:
Signature $\qquad$

## Midterm Exam

This exam contains 11 pages (including this cover page). You can earn 100 points.

## Instructions:

1. Print your Last name, First Name, Student ID Number and Signature at the top of this page.
2. The only items which should be on your desk are pencils and/or pens. NO other items are allowed. Place any other item UNDER your desk. Calculators are NOT allowed.
3. Once the exam begins, you are not allowed to leave the room until you hand in your exam.

Good luck! Budget your time wisely! (skip the question or even the exercise if you get stuck)

Do not write below this line (Grader use only)

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 40 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| Total: | 100 |  |

## 20 Multiple Choice Questions (40 points)

1. (40 points) Each multiple choice question has only one right answer. Use the Scantron to mark your answers.
(1) (2 points) Looking back at U.S. macroeconomic performance since 1929:
A. Real GDP growth was the same before and after 1971.
B. Real GDP growth was higher after 1971.
C. Real GDP growth was lower after 1971.
(2) (2 points) Which of the following is not a component of GDP according to the "product approach"?:
A. imports
B. investment
C. government spending
D. compensation of employees
E. none of the above
(3) (2 points) In Google Sheets, which built-in function did we use in Problem Set 1 to linearize the logarithm of GDP?
A. LINEARIZE
B. LOG-LINEARIZE
C. FORECAST
D. LOG
E. EXP
(4) (2 points) If the growth rate of $y_{t}$ after $T$ periods is $G$, then the average growth rate of $y_{t}$ per period is:
A. $(1+G)^{T}-1$
B. $(1+G)^{1 / T}$
C. $(1+G)^{T}$
D. $(1+G)^{1 / T}-1$
E. none of the above
(5) (2 points) Which component of GDP is most volatile?
A. Consumption
B. Investment
C. Non-durable goods expenditures
D. Services expenditures
(6) (2 points) Select the statement that best completes the following sentence. In the overlapping generations model that we saw in class, total investment in the economy will
A. Equal savings which is mostly made up of the precautionary savings of the old.
B. Equal the total income of the young minus total consumption of the young.
C. Equal firms' profits, which they reinvest.
D. Depend on the interest rate that the young get on their savings.
(7) (2 points) In the standard Solow model seen in class, an increase in the savings rate $s$ :
A. Has a positive effect on the consumption per capita, by increasing the steady state output per capita.
B. Has a negative effect on the consumption per capita, since the share consumed from the steady state output per capita is lower.
C. Has a negative effect on the consumption per capita, since it reduces the steady state stock of capital (decreasing returns to scale)
D. Both a. and b. are correct, so we cannot know the total effect without knowing the specific values of the parameters.
(8) (2 points) In the Neoclassical labor model (where $f(l)=A l^{1-\alpha}$ ), if we change $\alpha$, we will observe the following effect on the (log) labor demand curve:
A. The curve will change its slope.
B. The curve will be shifted.
C. The curve won't suffer any change.
D. Both a. and b. are correct.
(9) (2 points) Which of the following is a feature of the U.S. labor market?
A. During recessions, the unemployment rate decreases.
B. The labor force participation rate of men has continuously increased since 1990.
C. During the Great Recession, the number of job separations has decreased.
D. The labor force participation rate of women is currently around $20 \%$.
(10) (2 points) Which of the following cannot increase long-run growth?
A. Patents.
B. Capital accumulation.
C. Government funded research.
D. Prizes
E. Privately funded research.
(11) (2 points) The five following multiple choice questions are based on the Bathtub model. In the Bathtub model, what is the law of motion for unemployment?
A. $\Delta U_{t+1}=f U_{t}-s E_{t}$
B. $\Delta U_{t+1}=s E_{t}-f U_{t}$
C. $U_{t+1}=s E_{t}-f U_{t}$
D. $\Delta U_{t+1}=f L-s E_{t}$
E. $U_{t+1}=f U_{t}-s E_{t}$
(12) (2 points) What is the steady-state unemployment rate $u^{*}$ ?
A. $s L /(s+f)$
B. $s /(s+f)$
C. $f L /(f+s)$
D. $f /(f+s)$
(13) (2 points) Assume a monthly job separation rate equal to $s=1 \%$, and a monthly job finding rate equal to $f=19 \%$. Assume that the labor force is given by $L=100$ million. What is the steady-state unemployment rate?
A. $4.8 \%$
B. $4 \%$
C. $5 \%$
D. $5.8 \%$
E. $5.2 \%$
(14) (2 points) Assume that initially, the unemployment rate is given by $u_{0}=10 \%$. How many people lose their jobs each month initially?
A. 900,000
B. $1,000,000$
C. $10,000,000$
D. $1,900,000$
E. 190,000
(15) (2 points) Assume that initially, the unemployment rate is given by $u_{0}=10 \%$. How many people find a job each month initially?
A. 900,000
B. $1,000,000$
C. $10,000,000$
D. 1,900,000
E. 190,000
(16) (2 points) According to the Solow growth model, what is the effect of government deficits on the U.S. economy?
A. They reduce the private saving rate.
B. They lead to lower steady-state output.
C. They increase the private saving rate.
D. They lead to higher steady-state output.
(17) (2 points) If GDP per capita grows at $2 \%$ per year, how long does it take for it to double?
A. 50 years.
B. 35 years.
C. 20 years.
D. 65 years
E. 80 years.
(18) (2 points) In the two-period consumption model, by how much does utility vary when the consumer saves one more unit of income?
A. $u^{\prime}\left(c_{0}\right)$
B. $(1+r) \beta u^{\prime}\left(c_{1}\right)$
C. $(1+r) \beta u^{\prime}\left(c_{1}\right)-u^{\prime}\left(c_{0}\right)$
D. $u^{\prime}\left(c_{0}\right)-(1+r) \beta u^{\prime}\left(c_{1}\right)$
(19) (2 points) For the US economy, which of the following represents the largest component of GDP?
A. imports
B. investment
C. government spending
D. exports
E. none of the above: there exists a component of GDP that is greater than all the above in the US economy
(20) (2 points) Who has said: "The purpose of studying economics is not to acquire a set of ready-made answers to economic questions, but to learn how to avoid being deceived by economists."?
A. Paul Krugman
B. Karl Marx
C. Jérôme Powell
D. Joan Robinson
E. Donald Trump

## Exercise 1 (20 points)

2. (20 points) We consider the overlapping-generations model of Lecture 4 , with $\beta=1 / 2$ : $U=\log \left(c_{t}^{y}\right)+\frac{1}{2} \log \left(c_{t+1}^{o}\right)$ We denote the (net) real interest rate by $r_{t}$, and the wage by $w_{t}$. The production function is Cobb-Douglas with $\alpha=1 / 4: Y_{t}=K_{t}^{1 / 4} L_{t}^{3 / 4}$. The labor force is constant so that $L_{t}=1$. Moreover: $\delta=1=100 \%$.
(a) (2 points) Use the "intuitive" method to find a relation between $c_{t+1}^{o}, c_{t}^{y}$ and $r_{t}$, coming from utility maximization. (Give the economic intuition.)

Solution: By consuming one dollar less in period $t$, a consumer loses $1 / c_{t}^{y}$, and earns $1+r_{t}$, which gives him $1+r_{t} /\left(2 c_{t+1}^{o}\right)$ more in utility (marginal utility of an additional dollar, times the return on saving 1 dollar). As a consequence:

$$
1 / c_{t}^{y}=1+r_{t} /\left(2 c_{t+1}^{o}\right) \quad \Rightarrow \quad c_{t+1}^{o}=\frac{1}{2}\left(1+r_{t}\right) c_{t}^{y} \text {. }
$$

(b) (2 points) Use the intertemporal budget constraint to find $c_{t+1}^{o}$ and $c_{t}^{y}$.

Solution: Then plug back in the intertemporal budget constraint:

$$
c_{t}^{y}+\frac{c_{t+1}^{o}}{1+r_{t}}=w_{t} \quad \Rightarrow \quad c_{t}^{y}+\frac{1}{2} c_{t}^{y}=w_{t} \quad \Rightarrow \quad c_{t}^{y}=\frac{2 w_{t}}{3}, c_{t+1}^{o}=\left(1+r_{t}\right) \frac{w_{t}}{3} .
$$

(c) (4 points) What is the law of motion for the capital stock? Compute the steadystate capital stock $K^{*}$, the (net) steady-state real interest rate $r^{*}$.

Solution: The wage $w_{t}$ is equal to the marginal product of labor:

$$
w_{t}=\frac{\partial Y_{t}}{\partial L_{t}}=\frac{3}{4} K_{t}^{1 / 4} L_{t}^{-1 / 4}=\frac{3}{4} K_{t}^{1 / 4} .
$$

Saving is given by $w_{t}-c_{t}^{y}=w_{t} / 3$ and depreciation is $\delta=1$ thus:

$$
K_{t+1}-K_{t}=\frac{w_{t}}{3}-K_{t} \quad \Rightarrow \quad K_{t+1}=\frac{1}{4} K_{t}^{1 / 4} .
$$

The steady state capital stock is such that:

$$
K^{*}=\frac{1}{4}\left(K^{*}\right)^{1 / 4} \quad \Rightarrow \quad\left(K^{*}\right)^{3 / 4}=\frac{1}{4} \quad \Rightarrow \quad K^{*}=\frac{1}{4^{4 / 3}}
$$

The net real interest rate $r^{*}$ is the MPK minus $\delta=1$ :

$$
r^{*}=\frac{1}{4}\left(K^{*}\right)^{-3 / 4}-1=1-1=0 \quad \Rightarrow \quad r^{*}=0 \% \text {. }
$$

(d) (4 points) Compute the Golden Rule net interest rate $r_{g}^{*}$, capital $K_{g}^{*}$ and wage $w_{g}^{*}$.

Solution: The Golden Rule net interest rate is $r_{g}^{*}=0$, implying:

$$
r_{g}^{*}=\frac{1}{4}\left(K_{g}^{*}\right)^{-3 / 4}-1 \quad \Rightarrow \quad K_{g}^{*}=\frac{1}{4^{4 / 3}}
$$

The Golden Rule wage $w_{g}^{*}$ is such that:

$$
w_{g}^{*}=\frac{3}{4}\left(K^{*}\right)^{1 / 4}=\frac{3}{4} \frac{1}{4^{1 / 3}} \quad \Rightarrow \quad w_{g}^{*}=\frac{3}{4^{4 / 3}} .
$$

(e) (4 points) Compare the Golden Rule and steady-state levels of $r^{*}$ and $K^{*}$.

Solution: We have the following equalities:

$$
\begin{aligned}
r^{*} & =r_{g}^{*} \\
K_{g}^{*} & =K^{*}
\end{aligned}
$$

(f) (4 points) Compare this result from the one in the course. Why is it comparable, despite the fact that agents are more impatient?

Solution: On the one hand, agents are more impatient, which should lead the capital stock to exceed the Golden Rule level. This is the opposite of the case we saw in Problem Set 3, where people were less impatient, so that the capital stock was too high. At the same time, the returns to capital are decreasing faster than in the class, since $\alpha=1 / 4$ instead of $\alpha=1 / 3$, which implies that the demand for investment from firms is relatively lower. On the one hand, the supply of capital is lower because people are more impatient; on the other, the demand for capital is also lower. These two opposing forces exactly offset each other, so that on net, the market accumulation of capital corresponds to the Golden Rule level, "by chance".

## Exercise 2 (20 points)

3. (20 points) Consider the neoclassical labor market model. On the demand side, we assume a Cobb-Douglas production function for $f(l)$, such that: $f(l)=A l^{1-\alpha}$. On the supply side, we assume a linear utility for consumption as well as a power function of disutility for work $U(c, l)=c-B \cdot l^{1+\epsilon} /(1+\epsilon)$.
(a) (4 points) Assume that the price of consumption is $p$, and that the wage is $w$. Derive the labor demand curve assuming that firms maximize their profits $p f(l)-w l$.

Solution: We solve:

$$
\max _{l} \quad p f(l)-w l=\max _{l} \quad p A l^{1-\alpha}-w l
$$

This implies:

$$
p A(1-\alpha) l^{-\alpha}=w \quad \Rightarrow \quad \frac{w}{p}=A(1-\alpha) l^{-\alpha} .
$$

This is a labor demand curve: a (negative) relationship between the real wage and the quantity of employment.
(b) (4 points) Derive the labor supply curve assuming that workers' budget constraint is given by $p c=w l$ (you can use whichever of the 4 methods you prefer).

Solution: Let us replace out $c$ in the utility function using the budget constraint and the maximize over $l$ :

$$
\max _{l} \frac{w}{p} l-B \frac{l^{1+\epsilon}}{1+\epsilon}
$$

This implies:

$$
\frac{w}{p}=B l^{\epsilon} \text {. }
$$

(c) (4 points) Calculate the equilibrium quantity of labor $l$.

Solution: Equating the two above expressions, to find the intersection of the labor demand curve and the labor supply curve allows to find:

$$
\begin{aligned}
& A(1-\alpha) l^{-\alpha}=B l^{\epsilon} \Rightarrow \frac{A(1-\alpha)}{B}=l^{\epsilon+\alpha} \\
& \quad \Rightarrow \quad l=A^{1 /(\epsilon+\alpha)}(1-\alpha)^{1 /(\epsilon+\alpha)} B^{-1 /(\epsilon+\alpha)} .
\end{aligned}
$$

(d) (4 points) Calculate the equilibrium real wage $w / p$.

Solution: Substituting in either of the labor supply or labor demand curves, for example the labor supply curve:

$$
\begin{aligned}
\frac{w}{p} & =B l^{\epsilon} \\
& =B A^{\epsilon /(\epsilon+\alpha)}(1-\alpha)^{\epsilon /(\epsilon+\alpha)} B^{-\epsilon /(\epsilon+\alpha)} \\
\frac{w}{p} & =A^{\epsilon /(\epsilon+\alpha)}(1-\alpha)^{\epsilon /(\epsilon+\alpha)} B^{\alpha /(\epsilon+\alpha)}
\end{aligned}
$$

(e) (4 points) We consider a fall in $\log$ productivity $\Delta \log A$, where $\log$ is the natural log. What is the change in $\log$ employment $\Delta \log (l)$, and the change in the $\log$ real wage $\Delta \log (w / p)$, as a function of $\Delta \log A$ ?

Solution: Taking natural logs of the expression in question 3:

$$
\log l=\frac{1}{\epsilon+\alpha} \log A+\frac{1}{\epsilon+\alpha} \log (1-\alpha)-\frac{1}{\epsilon+\alpha} \log B
$$

This implies that the change in log employment is:

$$
\Delta \log l=\frac{\Delta \log A}{\epsilon+\alpha}
$$

Taking natural logs of the expression in question 4:

$$
\log \left(\frac{w}{p}\right)=\frac{\epsilon}{\epsilon+\alpha} \log A+\frac{\epsilon}{\epsilon+\alpha} \log (1-\alpha)+\frac{\alpha}{\epsilon+\alpha} \log B
$$

This implies that the change in the log real wage is:

$$
\Delta \log \left(\frac{w}{p}\right)=\frac{\epsilon \Delta \log A}{\epsilon+\alpha}
$$

## Exercise 3 (20 points)

4. (20 points) Consider the Solow growth model of Lecture 2, with however two small changes. Assume that the production function is given by $F\left(K_{t}, L_{t}\right)=A_{t} K_{t}^{\alpha} L_{t}^{1-\alpha}$, with $A_{t}$ such that $A_{t}=(1+g)^{t}$, and $L_{t}$ such that: $L_{t}=(1+n)^{t}$.
(a) (2 points) Write the law of motion for capital $K_{t}$.

Solution: The law of motion for capital is: $\Delta K_{t+1}=K_{t+1}-K_{t}=s Y_{t}-\delta K_{t}$. Using the value for $Y_{t}$, we get:

$$
K_{t+1}=s A_{t} K_{t}^{\alpha} L_{t}^{1-\alpha}+(1-\delta) K_{t}
$$

(b) (6 points) Define $k_{t}$ as: $k_{t} \equiv \frac{K_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}}$, and write a law of motion for $k_{t}$. Assume that $n$, and $g$ are small in order to simplify this law of motion. Hint: if $n$ and $g$ are small then: $(1+g)^{1 /(1-\alpha)}(1+n) \approx 1+\frac{1}{1-\alpha} g+n$.

Solution: Divide both the left-hand side and the right-hand side of the equation by $A_{t}^{1 /(1-\alpha)} L_{t}$. This gives:

$$
\begin{gathered}
\frac{K_{t+1}}{A_{t}^{1 /(1-\alpha)} L_{t}}=s \frac{A_{t} K_{t}^{\alpha} L_{t}^{1-\alpha}}{A_{t}^{1 /(1-\alpha)} L_{t}}+(1-\delta) \frac{K_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}} \\
\frac{K_{t+1}}{A_{t}^{1 /(1-\alpha)} L_{t}}=s \underbrace{\underbrace{A_{t}^{1 /(1-\alpha) L_{t}}}_{\frac{K_{t}}{A_{t}^{\alpha /(1-\alpha)} L_{t}^{\alpha}}})^{\alpha}=k_{t}^{\alpha}}_{k_{t}}+(1-\delta) \underbrace{\frac{K_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}}}
\end{gathered}
$$

The left-hand side can be expressed as:

$$
\begin{aligned}
& \frac{K_{t+1}}{A_{t}^{1 /(1-\alpha)} L_{t}}=\frac{A_{t+1}^{1 /(1-\alpha)} L_{t+1}}{A_{t}^{1 /(1-\alpha)} L_{t}} \cdot \underbrace{\frac{K_{t+1}}{A_{t+1}^{1 /(1-\alpha)} L_{t+1}}}_{k_{t+1}} \\
& \frac{K_{t+1}}{A_{t}^{1 /(1-\alpha)} L_{t}}=\underbrace{(1+g)^{1 /(1-\alpha)}(1+n)}_{\approx 1+\frac{1}{1-\alpha} g+n} \cdot k_{t+1} .
\end{aligned}
$$

Thus:

$$
\left(1+\frac{1}{1-\alpha} g+n\right) k_{t+1} \approx s k_{t}^{\alpha}+(1-\delta) k_{t} .
$$

A law of motion for $k_{t+1}$ is thus (we use equal signs now, even though it is really an approximation):

$$
k_{t+1}=\frac{s}{1+g /(1-\alpha)+n} k_{t}^{\alpha}+\frac{1-\delta}{1+g /(1-\alpha)+n} k_{t} .
$$

(c) (4 points) Compute $k^{*}$, the steady-state of $k_{t}$. Compute $y^{*}$ and $c^{*}$ corresponding to steady-state $k^{*}$ with: $y_{t} \equiv \frac{Y_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}}$ and $c_{t} \equiv \frac{C_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}}$.

Solution: The steady-state is such that:

$$
\begin{aligned}
& \left(1+\frac{1}{1-\alpha} g+n\right) k^{*}=s\left(k^{*}\right)^{\alpha}+(1-\delta) k^{*} \\
& \Rightarrow\left(\delta+\frac{1}{1-\alpha} g+n\right) k^{*}=s\left(k^{*}\right)^{\alpha} \Rightarrow k^{*}=\left(\frac{s}{\delta+g /(1-\alpha)+n}\right)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

Divide both sides of $Y_{t}=A_{t} K_{t}^{\alpha} L_{t}^{1-\alpha}$ and $C_{t}=(1-s) Y_{t}$ by $A_{t}^{1 /(1-\alpha)} L_{t}$ :

$$
\begin{aligned}
& \frac{Y_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}}=\frac{A_{t} K_{t}^{\alpha} L_{t}^{1-\alpha}}{A_{t}^{1 /(1-\alpha)} L_{t}} \Rightarrow \underbrace{\frac{Y_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}}}_{y_{t}}=\underbrace{\frac{K_{t}^{\alpha}}{A_{t}^{\alpha /(1-\alpha)} L_{t}^{\alpha}}}_{k_{t}^{\alpha}} \Rightarrow y^{*}=\left(k^{*}\right)^{\alpha} \\
& \frac{C_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}}=\frac{Y_{t}}{A_{t}^{1 /(1-\alpha)} L_{t}} \Rightarrow c_{t}=(1-s) y_{t} \quad \Rightarrow \quad c^{*}=(1-s) y^{*} .
\end{aligned}
$$

Finally:

$$
y^{*}=\left(\frac{s}{\delta+g /(1-\alpha)+n}\right)^{\frac{\alpha}{1-\alpha}} \quad c^{*}=(1-s) \cdot\left(\frac{s}{\delta+g /(1-\alpha)+n}\right)^{\frac{1}{1-\alpha}}
$$

(d) (4 points) What is the consumption-maximizing saving rate, which maximizes $c^{*}$ ?

## Solution:

$$
\max _{s}(1-s) s^{\frac{\alpha}{1-\alpha}} \Rightarrow s^{\frac{\alpha}{1-\alpha}}+\frac{\alpha}{1-\alpha}(1-s) s^{\frac{\alpha}{1-\alpha}-1}=0 \quad \Rightarrow \quad s=\alpha .
$$

(e) (4 points) What is then the value of the net interest rate $r^{*}$ ? Reminder: the net interest rate is the gross interest rate (the marginal product of capital) minus the depreciation rate.

Solution: The net interest rate $r^{*}$ is:

$$
\begin{aligned}
r_{t} & =\alpha \underbrace{A_{t} K_{t}^{\alpha-1} L_{t}^{1-\alpha}}_{k_{t}^{\alpha-1}}-\delta \quad \Rightarrow \quad r^{*}=\alpha\left(k^{*}\right)^{\alpha-1}-\delta \\
& \Rightarrow r^{*}=\alpha\left(\frac{s}{\delta+g /(1-\alpha)+n}\right)^{\frac{\alpha-1}{1-\alpha}}-\delta \quad \Rightarrow_{s=\alpha} \quad r^{*}=g /(1-\alpha)+n .
\end{aligned}
$$

