ECON 102 - Winter 2016
Instructor: François Geerolf

Midterm 1
January 27, 2016

Last Name: ____________________

First Name: ____________________

Student ID (UID): ____________________

Signature: ____________________

Test A

This exam contains 14 pages (including this cover page). The time limit is 75 minutes. You can earn 100 points.

Instructions:

1. Print your Last name, First Name, Student ID Number and Signature at the top right-hand corner of this page.
2. The only items that should be on your desk are pencils and/or pens, and the calculator **Canon LS-100TS**, described in the syllabus. NO other items are allowed. Place any other item ON THE STAGE.
3. Once the exam begins, you are not allowed to leave the room until you hand in your exam.

Good luck. Budget your time wisely. (skip the question or even the exercice if you get stuck)

Grade Table (FOR TEACHER USE ONLY)

Question	Points	Score
1	20	
2	10	
3	20	
4	20	
5	15	
6	15	
Total:	100	

1 Multiple Choice and Short Answers (20 points)

1. (20 points) These are **multiple choice** questions. Multiple responses may be correct. You get either 0 or 2 points.

(a) (2 points) In the Solow model, the level of GDP per capita is:

- ◯ A parameter.
- ◯ An exogenous variable.
- √ **An endogenous variable.**
- ◯ Constant always.
- √ **Constant in the long run.**

(b) (2 points) What was per capita GDP growth in the United States over 1870-2012?

- ◯ 1%
- √ **2%**
- ◯ 3%
- ◯ 4%
- ◯ 5%

(c) (2 points) What was GDP growth in the United States over 1870-2012?

- ◯ 1.5%
- ◯ 2.5%
- √ **3.5%**
- ◯ 4.5%
- ◯ 5.5%

(d) (2 points) What are the three different approaches for measuring GDP?

- √ **The production approach.**
- ◯ The nominal approach.
- √ **The expenditure approach.**
- ◯ The profit approach.
- √ **The income approach.**

(e) (2 points) In the Solow model, if we assume that capital depreciation rates are the same across all countries, differences in per capita output can be explained by:

- ◯ the steady state capital stock.
- ◯ the initial capital stock and saving rates.
- √ **differences in productivity and saving rates.**
- ◯ the labor stock and saving rates.
- ◯ None of these answers are correct.

(f) (2 points) An implication of the Solow model is that once an economy reaches the steady state:

- √ **per capita consumption is constant.**
- ◯ per capita output is constant, but per capita capital is not
- ◯ per capita capital is variable
- ◯ per capita consumption continues to grow
- ◯ per capita consumption is growing.

(g) (2 points) In the Solow model, if productivity of country A is about three times as large as the productivity in country B, and the savings rate of country A is about twice as large as the savings rate in country B then in the steady state output of country A is approximately:

- ◯ 6 times as large as in country B.
- ◯ 2 times as large as in country B.
- ◯ 5 times as large as in country B.
- √ **7 times as large as in country B.**
- ◯ 9 times as large as in country B.

Assume that production is given by the following production function: $Y_t = A_t K_t^a L_t^b$, and that $A_t = \bar{A}$. Give **short answers**, you won't be penalized if you do not write sentences.

(h) (1 point) What is the name of this production function?

Solution: This is a Cobb-Douglas production function.

(i) (1 point) What is the relationship between a and b in this production function, if returns to scale are constant? (with respect to capital and labor)

Solution: $a + b = 1$

(j) (2 points) Why do macroeconomists use this production function, a power function of the quantity of capital and the quantity of labor? What statistical regularity makes them confident to use this function?

Solution: Macroeconomists use this production function, because of the fact that the share of labor in GDP, and the share of capital in GDP are constants despite major changes in wages over time, the entering of women of the labor force, the changing of interest rates due to demographic change, etc. With Cobb-Douglas, the formula for the marginal product of capital shows that the share of capital in GDP is constant:

$$r_t = a\frac{Y_t}{K_t} \quad \Rightarrow \quad \frac{r_t K_t}{Y_t} = a.$$

Similarly, the share of labor in GDP is given by a constant:

$$w_t = b\frac{Y_t}{L_t} \quad \Rightarrow \quad \frac{w_t L_t}{Y_t} = b.$$

In fact, one can even show that **Cobb-Douglas** is the only production function which has this property, regarless of factor (labor, capital) prices.

(k) (1 point) What are a and b equal to typically?

Solution: Typical values are: $a = 1/3$ and $b = 2/3$.

(l) (1 point) Why are they equal to these numbers?

Solution: The share of capital in value added is approximately 1/3, and that of labor is approximately 2/3.

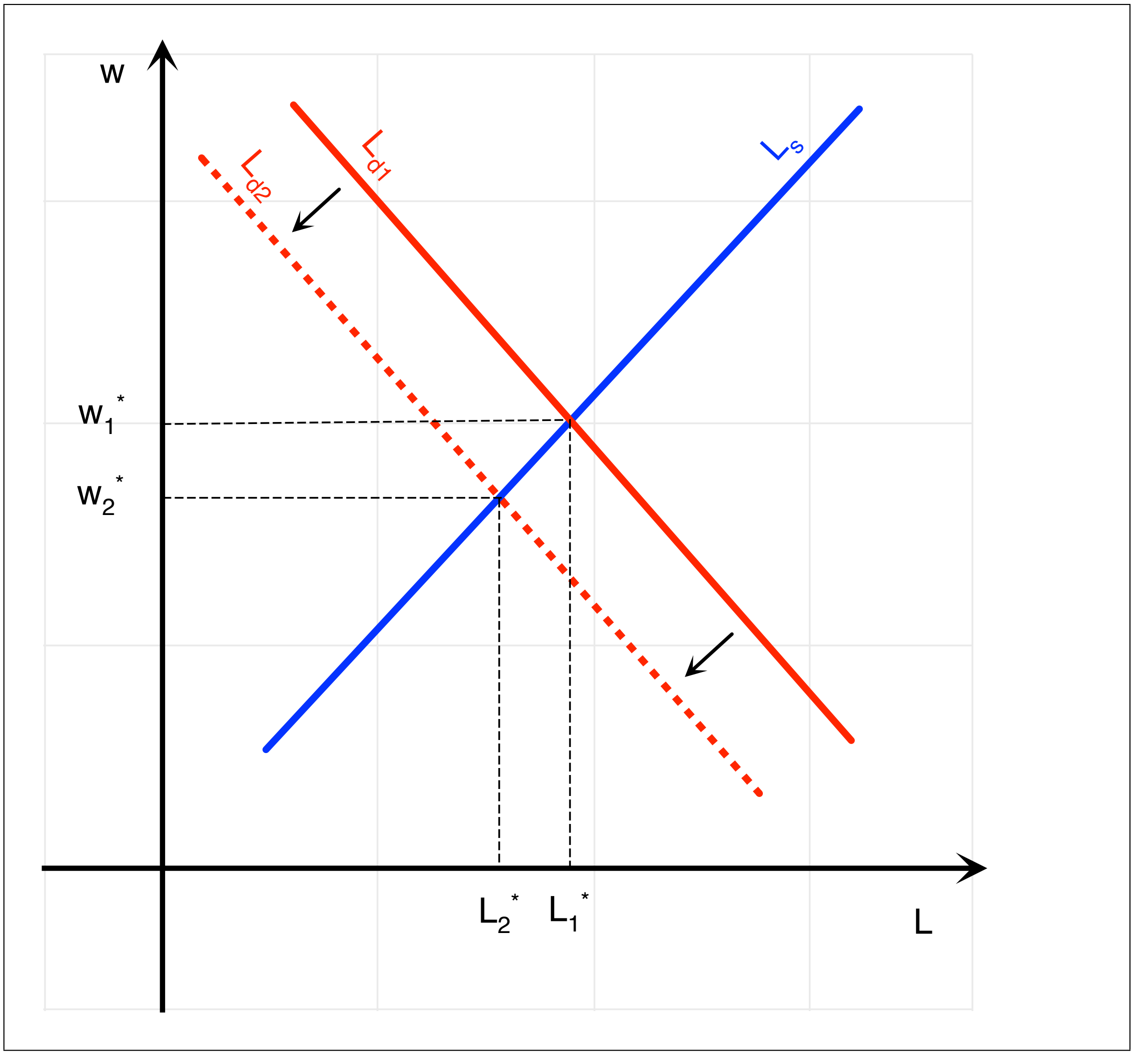
2 The Labor Market Model (10 points)

2. (10 points) Consider the following model of the labor market, with labor supply L^s and labor demand L^d respectively given as a function of the wage by:

$$L^s = \bar{a}w + \bar{l}$$
$$L^d = \bar{f} - w.$$

(a) (4 points) Represent the labor market model in the following box, showing what happens if we consider a decrease in firms' productivity $\bar{f}$. Represent the effect on the wage and the effect on employment.

Solution: See the graph below. One can see visually that both the price and the quantity of labor decrease.



(b) (3 points) Solve analytically for the equilibrium values of L^* and w^* in this model.

Solution: Using a little algebra, one can show that:

$$w^* = \frac{\bar{f} - \bar{l}}{1 + \bar{a}} \qquad L^* = \frac{\bar{a}\bar{f} + \bar{l}}{1 + \bar{a}}$$

(c) (3 points) Show analytically the effect on the L^* and w^* of the above decrease in $\bar{f}$.

Solution: When $\bar{f}$ decreases, it is clear from the above formula that both the equilibrium quantity of labor and the wage decrease.

3 Consumption Maximizing Savings Rate in the Solow Model (20 points)

3. (20 points) Consider the following Solow model. Assume that output Y_t is produced using capital K_t and labor L_t according to the following production function: $Y_t = \bar{A}K_t^{1/5}L_t^{4/5}$. (beware of the exponents !) The amount of labor is fixed $L_t = \bar{L}$, and a fraction of $\bar{s}$ of output is saved by agents in this economy, so that investment is $I_t = \bar{s}Y_t$. Capital depreciates every period at a rate $\bar{d}$.

   (a) (4 points) Write the law of motion for capital. (that is, K_{t+1} as a function of K_t) Solve for the steady state value of capital K^*, and the steady state value of Y^*, in this economy.

   Solution:

   $$\Delta K_{t+1} = K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t$$
   $$\Delta K_{t+1} = \bar{s}\bar{A}K_t^{1/5}\bar{L}^{4/5} - \bar{d}K_t$$

   Steady-state obtains with $\Delta K_{t+1} = 0$, so that:

   $$\bar{s}\bar{A}\left(K^*\right)^{1/5}\bar{L}^{4/5} - \bar{d}K^* = 0 \quad \Rightarrow \quad K^* = \bar{L}\left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{5/4}.$$

   Having found the steady state value of capital, we can calculate the output:

   $$Y^* = \bar{A}\left(K^*\right)^{1/5}\bar{L}^{4/5} = \bar{A}\left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{1/4}\bar{L} = \bar{A}^{5/4}\left(\frac{\bar{s}}{\bar{d}}\right)^{1/4}\bar{L}.$$

   (b) (4 points) Comment the dependence of Y^* with $\bar{A}$. Compare with the production model. Why is there a difference?

   Solution: There is an exponent 5/4. The production function has linearity in total factor productivity. The reason is that capital accumulation amplifies productivity differences, because more productive countries accumulate more capital, which increases their production further.

   (c) (4 points) Imagine that the government can enact policies aimed at targeting people's savings rate $\bar{s}$. Which savings rate would maximize steady-state production ? What would steady-state consumption then be equal to?

   Solution: Y^* is maximized when savings are maximum, so the savings rate maximizing steady state production would be $\bar{s} = 1$. However people would be starving in this economy. All its output would be used towards increased capital accumulation.

(d) (4 points) Using that the steady state value for consumption is equal to $C^* = (1-\bar{s})Y^*$, show that the savings rate $\bar{s}$ has two opposing effects on consumption. Give an intuition.

Solution: $(1-\bar{s})$ decreases in $\bar{s}$ while Y^* increases in $\bar{s}$. On the one hand, steady state consumption increases when savings increase because there is then more production. But on the other, consumption decreases because a higher fraction of production is then saved rather than consumed.

(e) (4 points) Use the previous expression for C^*, which is a function of $\bar{s}$, to calculate the steady-state consumption maximizing level of the savings rate $\bar{s}$ in the Solow model.

Solution: You need to maximize the following:

$$C^* = (1-\bar{s})\left(\frac{\bar{s}}{\bar{d}}\right)^{1/4}\left(\bar{A}\right)^{5/4}\bar{L}.$$

The first order condition gives (note you actually just need to maximize $(1-\bar{s})(\bar{s})^{1/4}$):

$$\frac{\partial Y^*}{\partial \bar{s}} = 0 \quad \Rightarrow \quad \frac{1}{4}\bar{s}^{-3/4} - \frac{5}{4}\bar{s}^{1/4} = 0 \quad \Rightarrow \quad \boxed{\bar{s} = \frac{1}{5} = 20\%}.$$

Adam Smith once wrote: "Consumption is the sole end and purpose of all production". Savings is good and beneficial to society to a certain limit, but there is no point in having a too high savings rate.

4 Solow Model with Land (20 points)

4. (20 points) We consider an economy with the following production function: $Y_t = \bar{A}K_t^{1/3}L_t^{2/3}N_t$, where the amount of land N_t is also entering as an input in the production function. (firms need land to produce, for example because they need offices) Assume that the number of people working is fixed and given by: $\bar{L}$. Capital depreciates at rate $\bar{d}$, and the savings rate is constant equal to $\bar{s}$. It is also assumed that there is a fixed quantity of land in the economy given by $\bar{N}$.

(a) (4 points) Express K_{t+1} as a function of as a function of Y_t and K_t.

Solution:

$$\Delta K_{t+1} = K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t.$$

(b) (6 points) Replace Y_t in the previous expression, to derive the law of motion for the capital stock, where only the capital stock and exogenous variables appear.

Solution:

$$\Delta K_{t+1} = \bar{s}\bar{A}\bar{L}^{2/3}\bar{N}K_t^{1/3} - \bar{d}K_t.$$

(c) (5 points) What is the marginal product of land?

Solution: The marginal product of land is given by:

$$\frac{\partial Y_t}{\partial N_t} = \bar{A}\bar{L}^{2/3}K_t^{1/3}.$$

(d) (5 points) Imagine the economy starts with an initially too low level of capital (relative to the steady state level). How does the marginal product of land varies over time? Why?

Solution: As the capital stock increases, the marginal product of land increases from the previous equation. This is intuitive: land being in fixed supply, its scarcity is becoming all the more limiting as the economy produces a lot. This helps explain why the price of land in practice covaries with GDP. (in booms, house prices are high, while they are low in recessions) It also helps explain why China has had soaring real estate prices recently as its economy was accumulating more capital, or as it was becoming more wealthy in general.

5 Some Numbers (15 points)

5. (15 points) On Table 1 below you may find some time series for US GDP, as well as for US CPI (indice) and US Population.

(a) (1 point) What does GDP stand for? (that is, what do the initials mean?)

Solution: GDP stands for Gross Domestic Product.

(b) (1 point) What does CPI stand for? (that is, what do the initials mean?)

Solution: CPI stands for Consumer Price Index

(c) (1 point) In which unit is US GDP expressed on Table 1?

Solution: In billion dollars.

(d) (1 point) In which unit is US Population expressed on Table 1?

Solution: In thousands.

(e) (1 point) What is US GDP per capita in 2014?

Solution: $\frac{\$17{,}348.10}{321{,}410.00} * \frac{1{,}000{,}000{,}000}{1{,}000} = \$53,974.99$

(f) (1 point) What was average annual US inflation during the period 1973-1975 ?

Solution: $\sqrt{\frac{53.83}{44.43}} - 1 = 10.07\%$

(g) (1 point) What was average annual US inflation during the period 2009-2011?

Solution: $\sqrt{\frac{224.93}{214.57}} - 1 = 2.39\%$

(h) (1 point) What was average GDP growth during the period 1973-1975 ?

Solution: $\sqrt{\frac{1{,}688.90}{1{,}428.50}} - 1 = 8.73\%$

(i) (1 point) What was average GDP growth during the period 2009-2011?

Solution: $\sqrt{\frac{15{,}517.90}{14{,}418.70}} - 1 = 3.74\%$

(j) (2 points) What was average annual Real Per Capita GDP growth during the period 1973-1975 ?

Solution: $\sqrt{\frac{1{,}688.90}{1{,}428.50}}\sqrt{\frac{44.43}{53.83}}\sqrt{\frac{211{,}857.00}{215{,}891.00}} - 1 = -2.14\%$

(k) (2 points) What was average annual Real Per Capita GDP growth during the period 2009-2011 ?

Solution: $\sqrt{\frac{15{,}517.90}{14{,}418.70}}\sqrt{\frac{214.57}{224.93}}\sqrt{\frac{307{,}374.00}{312{,}075.00}} - 1 = 0.56\%$

(l) (2 points) Your mother tells you she was earning \$10000 annually when she first started working in 1980. How much is this in 2014 dollars?

Solution: That is $\$10000 * \frac{236.71}{82.38} = \$28,733.11$ 2014 dollars. About the median wage for a single person in the US today.

Table 1: Time Series of US GDP, US CPI (indice), US Population

year	US GDP	US CPI (indice)	US Population
1962	605.10	30.25	186,475.00
1963	638.60	30.63	189,189.00
1964	685.80	31.04	191,820.00
1965	743.70	31.53	194,250.00
1966	815.00	32.47	196,508.00
1967	861.70	33.38	198,664.00
1968	942.50	34.79	200,664.00
1969	1,019.90	36.68	202,649.00
1970	1,075.90	38.84	204,982.00
1971	1,167.80	40.48	207,589.00
1972	1,282.40	41.81	209,838.00
1973	1,428.50	44.43	211,857.00
1974	1,548.80	49.32	213,815.00
1975	1,688.90	53.83	215,891.00
1976	1,877.60	56.93	217,999.00
1977	2,086.00	60.62	220,193.00
1978	2,356.60	65.24	222,525.00
1979	2,632.10	72.58	225,003.00
1980	2,862.50	82.38	227,622.00
1981	3,211.00	90.93	229,916.00
1982	3,345.00	96.53	232,128.00
1983	3,638.10	99.58	234,247.00
1984	4,040.70	103.93	236,307.00
1985	4,346.70	107.60	238,416.00
1986	4,590.20	109.69	240,593.00
1987	4,870.20	113.62	242,751.00
1988	5,252.60	118.28	244,968.00
1989	5,657.70	123.94	247,286.00
1990	5,979.60	130.66	250,047.00
1991	6,174.00	136.17	253,392.00
1992	6,539.30	140.31	256,777.00
1993	6,878.70	144.48	260,146.00
1994	7,308.80	148.23	263,325.00
1995	7,664.10	152.38	266,458.00
1996	8,100.20	156.86	269,581.00
1997	8,608.50	160.53	272,822.00
1998	9,089.20	163.01	276,022.00
1999	9,660.60	166.58	279,195.00
2000	10,284.80	172.19	282,296.00
2001	10,621.80	177.04	285,216.00
2002	10,977.50	179.87	288,019.00
2003	11,510.70	184.00	290,733.00
2004	12,274.90	188.91	293,389.00
2005	13,093.70	195.27	296,115.00
2006	13,855.90	201.56	298,930.00
2007	14,477.60	207.34	301,903.00
2008	14,718.60	215.25	304,718.00
2009	14,418.70	214.57	307,374.00
2010	14,964.40	218.08	309,761.00
2011	15,517.90	224.93	312,075.00
2012	16,155.30	229.60	314,402.00
2013	16,663.20	232.96	316,742.00
2014	17,348.10	236.71	321,410.00

6 A Simulated Solow Model (15 points)

6. (15 points) On Table 2 below you may find the time series of capital, production, investment, depreciation, generated by a simulation of the Solow growth model. Knowing that population is $\bar{L} = 1$, and that the share of capital in production is 1/3, you should be able to back out the parameters from the corresponding Solow model as well as guess what the missing values (denoted by dots) are. This is what the questions below ask you to do. (you do NOT need to use power functions for any of these)

   (a) (2 points) What is $\bar{A}$?

   Solution: $\bar{A} = \frac{\$7{,}922.96}{(\$1{,}450.00)^{1/3}}$

   (b) (2 points) What is $\bar{d}$?

   Solution: From one of the ratios from the Table you guess that: $\bar{d} = 5\%$

   (c) (2 points) What is $\bar{s}$?

   Solution: $\bar{s} = 30\%$

   (d) (1 point) What is K_0 ?

   Solution: \$1450

   (e) (1 point) Now you should be ready to fill in the dots. What is Y_3?

   Solution: \$15,315.40

   (f) (1 point) What is I_5?

   Solution: \$5,598.06

   (g) (1 point) What is K_{11}?

   Solution: \$48,896.90

   (h) (1 point) What is D_{11}?

   Solution: \$2,444.85

   (i) (1 point) What is K_{34}?

Solution: \$154,443.02

(j) (1 point) What is Y_{49}?

Solution: \$40,841.70

(k) (1 point) What is I_{49}?

Solution: \$12,252.51

(l) (1 point) What is D_{49}?

Solution: \$9,930.87

Table 2: WHAT PARAMETERS GENERATED THIS TABLE?

Time t	Capital K_t	Production Y_t	Investment I_t	Depreciation D_t	Capital K_{t+1}
0	$1,450.00	$7,922.96	$2,376.89	$72.50	$3,754.39
1	$3,754.39	$10,879.55	$3,263.87	$187.72	$6,830.53
2	$6,830.53	$13,281.58	$3,984.47	$341.53	$10,473.48
3	$10,473.48	....	$4,594.62	$523.67	$14,544.43
4	$14,544.43	$17,086.91	$5,126.07	$727.22	$18,943.28
5	$18,943.28	$18,660.21	....	$947.16	$23,594.18
6	$23,594.18	$20,077.04	$6,023.11	$1,179.71	$28,437.58
7	$28,437.58	$21,366.28	$6,409.88	$1,421.88	$33,425.58
8	$33,425.58	$22,548.85	$6,764.65	$1,671.28	$38,518.96
9	$38,518.96	$23,640.48	$7,092.14	$1,925.95	$43,685.15
10	$43,685.15	$24,653.35	$7,396.01	$2,184.26	$48,896.90
11	....	$25,597.16	$7,679.15	....	$54,131.21
12	$54,131.21	$26,479.75	$7,943.93	$2,706.56	$59,368.57
13	$59,368.57	$27,307.60	$8,192.28	$2,968.43	$64,592.42
14	$64,592.42	$28,086.13	$8,425.84	$3,229.62	$69,788.64
15	$69,788.64	$28,819.93	$8,645.98	$3,489.43	$74,945.19
16	$74,945.19	$29,512.95	$8,853.89	$3,747.26	$80,051.82
17	$80,051.82	$30,168.60	$9,050.58	$4,002.59	$85,099.80
18	$85,099.80	$30,789.85	$9,236.95	$4,254.99	$90,081.77
19	$90,081.77	$31,379.33	$9,413.80	$4,504.09	$94,991.48
20	$94,991.48	$31,939.36	$9,581.81	$4,749.57	$99,823.71
21	$99,823.71	$32,472.02	$9,741.61	$4,991.19	$104,574.13
22	$104,574.13	$32,979.15	$9,893.75	$5,228.71	$109,239.17
23	$109,239.17	$33,462.43	$10,038.73	$5,461.96	$113,815.94
24	$113,815.94	$33,923.38	$10,177.01	$5,690.80	$118,302.16
25	$118,302.16	$34,363.36	$10,309.01	$5,915.11	$122,696.06
26	$122,696.06	$34,783.63	$10,435.09	$6,134.80	$126,996.34
27	$126,996.34	$35,185.34	$10,555.60	$6,349.82	$131,202.13
28	$131,202.13	$35,569.55	$10,670.86	$6,560.11	$135,312.89
29	$135,312.89	$35,937.22	$10,781.16	$6,765.64	$139,328.41
30	$139,328.41	$36,289.24	$10,886.77	$6,966.42	$143,248.76
31	$143,248.76	$36,626.46	$10,987.94	$7,162.44	$147,074.26
32	$147,074.26	$36,949.64	$11,084.89	$7,353.71	$150,805.44
33	$150,805.44	$37,259.50	$11,177.85	$7,540.27	$154,443.02
34	$154,443.02	$37,556.70	$11,267.01	$7,722.15	....
35	$157,987.88	$37,841.87	$11,352.56	$7,899.39	$161,441.05
36	$161,441.05	$38,115.59	$11,434.68	$8,072.05	$164,803.68
37	$164,803.68	$38,378.41	$11,513.52	$8,240.18	$168,077.02
38	$168,077.02	$38,630.84	$11,589.25	$8,403.85	$171,262.42
39	$171,262.42	$38,873.36	$11,662.01	$8,563.12	$174,361.30
40	$174,361.30	$39,106.42	$11,731.93	$8,718.07	$177,375.16
41	$177,375.16	$39,330.46	$11,799.14	$8,868.76	$180,305.54
42	$180,305.54	$39,545.86	$11,863.76	$9,015.28	$183,154.02
43	$183,154.02	$39,753.03	$11,925.91	$9,157.70	$185,922.23
44	$185,922.23	$39,952.30	$11,985.69	$9,296.11	$188,611.81
45	$188,611.81	$40,144.03	$12,043.21	$9,430.59	$191,224.43
46	$191,224.43	$40,328.54	$12,098.56	$9,561.22	$193,761.77
47	$193,761.77	$40,506.13	$12,151.84	$9,688.09	$196,225.52
48	$196,225.52	$40,677.09	$12,203.13	$9,811.28	$198,617.37
49	$198,617.37	....	....	....	$200,939.01
50	$200,939.01	$41,000.21	$12,300.06	$10,046.95	$203,192.13