UCLA - Econ 102 - Fall 2018
Instructor: François Geerolf
Final Exam
December 13, 2018
Time Limit: 3 hours

Last Name: ____________________

First Name: ____________________

Student ID Number: ______________ Signature ____________________

Final Exam

This exam contains 21 pages (including this cover page). You can earn 100 points.

Instructions:

1. Print your Last name, First Name, Student ID Number and Signature at the top of this page.
2. The only items which should be on your desk are pencils and/or pens. NO other items are allowed. Place any other item UNDER your desk. Calculators are NOT allowed.
3. Once the exam begins, you are not allowed to leave the room until you hand in your exam.

Good luck ! Budget your time wisely ! (skip the question or even the exercise if you get stuck)

Do not write below this line (Grader use only)

Question	Points	Score
1	40	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	100	

40 Multiple Choice Questions (40 points)

1. (40 points) Each multiple choice question has only one right answer. Use the Scantron to mark your answers.

(1) (1 point) Which of the following would tend to make the government expenditure multiplier smaller?

A. an increase in the marginal propensity to consume
B. an increase in the marginal propensity to save
C. a reduction in taxes
D. a reduction in government spending
E. none of the above

(2) (1 point) Which of the following would tend to make the tax multiplier smaller?

A. an increase in the marginal propensity to consume
B. an increase in the marginal propensity to save
C. a reduction in taxes
D. a reduction in government spending
E. none of the above

(3) (1 point) Based on our understanding of the paradox of thrift, we know that a reduction in the desire to save will cause:

A. an increase in equilibrium GDP.
B. a reduction in GDP.
C. an increase in government spending.
D. no change in equilibrium GDP.
E. a permanent reduction in the level of saving.

(4) (1 point) If $C(Y_D) = 2000 + .9Y_D$, investment is exogenous, and the economy is closed, what decrease in taxes must occur for equilibrium output to increase by 1000?

A. 900.
B. 111.111**...**
C. 100.
D. 1000.
E. 500.

(5) (1 point) Which of the following is less efficient at increasing equilibrium output?

A. increases in consumer confidence
B. increases in investment
C. increases in government spending
D. decreases in taxes

(6) (1 point) Based on the Keynesian Cross model, an equal and simultaneous reduction in G and T will cause:

A. an increase in output.

B. no change in output.

C. a reduction in output.

D. an increase in investment.

(7) (1 point) Suppose that the marginal propensity to consume is 0.8. Given this information, which of the following events will cause the largest increase in output?

A. G increases by 200.

B. T decreases by 200.

C. I increases by 150.

D. both A and B.

(8) (1 point) In the overlapping generations model, why do people save?

A. To leave bequests.

B. For prestige.

C. Because of religious beliefs.

D. They have too much money.

E. To plan for retirement.

(9) (1 point) Which of the following cannot increase long-run growth?

A. Patents.

B. Capital accumulation.

C. Government funded research.

D. Prizes.

E. Privately funded research.

(10) (1 point) Assume that the uncovered interest parity condition holds. Also assume that the U.S. nominal interest rate is less than the U.K. nominal interest rate. Given this information, we know that investors expect:

A. the pound to depreciate.

B. the pound to appreciate.

C. the dollar-pound exchange rate to remain fixed.

D. the U.S. interest rate to fall.

E. none of the above.

(11) (1 point) The **four** following multiple choice questions are based on the Bathtub model. In the Bathtub model, what is the law of motion for unemployment?

A. $\Delta U_{t+1} = fU_t - sE_t$
B. $\Delta U_{t+1} = sE_t - fU_t$
C. $U_{t+1} = sE_t - fU_t$
D. $\Delta U_{t+1} = fL - sE_t$
E. $U_{t+1} = fU_t - sE_t$

(12) (1 point) Assume a monthly job separation rate equal to $s = 1\%$, and a monthly job finding rate equal to $f = 19\%$. Assume that the labor force is given by $L = 100$ million. What is the steady-state unemployment rate?

A. 4.8%
B. 4%
C. 5%
D. 5.8%
E. 5.2%

(13) (1 point) Assume that initially, the unemployment rate is given by $u_0 = 10\%$. How many people lose their jobs each month initially?

A. 900,000
B. 1,000,000
C. 10,000,000
D. 1,900,000
E. 190,000

(14) (1 point) Assume that initially, the unemployment rate is given by $u_0 = 10\%$. How many people find a job each month initially?

A. 900,000
B. 1,000,000
C. 10,000,000
D. 1,900,000
E. 190,000

(15) (1 point) Assume a country is closed. Given this information, which of the following must occur?

A. demand for domestic goods will be less than the domestic demand for goods.
B. demand for domestic goods will be greater than the domestic demand for goods.
C. $S + T = I + G$**.**
D. a budget surplus exists.
E. $S = I$.

(16) (1 point) In an open economy, which of the following will cause an increase in the size of the multiplier?

A. a reduction in the marginal propensity to import

B. a reduction in foreign output

C. an increase in the marginal propensity to save

D. all of the above

E. none of the above

(17) (1 point) Suppose there is a reduction in foreign output (Y^*). This reduction in Y^* will cause which of the following in the domestic country?

A. a reduction in output.

B. a reduction in consumption.

C. a reduction in net exports.

D. all of the above.

E. none of the above.

(18) (1 point) An increase in government spending will have a greater impact on net exports when:

A. the marginal propensity to save is smaller.

B. the economy is closed.

C. the sensitivity of investment to income is smaller.

D. all of the above.

E. none of the above.

(19) (1 point) Which of the following will occur in a small country with a high marginal propensity to import?

A. Changes in government spending will cause large changes in output.

B. Changes in government spending will cause large changes in the trade balance.

C. A depreciation will cause only small changes in the trade balance.

D. There is no combination of policies that can eliminate the trade deficit.

E. all of the above.

(20) (1 point) Which of the following would make the government spending multiplier smaller?

A. a reduction in marginal propensity to save.

B. a small initial trade deficit.

C. a reduction in the marginal propensity to import.

D. a real appreciation.

E. none of the above.

(21) (1 point) Which of the following will occur as a result of a tax increase?

A. private saving increases.

B. investment increases.

C. the trade balance improves.

D. the trade balance worsens.

E. the budget deficit increases.

(22) (1 point) An open economy with a low total level of saving (private and public) must have:

A. low investment only.

B. high investment only.

C. a trade surplus only.

D. low investment or a trade deficit.

E. low investment or a trade surplus.

(23) (1 point) Which of the following is not a result of a reduction in short-term interest rates?

A. More competitive exports.

B. More refinancing of fixed-rate mortgages.

C. Increase in creditors' disposable income.

D. All of A, B, C.

E. None of A, B, C.

(24) (1 point) According to the neoclassical synthesis:

A. There is a positive correlation between unemployment and inflation.

B. There is an excess of saving over investment.

C. Prices are sticky.

D. Aggregate demand has long-run effects.

E. Output is always determined by technology.

(25) (1 point) Who in history implemented the first Keynesian stimulus?

A. Franklin D. Roosevelt

B. Herbert Hoover

C. John Maynard Keynes

D. Donald Trump

E. Adolf Hitler

(26) (1 point) Which of the following methods cannot be used for empirical macroeconomics?

A. Narrative Approaches.
B. Cross-sectional studies.
C. Individual-level studies on the Marginal Propensity to Consume.
D. Randomized experiments.
E. Individual-level studies on the Labor Supply Elasticity.

(27) (1 point) If Jay Powell, the Chairman of the Federal Reserve Bank, decides to raise short-term interest rates unexpectedly, what will happen to the euro?

A. It would appreciate relative to the dollar.
B. It would appreciate relative to the Yen.
C. It would depreciate relative to the dollar.
D. It would depreciate relative to the Yen.
E. None of the above.

(28) (1 point) If Jay Powell, the Chairman of the Federal Reserve Bank, decides to raise short-term interest rates unexpectedly, what else will happen, at least in the short run?

A. Manufacturing employment will increase.
B. Government spending will decline.
C. Consumption will rise.
D. Exports will decline.
E. None of the above.

(29) (1 point) Despite the first budget surplus in over 20 years, the U.S. still experienced a large trade deficit at the end of Bill Clinton's presidency from 1998 to 2001. Which of the following is incorrect about this period?

A. Investment boomed.
B. Fiscal policy was redistributive.
C. Consumption increased.
D. Imports Rose.
E. None of the above.

(30) (1 point) Which of the following does Paul Krugman not believe?

A. Output is mostly determined by the demand side.
B. We should maintain Pay-As-You-Go (PAYG) systems.
C. Public debt is worrying when it comes from tax cuts to the wealthy.
D. We need to "starve the beast".
E. We should send out checks to lower income people to boost consumption.

(31) (1 point) Which of the following does Robert Barro not believe?

A. Taxes need to be cut where marginal rates are initially the highest.

B. Output is mostly determined by the supply-side.

C. Social security needs to be replaced by private accounts.

D. Public deficits associated to marginal tax rate cuts are an important issue.

E. Tax cuts increase people's incentives to work.

The following four questions assume that $C = c_0 + c_1(Y - T)$, $T = t_0 + t_1 Y$, $I = b_0 + b_1 Y$, $M = m_1 Y$, and $X = x_1 Y^*$, $G = g_0 + g_1 Y$, with $c_1 = 2/3$, $t_1 = 1/4$, $b_1 = 1/6$, $g_1 = 1/6$, $m_1 = 1/6$, and $x_1 = 1/4$. You need to first compute Y using that Z is the sum of all components of aggregate demand, and $Y = Z$.

(32) (1 point) What is the government spending multiplier $\Delta Y / \Delta g_0$?

A. 1/2

B. 3/4

C. 1

D. 2

E. 3

(33) (1 point) What is the tax multiplier $\Delta Y / (-\Delta t_0)$?

A. 1/2

B. 3/4

C. 1

D. 2

E. 3

(34) (1 point) What is $\Delta Y / \Delta Y^*$?

A. 1/2

B. 3/4

C. 1

D. 2

E. 3

(35) (1 point) What is $\Delta Y / \Delta c_0$?

A. 1/2

B. 3/4

C. 1

D. 2

E. 3

The following five questions assume 2 groups, the bottom 90% low-income group paying $\underline{T}_0$ with MPC $\underline{c}_0$, and the top 10% high-income group richer by $\gamma = 9$, paying $\bar{T}_0$ with MPC $\bar{c}_0$. Aggregate consumption is $C = C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + c_1(1 - t_1)Y$, $T = \underline{T}_0 + \bar{T}_0 + t_1 Y$, $I = b_0 + b_1 Y$, $M = m_1 Y$, and $X = x_1 Y^*$, $G = g_0 + g_1 Y$, with $\underline{c}_1 = 1/2$, $\bar{c}_1 = 1$, $t_1 = 1/3$, $b_1 = 1/6$, $g_1 = 1/6$, $m_1 = 1/3$, and $x_1 = 1/3$.

(36) (1 point) What is the average propensity to consume c_1 in this economy?

A. 1/3
B. 1/2
C. 2/3
D. 3/4
E. 1

(37) (1 point) **You need to first compute Y using that Z is the sum of all components of aggregate demand, and $Y = Z$.** What is the government spending multiplier $\Delta Y / \Delta g_0$?

A. 1/2
B. 1
C. 3/2
D. 2
E. 3

(38) (1 point) What is the "tax cuts for the rich" multiplier $\Delta Y / \Delta(-\bar{T}_0)$?

A. 1/2
B. 1
C. 3/2
D. 2
E. 3

(39) (1 point) What is the "tax cuts for the poor" multiplier $\Delta Y / \Delta(-\underline{T}_0)$?

A. 1/2
B. 1
C. 3/2
D. 2
E. 3

(40) (1 point) What is the "redistribution multiplier" $\Delta Y / \Delta \underline{T}_0$ if $\Delta \bar{T}_0 = -\Delta \underline{T}_0$?

A. 1/2
B. 1
C. 3/2
D. 2
E. 3

Exercise 1 (10 points)

2. (10 points) Consider the standard Solow growth model. We assume that the economy's production function is $Y = F(K, L) = K^{1/4}L^{3/4}$. Assume no population growth.

   (a) (1 point) What is the name of this production function?

   Solution: This is a Cobb-Douglas production function.

   (b) (1 point) Show that this production function has constant returns to scale.

   Solution: This production function has constant returns to scale as for all x:

   $$F(xK, xL) = (xK)^{1/4}(xL)^{3/4} = xK^{1/4}L^{3/4} = xF(K, L)$$

   (c) (2 points) For a given saving rate, s, and depreciation rate, δ, derive an expression for capital per worker in the steady state. Give the intermediate steps.

   Solution: We write the evolution of the capital stock as:

   $$\begin{aligned} K_{t+1} &= (1-\delta)K_t + I_t \\ &= (1-\delta)K_t + sY_t \\ K_{t+1} &= (1-\delta)K_t + sK_t^{1/4}L^{3/4} \end{aligned}$$

   Dividing both sides by L:

   $$\frac{K_{t+1}}{L} = (1-\delta)\frac{K_t}{L} + s\left(\frac{K_t}{L}\right)^{1/4}$$

   In steady state, $\frac{K_{t+1}}{L} = \frac{K_t}{L} = \frac{K^*}{L}$, so we have:

   $$\delta\frac{K^*}{L} = s\left(\frac{K^*}{L}\right)^{1/4}$$

   Therefore:

   $$\left(\frac{K^*}{L}\right)^{3/4} = \frac{s}{\delta}$$

   Finally, the capital per worker in the steady state is given by:

   $$\boxed{\frac{K^*}{L} = \left(\frac{s}{\delta}\right)^{4/3}}.$$

(d) (2 points) Derive an expression for output per worker in the steady state. What is it equal to if $s = 24\%$ and $\delta = 3\%$?

Solution: Using that: $Y^* = K^{*1/4} L^{3/4}$, we have:

$$\frac{Y^*}{L} = \left(\frac{K^*}{L}\right)^{1/4} \quad \Rightarrow \quad \boxed{\frac{Y^*}{L} = \left(\frac{s}{\delta}\right)^{1/3}}.$$

The numerical application gives:

$$\frac{Y^*}{L} = \left(\frac{s}{\delta}\right)^{1/3} = \left(\frac{0.24}{0.03}\right)^{1/3} = 8^{1/3} = 2.$$

(since $2^3 = 8$)

(e) (2 points) Give an expression for consumption per worker in the steady state. What is it equal to if $s = 24\%$ and $\delta = 3\%$?

Solution:

$$\frac{C^*}{L} = (1-s)\frac{Y^*}{L} = (1-s)\left(\frac{s}{\delta}\right)^{1/3}.$$

The numerical application gives:

$$\frac{C^*}{L} = (1-s)\frac{Y^*}{L} = \frac{76}{100} * 2 = \frac{152}{100} = 1.52.$$

(f) (2 points) Derive the consumption-maximizing saving rate. Show the intermediate steps.

Solution: The consumption-maximizing saving rate is such that the level of steady-state consumption per capita C^*/L is maximized. Therefore, the saving rate must maximize:

$$\max_s \quad (1-s)s^{1/3} = s^{1/3} - s^{4/3}$$

Setting the derivative to zero (using $(s^a)' = as^{a-1}$ for $a = 1/3$ and $a = 4/3$):

$$\frac{1}{3}s^{-2/3} - \frac{4}{3}s^{1/3} = 0 \quad \Rightarrow \quad \boxed{s = \frac{1}{4} = 25\%}.$$

<u>Comment</u>: you should have gotten the same if you maximized $(1-s)\left(\frac{s}{\delta}\right)^{1/3}$ instead.

Exercise 2 (10 points)

3. (10 points) Consider the closed economy goods market model where consumption is linear in disposable income with $C(Y_D) = c_0 + c_1 Y_D$, disposable income is income minus taxes, government spending and taxes are exogenous and equal to G and T respectively, but investment depends on output through $I = b_0 + b_1 Y$.

   (a) (2 points) Solve for equilibrium output.

   Solution: Total aggregate demand Z in the closed economy is given by:

$$
\begin{aligned}
Z &= C + I + G \\
Z &= c_0 + c_1(Y - T) + b_0 + b_1 Y + G \\
Z &= c_0 + b_0 - c_1 T + G + (c_1 + b_1)\, Y.
\end{aligned}
$$

   Thus, using $Z = Y$:

$$
\boxed{Y = \frac{1}{1 - c_1 - b_1}\,(c_0 + b_0 - c_1 T + G)}
$$

   (b) (2 points) Assume that there is a fall in autonomous spending given by $\Delta c_0 < 0$. Show that there is a direct effect on private saving of the change in autonomous spending as well as an indirect effect. What is the sign of the direct effect? (Hint: the direct effect is $\Delta(-c_0)$)

   Solution: Private saving is equal to disposable income minus consumption, as always:

$$
\begin{aligned}
S &= (Y - T) - C \\
&= Y - T - (c_0 + c_1(Y - T)) \\
&= -c_0 + (1 - c_1)\,(Y - T)
\end{aligned}
$$

   This allows to decompose into a direct and an indirect effect (the hint allows us to recognize the direct effect, so the other part has to be the indirect effect):

$$
\boxed{\Delta S = \underbrace{\Delta(-c_0)}_{\text{direct effect}} + \underbrace{\Delta\left[(1 - c_1)\,(Y - T)\right]}_{\text{indirect effect}}}
$$

   Note that the indirect effect is also equal to $(1 - c_1)\Delta Y$, since taxes are assumed to be fixed. This was an equally valid answer to the computation of the indirect effect. Obviously, since $\Delta(-c_0) > 0$, the direct effect is positive. There is a straightforward economic interpretation: a decrease in consumption leads to an increase in saving.

(c) (2 points) What is the value of the indirect effect, as a function of $\Delta c_0 < 0$?

Solution: We see that the indirect effect involves the change in income, which we therefore need to calculate. From the above equation, a given change in $\Delta c_0 < 0$ leads to decline in output given by:

$$\Delta Y = \frac{\Delta c_0}{1 - c_1 - b_1}$$

This allows to calculate the magnitude of the indirect effect.

$$\Delta\left[(1 - c_1)(Y - T)\right] = (1 - c_1)\Delta Y$$
$$\Delta\left[(1 - c_1)(Y - T)\right] = (1 - c_1)\frac{\Delta c_0}{1 - c_1 - b_1}$$

$$\boxed{\Delta\left[(1 - c_1)(Y - T)\right] = \frac{1 - c_1}{1 - c_1 - b_1}\Delta c_0}.$$

(d) (2 points) Compute the total effect of the change $\Delta c_0 < 0$ on private saving S (direct + indirect effect).

Solution: Therefore, the total effect on saving is:

$$\begin{aligned}\Delta S &= \Delta(-c_0) + \Delta\left[(1 - c_1)(Y - T)\right] \\ &= -\Delta c_0 + \frac{1 - c_1}{1 - c_1 - b_1}\Delta c_0 \\ &= \frac{1 - c_1 - (1 - c_1 - b_1)}{1 - c_1 - b_1}\Delta c_0\end{aligned}$$

Finally:

$$\boxed{\Delta S = \frac{b_1}{1 - c_1 - b_1}\Delta c_0}.$$

(e) (2 points) Why is the total effect on private saving a paradox?

Solution: Total saving falls, as $\Delta S < 0$. This phenomenon is a paradox (of thrift, or of saving) because a fall in consumption $\Delta c_0 < 0$ should intuitively lead to an increase in saving, but in fact leads to a decrease in saving. This is explained by the fact that the indirect effect going through a decline in income which reduces saving, more than offsets the direct effect.

Exercise 3 (10 points)

4. (10 points) Consider the Solow growth model of Lecture 2, with however two small changes. Assume that the production function is given by $F(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}$, with A_t such that $A_t = (1+g)^t$, and L_t such that: $L_t = (1+n)^t$.

   (a) (1 point) Write the law of motion for capital K_t.

   Solution: The law of motion for capital is: $\Delta K_{t+1} = K_{t+1} - K_t = sY_t - \delta K_t$. Using the value for Y_t, we get:

   $$\boxed{K_{t+1} = sA_t K_t^\alpha L_t^{1-\alpha} + (1-\delta)K_t}.$$

   (b) (3 points) Define k_t as: $k_t \equiv \frac{K_t}{A_t^{1/(1-\alpha)} L_t}$, and write a law of motion for k_t. Assume that n, and g are small in order to simplify this law of motion. Hint: if n and g are small then: $(1+g)^{1/(1-\alpha)}(1+n) \approx 1 + \frac{1}{1-\alpha} g + n$.

   Solution: Divide both the left-hand side and the right-hand side of the equation by $A_t^{1/(1-\alpha)} L_t$. This gives:

   $$\frac{K_{t+1}}{A_t^{1/(1-\alpha)} L_t} = s\frac{A_t K_t^\alpha L_t^{1-\alpha}}{A_t^{1/(1-\alpha)} L_t} + (1-\delta)\frac{K_t}{A_t^{1/(1-\alpha)} L_t}$$

   $$\frac{K_{t+1}}{A_t^{1/(1-\alpha)} L_t} = s \underbrace{\frac{K_t^\alpha}{A_t^{\alpha/(1-\alpha)} L_t^\alpha}}_{\left(\frac{K_t}{A_t^{1/(1-\alpha)} L_t}\right)^\alpha = k_t^\alpha} + (1-\delta) \underbrace{\frac{K_t}{A_t^{1/(1-\alpha)} L_t}}_{k_t}$$

   The left-hand side can be expressed as:

   $$\frac{K_{t+1}}{A_t^{1/(1-\alpha)} L_t} = \frac{A_{t+1}^{1/(1-\alpha)} L_{t+1}}{A_t^{1/(1-\alpha)} L_t} \cdot \underbrace{\frac{K_{t+1}}{A_{t+1}^{1/(1-\alpha)} L_{t+1}}}_{k_{t+1}}$$

   $$\frac{K_{t+1}}{A_t^{1/(1-\alpha)} L_t} = \underbrace{(1+g)^{1/(1-\alpha)}(1+n)}_{\approx 1 + \frac{1}{1-\alpha} g + n} \cdot k_{t+1}.$$

   Thus:

   $$\left(1 + \frac{1}{1-\alpha} g + n\right) k_{t+1} \approx s k_t^\alpha + (1-\delta) k_t.$$

   A law of motion for k_{t+1} is thus (we use equal signs now, even though it is really an approximation):

   $$\boxed{k_{t+1} = \frac{s}{1 + g/(1-\alpha) + n} k_t^\alpha + \frac{1-\delta}{1 + g/(1-\alpha) + n} k_t}.$$

(c) (2 points) Compute k^*, the steady-state of k_t. Compute y^* and c^* corresponding to steady-state k^* with: $y_t \equiv \frac{Y_t}{A_t^{1/(1-\alpha)} L_t}$ and $c_t \equiv \frac{C_t}{A_t^{1/(1-\alpha)} L_t}$.

Solution: The steady-state is such that:

$$\left(1 + \frac{1}{1-\alpha} g + n\right) k^* = s(k^*)^\alpha + (1-\delta) k^*$$

$$\Rightarrow \quad \left(\delta + \frac{1}{1-\alpha} g + n\right) k^* = s(k^*)^\alpha \quad \Rightarrow \quad \boxed{k^* = \left(\frac{s}{\delta + g/(1-\alpha) + n}\right)^{\frac{1}{1-\alpha}}}$$

Divide both sides of $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$ and $C_t = (1-s) Y_t$ by $A_t^{1/(1-\alpha)} L_t$:

$$\frac{Y_t}{A_t^{1/(1-\alpha)} L_t} = \frac{A_t K_t^\alpha L_t^{1-\alpha}}{A_t^{1/(1-\alpha)} L_t} \quad \Rightarrow \quad \underbrace{\frac{Y_t}{A_t^{1/(1-\alpha)} L_t}}_{y_t} = \underbrace{\frac{K_t^\alpha}{A_t^{\alpha/(1-\alpha)} L_t^\alpha}}_{k_t^\alpha} \quad \Rightarrow \quad y^* = (k^*)^\alpha$$

$$\frac{C_t}{A_t^{1/(1-\alpha)} L_t} = \frac{Y_t}{A_t^{1/(1-\alpha)} L_t} \quad \Rightarrow \quad c_t = (1-s) y_t \quad \Rightarrow \quad c^* = (1-s) y^*.$$

Finally:

$$\boxed{y^* = \left(\frac{s}{\delta + g/(1-\alpha) + n}\right)^{\frac{\alpha}{1-\alpha}}} \quad \boxed{c^* = (1-s) \cdot \left(\frac{s}{\delta + g/(1-\alpha) + n}\right)^{\frac{\alpha}{1-\alpha}}}$$

(d) (2 points) What is the consumption-maximizing saving rate, which maximizes c^* ?

Solution:

$$\max_s \quad (1-s) s^{\frac{\alpha}{1-\alpha}} \quad \Rightarrow \quad s^{\frac{\alpha}{1-\alpha}} + \frac{\alpha}{1-\alpha} (1-s) s^{\frac{\alpha}{1-\alpha} - 1} = 0 \quad \Rightarrow \quad \boxed{s = \alpha.}$$

(e) (2 points) What is then the value of the net interest rate r^*? Reminder: the net interest rate is the gross interest rate (the marginal product of capital) minus the depreciation rate.

Solution: The net interest rate r^* is:

$$r_t = \alpha \underbrace{A_t K_t^{\alpha-1} L_t^{1-\alpha}}_{k_t^{\alpha-1}} - \delta \quad \Rightarrow \quad r^* = \alpha (k^*)^{\alpha-1} - \delta$$

$$\Rightarrow \quad r^* = \alpha \left(\frac{s}{\delta + g/(1-\alpha) + n}\right)^{\frac{\alpha-1}{1-\alpha}} - \delta \quad \Rightarrow_{s=\alpha} \quad \boxed{r^* = g/(1-\alpha) + n}.$$

Exercise 4 (10 points)

5. (10 points) We consider again the schematic redistributive policies from the rich to the poor in the open economy, which we studied in lecture 11. We assume that aggregate consumption is $C = C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + c_1(1-t_1)Y$, $T = \left(\underline{T}_0 + \bar{T}_0\right) + t_1 Y$, $I = b_0 + b_1 Y$, G fixed, $M = m_1 Y$, $X = x_1 Y^*$. We consider a reduction in taxes on the poor $\Delta \underline{T}_0 < 0$, with an offsetting increase in taxes on high income earners such that $\Delta T_0 = \Delta \underline{T}_0 + \Delta \bar{T}_0 = 0$. We then have that $\Delta \bar{T}_0 = -\Delta \underline{T}_0 > 0$.

   (a) (2 points) What is the effect of redistribution on ΔY? What is the effect on the budget deficit of such a redistribution?

   Solution: Computing aggregate demand Y with $Z = Y$:

$$\begin{aligned} Y &= C + I + G + X - M \\ &= C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + c_1(1-t_1)Y + b_0 + b_1 Y + G + x_1 Y^* - m_1 Y \\ Y &= \left[C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + b_0 + G + x_1 Y^*\right] + \left(c_1(1-t_1) + b_1 - m_1\right) Y \\ \Rightarrow \quad Y &= \frac{1}{1-(1-t_1)c_1 - b_1 + m_1}\left[C_0 - \underline{c}_1 \underline{T}_0 - \bar{c}_1 \bar{T}_0 + b_0 + G + x_1 Y^*\right]. \end{aligned}$$

   With $\Delta \underline{T}_0 < 0$, and an offsetting $\Delta \bar{T}_0 = -\Delta \underline{T}_0$ and using $\Delta(T-G) = \Delta T - \Delta G$:

$$\boxed{\Delta Y = \frac{\underline{c}_1 - \bar{c}_1}{1-(1-t_1)c_1 - b_1 + m_1}\Delta \bar{T}_0} \qquad \boxed{\Delta(T-G) = t_1 \Delta Y}$$

   (b) (1 point) Using that $\Delta NX = -m_1 \Delta Y$, compute the change in the trade deficit.

   Solution: The change in the trade deficit is:

$$\boxed{\Delta NX = -\frac{m_1\left(\underline{c}_1 - \bar{c}_1\right)}{1-(1-t_1)c_1 - b_1 + m_1}\Delta \bar{T}_0}$$

   (c) (2 points) We shall now decompose the trade deficit into saving and investment, to better understand the intuition behind the counterintuitive result. Compute first ΔI. Does it contribute to explaining the puzzle?

   Solution: The change in investment is such that $\Delta I = b_1 \Delta Y$ so:

$$\boxed{\Delta I = \frac{b_1\left(\underline{c}_1 - \bar{c}_1\right)}{1-(1-t_1)c_1 - b_1 + m_1}\Delta \bar{T}_0}.$$

   This contributes to explaining the puzzle as it moves opposite of public saving.

(d) (2 points) Compute now private saving ΔS.

Solution:

$$\begin{aligned} S &= Y - T - C \\ &= Y - \left(\left(\underline{T}_0 + \bar{T}_0\right) + t_1 Y\right) - \left(C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + c_1(1-t_1)Y\right) \\ S &= (1-t_1)(1-c_1)Y - C_0 - \left(\underline{T}_0 + \bar{T}_0\right) + \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) \end{aligned}$$

Therefore, the change in saving is such that:

$$\begin{aligned} \Delta S &= (1-t_1)(1-c_1)\Delta Y + \underline{c}_1 \Delta \underline{T}_0 + \bar{c}_1 \Delta \bar{T}_0 \\ &= \frac{(1-t_1)(1-c_1)\left(\underline{c}_1 - \bar{c}_1\right)}{1-(1-t_1)\,c_1 - b_1 + m_1} \Delta \bar{T}_0 - (\underline{c}_1 - \bar{c}_1)\Delta \bar{T}_0 \\ &= \left[\frac{(1-t_1)(1-c_1) - (1-(1-t_1)\,c_1 - b_1 + m_1)}{1-(1-t_1)\,c_1 - b_1 + m_1}\right](\underline{c}_1 - \bar{c}_1)\Delta \bar{T}_0 \\ \Delta S &= \frac{(-t_1 + b_1 - m_1)\left(\underline{c}_1 - \bar{c}_1\right)}{1 - c_1(1-t_1) - b_1 + m_1} \Delta \bar{T}_0 \end{aligned}$$

(e) (2 points) Compute the net effect, total saving minus investment.

Solution: The net effect on net exports can be given as:

$$\begin{aligned} \Delta NX &= \Delta S + \Delta(T-G) + \Delta I \\ &= \frac{(-t_1 + b_1 - m_1) + t_1 - b_1}{1-(1-t_1)\,c_1 - b_1 + m_1}\left(\underline{c}_1 - \bar{c}_1\right)\Delta \bar{T}_0 \\ \Delta NX &= -\frac{m_1}{1-(1-t_1)\,c_1 - b_1 + m_1}\left(\underline{c}_1 - \bar{c}_1\right)\Delta \bar{T}_0 \end{aligned}$$

(f) (1 point) Assuming that $m_1 = 1/6$, $b_1 = 1/6$, $t_1 = 1/4$, $\underline{c}_1 = 1$, $\bar{c}_1 = 1/3$, $c_1 = 2/3$, what is the size of each component?

Solution:

$$\Delta Y = \frac{1}{1-(2/3)\cdot(1-1/4) - 1/6 + 1/6} \cdot \left(1 - \frac{1}{3}\right) \cdot 1 = \frac{4}{3}$$

$$\Delta S = (-t_1 + b_1 - m_1) \cdot \Delta Y = \left(-\frac{1}{4} + \frac{1}{6} - \frac{1}{6}\right) \cdot \frac{4}{3} = -\frac{1}{3}$$

$$\Delta(T-G) = t_1 \cdot \Delta Y = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3} \qquad \Delta I = b_1 \cdot \Delta Y = \frac{1}{6} \cdot \frac{4}{3} = \frac{2}{9}$$

$$\Delta NX = \Delta S + \Delta(T-G) - \Delta I = -\frac{1}{3} + \frac{1}{3} - \frac{2}{9} = -\frac{2}{9}$$

Exercise 5 (10 points)

6. (10 points) In this exercise, we consider the same problem as in lecture 10, except that lifetime utility is logarithmic with $\beta = 3$ (that is, people are patient instead of impatient, so they tend to save a lot): $U = \log(c_t^y) + 3\log(c_{t+1}^o)$ We denote the (net) real interest rate by r_t so that the intertemporal budget constraint is: $c_t^y + \frac{c_{t+1}^o}{1+r_t} = w_t$. Other than that, we still assume that: $Y_t = K_t^{1/3} L_t^{2/3}$. We assume that the labor force is constant so that $L_t = 1$. The depreciation rate is $\delta = 1 = 100\%$.

   (a) (2 points) Using your preferred method, compute c_{t+1}^o and c_t^y as a function of w_t.

   Solution: We use the ratio of marginal utilities:

$$\frac{1/c_t^y}{3/c_{t+1}^o} = 1 + r_t \quad \Rightarrow \quad c_{t+1}^o = 3(1+r_t)c_t^y.$$

   Then plug back in the intertemporal budget constraint:

$$c_t^y + \frac{c_{t+1}^o}{1+r_t} = w_t \quad \Rightarrow \quad c_t^y + 3c_t^y = w_t$$

$$\Rightarrow \quad \boxed{c_t^y = \frac{w_t}{4}} \quad \Rightarrow \quad \boxed{c_{t+1}^o = (1+r_t)\frac{3w_t}{4}}.$$

   (b) (2 points) What is the law of motion for the capital stock? Compute the steady-state capital stock K^*, the (net) steady-state real interest rate r^*.

   Solution: The wage w_t is:

$$w_t = \frac{\partial Y_t}{\partial L_t} = \frac{2}{3} K_t^{1/3} L_t^{-1/3} = \frac{2}{3} K_t^{1/3}.$$

   Saving is given by $w_t - c_t^y = 3w_t/4$ and depreciation is $\delta = 1$ thus:

$$K_{t+1} - K_t = \frac{3w_t}{4} - K_t \quad \Rightarrow \quad \boxed{K_{t+1} = \frac{1}{2} K_t^{1/3}}.$$

   The steady state capital stock is such that:

$$K^* = \frac{1}{2}(K^*)^{1/3} \quad \Rightarrow \quad (K^*)^{2/3} = \frac{1}{2} \quad \Rightarrow \quad \boxed{K^* = \frac{1}{2^{3/2}}}$$

   The net real interest rate r^* is the MPK minus $\delta = 1$:

$$r^* = \frac{1}{3}(K^*)^{-2/3} - 1 = \frac{1}{3} \cdot 2 - 1 \quad \Rightarrow \quad \boxed{r^* = -\frac{1}{3} \approx -33.3\%}.$$

(c) (2 points) Compute the Golden Rule net interest rate r_g^*, capital K_g^* and wage w_g^*.

Solution: The Golden Rule net interest rate is $r_g^* = 0$, implying:

$$r_g^* = \frac{1}{3}(K_g^*)^{-2/3} - 1 \quad \Rightarrow \quad K_g^* = \frac{1}{3^{3/2}} \quad \Rightarrow \quad \boxed{K_g^* = \frac{1}{3\sqrt{3}}}$$

The Golden Rule wage w_g^* is such that:

$$w_g^* = \frac{2}{3}(K^*)^{1/3} = \frac{2}{3}\frac{1}{3^{1/2}} \quad \Rightarrow \quad \boxed{w_g^* = \frac{2}{3\sqrt{3}}}.$$

(d) (2 points) Compare the Golden Rule and steady-state levels of r^* and K^*, and give an economic intuition.

Solution: We have the following inequalities:

$$-\frac{1}{3} < 0 \quad \Rightarrow \quad \boxed{r^* < r_g^*},$$

$$\frac{1}{3^{3/2}} < \frac{1}{2^{3/2}} \quad \Rightarrow \quad \boxed{K_g^* < K^*}.$$

First, the steady-state level of the interest rate is $r^* = -33.3\%$ - which as we know now is below the Golden Rule interest rate, which is 0, in an economy which has zero growth in the long run. Moreover, we also know that the gross interest rate $R^* = 1 + r^*$, which is the MPK, is decreasing in the quantity of capital, because of decreasing returns to capital. Therefore, the same is true of the net interest rate. Higher net interest rate thus implies a lower capital stock.

(e) (2 points) What level of government debt B_g^* brings the capital stock to the Golden Rule level ?

Solution: We know that the level of government debt B_g^* which brings the capital stock to the Golden Rule level is that which implies that when savers save 3/4 of their young age wage, they are able to exactly buy the quantity of capital corresponding to the Golden Rule level, as well as that public debt. Thus, we get an equation that B_g^* must satisfy:

$$B_g^* + K_g^* = \frac{3}{4}w_g^* \quad \Rightarrow \quad B_g^* = \frac{3}{4}w_g^* - K_g^*$$

$$\Rightarrow \quad B_g^* = \frac{3}{4} \cdot \frac{2}{3\sqrt{3}} - \frac{1}{3\sqrt{3}} = \left(\frac{1}{2} - \frac{1}{3}\right) \cdot \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \boxed{B_g^* = \frac{1}{6\sqrt{3}}}$$

Exercise 6 (10 points)

7. (10 points) Consider an open economy where consumption is given by $C(Y_D) = 10 + 0.8Y_D$, investment is given by $I = 8 + 0.1Y$, government spending is given by $G = g_0 + 0.1Y$, taxes are given by $T = 10 + 0.5Y$, and imports and exports are given by $M = 0.1Y$ and $X = 0.1Y^*$ respectively, where Y^* denotes foreign output.

   (a) (2 points) Solve for equilibrium output in the domestic economy, given Y^*. What is the multiplier in this economy?

   Solution: Total aggregate demand Z is:

   $$\begin{aligned} Z &= C + I + G + X - M \\ &= 10 + 0.8(Y - 10 - 0.5Y) + 8 + 0.1Y + g_0 + 0.1Y + 0.1Y^* - 0.1Y \\ Z &= 0.5Y + 10 + g_0 + 0.1Y^* \end{aligned}$$

   Setting $Y = Z$ gives:

   $$\boxed{Y = 20 + 2g_0 + 0.2Y^*}.$$

   Therefore, <u>the multiplier is 2</u> since:

   $$\Delta Y = 2\Delta g_0.$$

   (b) (2 points) If we were to close the economy - so exports and imports were identical and equal to zero - what would be the multiplier be? Why would the multiplier be different in a closed economy?

   Solution: If we were to close the economy, then demand would be:

   $$\begin{aligned} Z &= C + I + G \\ &= 10 + 0.8(Y - 10 - 0.5Y) + 8 + 0.1Y + g_0 + 0.1Y \\ Z &= 0.6Y + 10 + g_0 \end{aligned}$$

   Setting $Y = Z$ gives: $Y = 25 + 2.5g_0$. Therefore, <u>the multiplier is 2.5</u> since $\Delta Y = 2.5\Delta g_0$.
   In a closed economy, the multiplier would be higher because all the increase in income would be feeding domestic demand: there would be no "leakage" of aggregate demand.

   (c) (2 points) **From now on, you may keep fractions for numbers, since you do not have a calculator.** Assume that the foreign economy is characterized by the same equations as the domestic economy (with asterisks reversed). Use the two sets of equations to solve for the equilibrium output of each country.

Solution: We similarly have: $Y^* = 20 + 2g_0^* + 0.2Y$. Therefore:

$$\begin{aligned} Y &= 20 + 2g_0 + 0.2Y^* \\ &= 20 + 2g_0 + 0.2\left(20 + 2g_0^* + 0.2Y\right) \\ Y &= 24 + 2g_0 + 0.4g_0^* + 0.04Y \end{aligned}$$

This gives output Y, and symmetrically foreign output Y^*:

$$\boxed{Y = \frac{24}{0.96} + \frac{2}{0.96}g_0 + \frac{0.4}{0.96}g_0^*} \qquad \boxed{Y^* = \frac{24}{0.96} + \frac{2}{0.96}g_0^* + \frac{0.4}{0.96}g_0}.$$

(d) (2 points) What is the multiplier in each country now? Why is it different from the open economy multiplier in part (a)?

Solution: The multiplier is now given by 2/0.96. Indeed:

$$\Delta Y = \frac{2}{0.96}\Delta g_0.$$

This is higher than 2. The reason is that increasing G in the home economy increases M from the foreign economy and therefore, Y in the foreign economy, which in turn increases demand for X in the home economy.

(e) (1 point) What is the multiplier for a coordinated increase in government spending, such that $\Delta g_0 = \Delta g_0^*$?

Solution:

$$\Delta Y = \frac{2}{0.96}\Delta g_0 + \frac{0.4}{0.96}\Delta g_0^* = \frac{2.4}{0.96}\Delta g_0$$

The multiplier for a coordinated increase in government spending is 2.4/0.96. Comment. This is also the closed economy multiplier, which you could have guessed from the economic intuition (not required to get full credit).

(f) (1 point) Is the multiplier then higher or lower than in the previous question? What is the economic intuition?

Solution: The multiplier is higher. If government spending is coordinated, then exports in the home economy increase, which contributes to boosting output further. The aggregate demand leakage, increasing imports, is offset by an expansion abroad, increasing exports.