ECON 102 - Winter 2016
Instructor: François Geerolf

Last Name: ____________

First Name: ____________

Student ID (UID): ____________

Final Exam
March 17, 2016

Signature: ____________

Time Limit: 180 Minutes

TA Name: ____________

Test A

This exam contains 25 pages, including this cover page and **4 pages of scratch paper** at the end. The time limit is **180 Minutes**. You can earn 100 points. There are 40 multiple choice questions, worth 1 point each. There are then 30 short answer questions, worth 2 points each, divided in three problems.

Instructions:

1. Print your Last name, First Name, Student ID Number, Signature, and TA Name (Flavien Moreau, Giovanni Nicolo, Santiago Justel, or Shuo Liu) at the top right-hand corner of this page, as well as on your scantron.

2. The only items that should be on your desk are pencils and/or pens, a Scantron 882-E, and the calculator Canon LS-100TS, described in the syllabus. NO other items are allowed.

Good luck. Budget your time wisely. (skip the question or even the exercice if you get stuck)

Grade Table (FOR GRADER USE ONLY)

Question	Points	Score
A	40	
B	20	
C	20	
D	20	
Total:	100	

Multiple Choice (40 pts)

A. (40 pts) There are 40 multiple choice questions. **Multiple responses may be correct, and at least one response is correct**. Use your scantron (Results not reported on the scantron will not be taken into account). Again, write your first and last name on the scantron. You get either 0 or 1 point: if multiple answers are correct you get the one point only if you have them all ticked.

---

(1) (1 pt) What is an endogenous variable?

A. An input that can change over time, but determined ahead of time by the model builder.

B. An outcome of the model.

C. An input that is fixed over time, except when the model builder changes it for an experiment.

D. Total factor productivity in the Solow model.

E. The stock of capital in the Solow model.

---

(2) (1 pt) What was per capita GDP growth in the United States over 1870-2012?

A. 1.5%

B. 2%

C. 2.5%

D. 3%

E. 3.5%

---

(3) (1 pt) What was GDP growth in the United States over 1870-2012?

A. 0.5%

B. 1.5%

C. 2.5%

D. 3.5%

E. 4.5%

Long Run

(4) (1 pt) Suppose a worker replaces his computer with a better one; this would:

A. decrease the nominal wage

B. increase the marginal product of labor

C. increase total factor productivity

D. increase the amount of leisure demanded

E. None of these answers are correct

(5) (1 pt) In the Solow model, the level of GDP per capita is:

A. A parameter.

B. An exogenous variable.

C. An endogenous variable.

D. Constant always.

E. Constant in the long run.

(6) (1 pt) What are the three different approaches for measuring GDP?

A. The production approach.

B. The nominal approach.

C. The expenditure approach.

D. The income approach.

E. The profit approach.

(7) (1 pt) In the Solow model, if we assume that capital depreciation rates are the same across all countries, differences in per capita output can be explained by:

A. the steady state capital stock.

B. the initial capital stock and saving rates.

C. differences in productivity and saving rates.

D. the labor stock and saving rates.

E. None of these answers are correct.

(8) (1 pt) In the Solow model, once the economy reaches a steady state:

A. per capita output is constant, but per capita capital is not

B. per capita capital is variable

C. per capita consumption continues to grow

D. per capita consumption is constant.

E. per capita consumption is growing.

(9) (1 pt) In the Solow model, if the share of capital in production is 1/3, and if the productivity of country A is about four times as large as the productivity in country B, and the savings rate of country A is about twice as large as the savings rate in country B then in the steady state output of country A is approximately:

A. 5 times as large as in country B.

B. 7 times as large as in country B.

C. 9 times as large as in country B.

D. 11 times as large as in country B.

E. 13 times as large as in country B.

Short Run

(10) (1 pt) When we add the financial friction to the AD curve it:

A. is represented by a downward movement along the AD curve

B. is represented by a downward movement along the AS curve

C. shifts the AD curve down

D. shifts the AS curve up

E. has no impact on the AD curve.

---

(11) (1 pt) "DSGE" stands for:

A. deterministic simulated generalized estimation.

B. demand supply general estimation.

C. dynamic stationary generated equilibrium.

D. dynamic stochastic general equilibrium.

E. demand and supply of government expenditures.

---

(12) (1 pt) Consider the consumption function $C_t/\bar{Y}_t = \bar{a}_c + \bar{x}\tilde{Y}_t$. If the marginal propensity to consume is $\bar{x} = 25\%$, then a 3 percent positive demand shock:

A. raises short-run output by 0.75 percent

B. raises short-run output by 2.25 percent

C. raises short-run output by 4 percent

D. raises short-run output by 5 percent

E. has no impact on short-run output

---

(13) (1 pt) With adaptive expectations, the Phillips curve can be written as:

A. $\Delta\pi_t = \pi_{t-1} + n\bar{u}\tilde{Y}_t$

B. $\Delta\pi_t = \bar{\nu}u_t$

C. $\pi_t = \pi_{t-1} + \bar{\nu}\tilde{Y}_t + \bar{o}$

D. $\pi_t = \pi_{t-1}$

E. $\Delta\pi_t = \bar{\nu}\tilde{Y}_t + \bar{o}$

---

(14) (1 pt) Which of the following best describes why the aggregate demand curve slopes downward ?

A. If the central bank observes a high rate of inflation, the monetary policy rule dictates an increase in the real interest rate. The high interest rate reduces output by reducing investment demand in the economy.

B. If the central bank observes a low rate of inflation, the monetary policy rule dictates an increase in the real interest rate. The high interest rate reduces output by reducing investment demand in the economy.

C. If the central bank observes a high rate of inflation, the monetary policy rule dictates a decrease in the real interest rate. The low interest rate increases output by reducing investment demand in the economy.

D. If the central bank observes a low rate of inflation, the monetary policy rule dictates a decrease in the real interest rate. The low interest rate reduces output by reducing investment demand in the economy.

E. None of these answers is correct.

---

(15) (1 pt) According to the Fisher equation, if the nominal interest rate is 0% and there is 2% expected inflation, then the expected real interest rate is:

A. -4%

B. -2%

C. 0%

D. 2%

E. None of the above.

---

(16) (1 pt) Assume that investment responds to interest rates in such a way that $\bar{b} = 0.5$. If $\bar{a}$ rises by 2% and the real interest rate falls by 2%, short-run output:

A. falls by 2 percent

B. rises by 1 percent

C. rises by 3 percent

D. falls by 1 percent

E. does not change

---

(17) (1 pt) If $\bar{m} = 5$, how much does inflation need to be above target to justify raising interest rates above the marginal product of capital by 1%?

A. 0.1%

B. 0.2%

C. 0.25%

D. 2.5%

E. 5%

---

(18) (1 pt) The (AD) curve can be seen as:

A. A combination of the (IS) curve and the Phillips curve.

B. A combination of the (IS) curve and the (LM) curve.

C. A combination of the (IS) curve and the monetary policy rule.

D. A combination of the demand curve for labor and the demand curve for investment.

E. A combination of the (AS) curve and the monetary policy rule.

---

(19) (1 pt) Assume that the central bank decides to raise its inflation target from 2% to 4%, and that people have adaptive expectations. What happens to inflation and why?

A. Inflation rises because the central bank said so.

B. Inflation rises because the central bank is now responding less to deviations of inflation from the target inflation.

C. Inflation decreases because aggregate demand is too low at this level of inflation.

D. Inflation rises because the central bank now sets a lower interest rate for a given level of inflation.

E. Inflation decreases because of sticky expectations.

---

(20) (1 pt) What happens if there is a boom in China, which leads China to import more goods from the US?

A. The US faces an positive aggregate demand shock.

B. The US faces a negative aggregate demand shock.

C. The (IS) curve shifts right.

D. The (AD) curve shifts right.

E. This does not change anything for the US.

---

(21) (1 pt) Assume that firms start increasing their prices more when they are faced with a higher demand from consumers. What happens?

A. The (AD) curve shifts right.

B. The (AD) curve steepens.

C. The (AD) curve flattens.

D. The (AS) curve flattens.

E. The (AS) curve steepens.

---

(22) (1 pt) What happens in the (AS)/(AD) framework if investment starts to respond more to interest rate changes?

A. $\bar{m}$ increases.

B. $\bar{b}$ increases.

C. The (AS) curve steepens.

D. The (AD) curve flattens.

E. The (AD) curve steepens.

---

(23) (1 pt) Assume that the central bank decides to raise its inflation target from 2% to 4%. What happens to the (AD) curve?

A. The (AD) curve does not move.
B. The (AD) curve steepens.
C. The (AD) curve flattens.
D. The (AD) curve shifts right.
E. The (AD) curve shifts left.

---

(24) (1 pt) Assume that your first salary coming out of UCLA is \$42,000, that your boss gives you a promotion every year of 2%. Assuming that the interest rate is 5%, how much would your human capital be worth if you were able to live forever?
A. \$1,200,000.
B. \$1,270,000.
C. \$1,400,000.
D. \$1,470,000.
E. \$1,600,000.

---

(25) (1 pt) Assume that your first salary coming out of UCLA is \$70,000, that your boss gives you a promotion every year of 10%. Assuming that the interest rate is 3%, how much will you earn approximately over the next 8 years, in present value?
A. \$663,000.
B. \$713,000.
C. \$763,000.
D. \$813,000.
E. \$863,000.

Bathtub Model

In subsequent multiple choice questions, we consider the following "bathtub" model of unemployment. There are $\bar{L} = 161,000,000$ people in the labor force. Jobs separate at a rate $\bar{s} = 3\%$ every year, and the annual rate at which unemployed people find new jobs is $\bar{f} = 60\%$. Finally, the number of people unemployed is denoted by U_t and the number of people employed as E_t. The unemployment rate is denoted by u_t, and the long run unemployment rate by u^*.

---

(26) (1 pt) Which equality (equalities) is (are) true?
A. $U_{t+1} = \bar{f}U_t - \bar{s}E_t$.
B. $\Delta U_{t+1} = \bar{f}U_t - \bar{s}E_t$.
C. $U_{t+1} - U_t = \bar{f}U_t - \bar{s}(\bar{L} - U_t)$.
D. $U_{t+1} - U_t = \bar{f}U_t - \bar{s}E_t$.
E. $\Delta U_{t+1} = \bar{s}E_t - \bar{f}U_t$**.**

---

(27) (1 pt) Assume that $u_0 = 10\%$. What is the value of u^*?

A. 3.51%.

B. 4.22%.

C. 4.76%**.**

D. 5.02%.

E. 6.16%.

---

(28) (1 pt) Assume that $u_0 = 12\%$. What is the unemployment rate at time 1 in this economy?

A. 4.22%.

B. 5.44%.

C. 6.22%.

D. 7.44%

E. 8.22%.

---

(29) (1 pt) Assume that $u_0 = 12\%$. Approximately how many people are unemployed at time 2 in this economy?

A. 8,262,000.

B. 8,762,000.

C. 9,262,000.

D. 9,762,000.

E. 10,762,000.

---

(30) (1 pt) How many people find a new job every year in the long run steady state?

A. 0.

B. 2,300,000.

C. 4,600,000.

D. 6,900,000.

E. With the given elements, one cannot know for sure.

---

(31) (1 pt) How many people lose their job every year in the long run steady state?

A. 0.

B. 2,300,000.

C. 4,600,000.

D. 6,900,000.

E. With the given elements, one cannot know for sure.

Two-Period Consumption Model

Consider the two period consumption model solved during the class, and assume that $\beta = \frac{1}{3}$ and utility takes the log form, that is $u(c) = \log(c)$. Suppose the real interest rate is 5 percent, that is $R = 5\%$. Assume that initial assets are $f_0 = \$50,000$, and the path of labor income is $y_0 = \$100,000$ and $y_1 = \$1,000,000$.

---

(32) (1 pt) How many different methods can be used to derive the optimality condition for the consumer?

A. 1.

B. 2.

C. 3.

D. 4.

E. 5.

---

(33) (1 pt) What fraction of his total wealth does the consumer consume in period 0?

A. 0%

B. 25%

C. 50%.

D. 75%.

E. 100%.

---

(34) (1 pt) What is the consumer's approximate human wealth?

A. $942,000.

B. $990,000.

C. $1,052,000.

D. $1,102,000.

E. $1,150,000.

---

(35) (1 pt) How much does the consumer save approximately (in addition to his initial assets) in period 0?

A. $193,000.

B. $204,000.

C. $215,000.

D. $226,000.

E. $236,000.

---

(36) (1 pt) How much does the consumer approximately consume in period 1?

A. $237,000.

B. \$268,000.
C. \$289,000.
D. \$320,000.
E. \$345,000.

Government and the Macroeconomy

(37) (1 pt) Which of the following statement(s) is (are) true?
A. The real interest rate is always non negative.
B. Public debt cannot be higher than the level of GDP.
C. Government debt reduces capital accumulation.
D. Government debt is always a problem.
E. Government debt always leads to future higher taxes.

---

(38) (1 pt) In a two period model, the government budget constraint writes:
A. $T_1 + T_2 = G_1 + G_2 + B_1$.
B. $T_1 - G_1 = (1+i)(G_2 - T_2) + B_1$.
C. $(1+i)B_1 = (T_1 - G_1) + \frac{T_2 - G_2}{1+i}$**.**
D. $T_1 - G_1 = (1+i)(G_2 - T_2) - (1+i)B_1$
E. $T_1 + \frac{T_2}{1+i} = G_1 + \frac{G_2}{1+i} + (1+i)B_1$.

---

(39) (1 pt) Who studied the impact of government debt in an infinite horizon model?
A. John M. Keynes.
B. Milton Friedman.
C. Peter A. Diamond.
D. John B. Taylor.
E. Ben S. Bernanke.

---

(40) (1 pt) In an infinite horizon model, long-run government debt:
A. Will always equal to zero.
B. Can raise every generation's consumption.
C. May increase production.
D. Will increase capital accumulation.
E. Will decrease capital accumulation.

An Oil Price Shock in the AS/AD Model (20 points)

B. (20 pts) Take the usual AS/AD model, ruling out Aggregate Demand shocks, so with $\bar{a} = 0$, but assuming a one-time, unexpected oil price shock $\bar{o}_0 > 0$. One time means that the oil price shock lasts only for one period, in period $t = 0$, and that $\bar{o}_t = 0$ for all subsequent $t \in \{1, 2, ...\}$. Unexpected means that the economy was originally in steady-state, and in particular that $\pi_0 = \bar{\pi}$. Unless otherwise noted, agents have adaptive expectations about inflation. The economy is described by an AS/AD model.

(1) (2 pts) What are the values of π_1 and $\tilde{Y}_1$ in terms of the parameters of the model? (in particular the size of the oil price shock, $\bar{o}_0$)

Solution: The AS/AD equations are:

$$\pi_1 = \bar{\pi} + \bar{\nu}\tilde{Y}_1 + \bar{o}_0$$
$$\tilde{Y}_1 = -\bar{b}\bar{m}\left(\pi_1 - \bar{\pi}\right).$$

Using the second equation to plug in the first, one gets:

$$\tilde{Y}_1 = -\bar{b}\bar{m}\left[\bar{\nu}\tilde{Y}_1 + \bar{o}_0\right] \quad \Rightarrow \quad \tilde{Y}_1 = -\frac{\bar{b}\bar{m}}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{o}_0.$$

Using this to replace in one or the other equation, one gets:

$$\boxed{\pi_1 = \bar{\pi} + \frac{1}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{o}_0}.$$

(2) (2 pts) Show in mathematical terms the effect of a more agressive monetary policy on inflation and short-run output in period 1: do inflation and short-run output increase or decrease with a more agressive monetary policy?

Solution:

$$\frac{\partial \pi_1}{\partial \bar{m}} = -\frac{\bar{b}\bar{\nu}}{(1 + \bar{b}\bar{m}\bar{\nu})^2}\bar{o}_0.$$
$$\frac{\partial \tilde{Y}_1}{\partial \bar{m}} = -\frac{\bar{b}}{(1 + \bar{b}\bar{m}\bar{\nu})^2}\bar{o}_0.$$

Both decrease. Note that you could see directly from the formula for π_1 that it is decreasing in $\bar{m}$. For $\tilde{Y}_1$, there is a bit more work as:

$$\tilde{Y}_1 = -\frac{\bar{b}\bar{m}}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{o}_0 = -\frac{\bar{b}\bar{m}\bar{\nu}}{1 + \bar{b}\bar{m}\bar{\nu}}\frac{\bar{o}_0}{\bar{\nu}} = \left(\frac{1}{1 + \bar{b}\bar{m}\bar{\nu}} - 1\right)\frac{\bar{o}_0}{\bar{\nu}}.$$

Since inflation was increasing initially, this means there is a more muted response of inflation. To the contrary, since output was already decreasing, this means that the response of short-run output is actually more important. With a more agressive monetary policy, the bulk of the adjustment goes through unemployment, and a decrease in short-run output. (Note: of course, adaptive expectations neglect the fact that if monetary policy was expected to be agressive, then people may anticipate that inflation will be lower in future periods, which on the contrary mitigates the needed adjustment of short-run output).

(3) (2 pts) Illustrate this on two graphs with the AS/AD curves: show one AS/AD diagram with a soft monetary policy, and next to it another AS/AD diagram with an agressive monetary policy. (**please indicate explicitly which is which**) Show π_1 and $\tilde{Y}_1$ as well as the long run values of inflation and short-run output on these graphs.

Solution: See TA section.

(4) (2 pts) What are the values of $\tilde{Y}_1$ and π_1 when the central bank does not respond at all to changes in inflation?

Solution: When the central bank does not respond at all to changes in inflation, then the parameter in the monetary policy rule is $\bar{m} = 0$. Then, from the previous formula we get:

$$\tilde{Y}_1 = 0, \qquad \pi_1 = \bar{\pi} + \bar{o}_0.$$

(5) (2 pts) What are the values of $\tilde{Y}_1$ and π_1 when the central bank responds with a parameter $\bar{m} = +\infty$ in the monetary policy rule?

Solution: When the central bank responds very agressively with $\bar{m} = +\infty$, we get for short-run output:

$$\tilde{Y}_1 = -\frac{\bar{b}\bar{m}}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{o}_0 = -\frac{\bar{b}}{\frac{1}{\bar{m}} + \bar{b}\bar{\nu}}\bar{o}_0 = -\frac{\bar{o}_0}{\bar{\nu}},$$

since $1/\bar{m}$ goes to 0 as $\bar{m}$ goes to infinity (i derived this during the class). For inflation we get:

$$\pi_1 = \bar{\pi}.$$

(6) (2 pts) What is the intuition behind the result in the previous question?

Solution: When the central bank responds very aggressively to any change in inflation, all the effect of the oil price shock is accommodated by lower short run output which, through the (AS) curve, can bring inflation to target if it decreases by a sufficient amount. Note that the recession is all the more severe that $\bar{\nu}$ is small: if firms do not respond a lot to variations in short-run output by lowering their prices, then it needs a large recession to bring inflation back to target.

(7) (2 pts) Find a difference equation for π_t, for $t \in \{2, 3, ...\}$.

Solution: Because people have adaptive expectations, he have the following equations:

$$\pi_t = \pi_{t-1} + \bar{\nu}\tilde{Y}_t + \bar{o}_{t-1}$$
$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\pi_t - \bar{\pi}\right).$$

Solving this system of two equations and two unknowns (where the unknowns are $(\tilde{Y}_t, \pi_t)$), it is easy to show that inflation follows the following difference equation:

$$\pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi} + \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\bar{o}_{t-1}.$$

Moreover we have that $\bar{o}_{t-1} = 0$ for all $t \in \{2, 3, ...\}$, because the shock is a one time shock. So finally:

$$\forall t \in \{2, 3, ...\}, \quad \pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi}$$

(8) (2 pts) Solve for the previous difference equation, and get π_t as a function of time and parameters.

Solution: We have:

$$\pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi} \quad \Rightarrow \quad \pi_t - \bar{\pi} = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\left(\pi_{t-1} - \bar{\pi}\right).$$

This difference equation iterates (just as those in the Romer model) through:

$$\pi_t - \bar{\pi} = \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t-1}(\pi_1 - \bar{\pi}).$$

Using the expression for π_1 in the previous question:

$$\pi_1 = \bar{\pi} + \frac{\bar{o}_0}{1+\bar{b}\bar{m}\bar{\nu}},$$

This gives:

$$\pi_t = \bar{\pi} + \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^t \bar{o}_0.$$

(9) (2 pts) Use the (AD) curve to then calculate $\tilde{Y}_t$ as a function of time and the parameters of the model.

Solution: We use the AD curve which is:

$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\pi_t - \bar{\pi}\right).$$

with the expression found in the previous question:

$$\pi_t = \bar{\pi} + \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^t \bar{o}_0 \qquad \Rightarrow \qquad \tilde{Y}_t = -\bar{b}\bar{m}\left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^t \bar{o}_0.$$

(10) (2 pts) Numerical Application: Suppose the parameters of the (AS) and (AD) curves take the following values: $\bar{o}_0 = 8\%$, $\bar{a} = 0$, $\bar{b} = 2$, $\bar{m} = 1$, $\bar{\nu} = 1$, and $\bar{\pi} = 4\%$. Solve for the value of short-run output and the inflation rate for the first 2 years after the shock. (**give an approximation**)

Solution: Therefore, numerically:

$$\frac{1}{1+\bar{b}\bar{m}\bar{\nu}} = 0.3333.$$

Time	**Inflation** π_t	**Short-Run Output** $\tilde{Y}_t$
0	4 %	0 %
1	6.67%	-5.33%
2	4.89%	-1.78%

Long-Run / Microfoundations: a Growth model with an endogenous saving behavior (20 points)

C. Consider a closed economy with the following Cobb-Douglas, constant returns to scale, production function: (beware of the exponents !)

$$Y_t = \bar{A}K_t^{1/4}L_t^{3/4}.$$

Assume that the labor force is fixed to unity: $L_t = \bar{L} = 1$. Assume that labor is taxed at rate τ (that is, if the employer pays w_t in wages, the worker receives only $(1-\tau)w_t$). Assume that capital depreciates at rate $\bar{d} = 1 = 100\%$. (that is, capital fully depreciates each period)

(1) (2 pts) Assuming firms are competitive and maximize profits, what is the wage w_t paid by employers in period t as a function of $\bar{A}$ and the level of the capital stock in period t, K_t ?

Solution: The wage paid by employers is given by:

$$w_t = \frac{3}{4}\bar{A}K_t^{1/4}\bar{L}^{-1/4}$$

Therefore, because $\bar{L} = 1$, we have:

$$w_t = \frac{3}{4}\bar{A}K_t^{1/4}.$$

(2) (2 pts) What is the wage received by workers, as a function of τ, $\bar{A}$ and the level of the capital stock in period t, K_t ?

Solution: The net of tax wage is given by:

$$(1-\tau)w_t = (1-\tau)\frac{3}{4}\bar{A}K_t^{1/4}.$$

(3) (2 pts) Represent the labor market (labor supply and labor demand), with employment on the x axis, and the wage w_t on the y axis. Show the effect of the imposition of a tax. Show the deadweight loss.

Solution: See TA section

(4) (2 pts) Who effectively pays the tax? What is the economic intuition for that? Taking K_t as given, what is the effect of the tax on output Y_t?

Solution: The worker pays the tax. This is because we assumed that the labor force was fixed. There is no effect of the tax on output, as given K_t, output is given by:

$$Y_t = \bar{A}K_t^{1/4},$$

where τ does not appear.

From now on, and until the end of this exercice, we assume that $\tau = 0$. Moreover, instead of assuming that savings are a fixed fraction $\bar{s}$ of output, we will instead derive the saving behavior from microfoundations. We assume that people in this economy live only for two periods. People from generation t are young in period t, and old in period $t+1$. We denote their consumption when young by c_t^y and their consumption when old by c_{t+1}^o. Assume that lifetime utility is logarithmic with $\beta = 1$:

$$U = \log(c_t^y) + \log(c_{t+1}^o).$$

There is always two generations living in period t: the previous period's young, born in period $t-1$, now old, consuming the return from their savings; and this period's young, newly born (in period t). In questions 5-8, assume that people work when young, and then receive a wage given by w_t. They retire when old, and then do not work.

(5) (2 pts) Calculate c_t^y and c_{t+1}^o as a function of w_t. (you can give the solution without an explanation) Calculate saving by the young, given by $w_t - c_t^y$.

Solution: We have:

$$c_t^y = \frac{w_t}{2} \qquad c_{t+1}^o = (1+R)\frac{w_t}{2}.$$

Saving is given by:

$$w_t - c_t^y = \frac{w_t}{2}.$$

(6) (2 pts) What is the relationship between savings by the young and investment in this economy? (Hint: remember that the economy is closed !) Write the law of motion for capital as a function of the parameters of the model. Note that it takes a very similar form as that in the Solow model. What is the savings rate in this model corresponding to $\bar{s}$ in the Solow model?

Solution: Because it is a closed economy, we have that net exports are zero, and savings must finance investment at home:

$$I_t = s_t = w_t - c_t^y = \frac{w_t}{2}.$$

We have that (since $\bar{d} = 1$):

$$\Delta K_{t+1} = I_t - K_t$$

Therefore:

$$\Delta K_{t+1} = \frac{w_t}{2} - K_t.$$

Because we have:

$$w_t = \frac{3}{4} Y_t.$$

The law of motion for capital is therefore:

$$\Delta K_{t+1} = \frac{1}{4} \bar{A} K_t^{1/4} - \bar{d} K_t.$$

This is the capital accumulation equation (or law of motion for capital) of the Solow model, with $\bar{s} = 1/4$.

(7) (2 pts) Assume now (in questions 7-10) that people work two thirds of the time when young, and another third when old, earning wage $2w_t/3$ in the first period, and $w_t/3$ in the second period. Write the arbitrage condition for the firm between the marginal product of capital, depreciation, and the interest rate R, assuming that the price of capital is constant and equal to one.

Solution: The arbitrage condition is: $R = MPK - \bar{d} = MPK - 1$.

(8) (2 pts) What is an expression for R as a function of K_t and the parameters of the model?

Solution: We have that:

$$MPK = \frac{1}{4} \bar{A} K_t^{-3/4}.$$

Therefore:

$$R = \frac{1}{4} \bar{A} K_t^{-3/4} - \bar{d} = \frac{1}{4} \bar{A} K_t^{-3/4} - 1.$$

(9) (2 pts) Write the law of motion for capital ONLY as a function of the parameters of the model. (in particular, substitute out R)

Solution: Savings are now given by:

$$\frac{2w_t}{3} - \frac{1}{2}\left(\frac{2w_t}{3} + \frac{w_t}{3(1+R)}\right) = \frac{w_t}{3} - \frac{w_t}{6(1+R)}.$$

Change in the capital stock is savings minus depreciation:

$$\Delta K_{t+1} = \frac{w_t}{3} - \frac{w_t}{6(1+R)} - \bar{d} K_t.$$

One uses:

$$1 + R = \frac{1}{4}\bar{A}K_t^{-3/4} \qquad w_t = \frac{3}{4}\bar{A}K_t^{1/4}.$$

Therefore, the law of motion for capital is, with $\bar{d} = 1$:

$$\Delta K_{t+1} = \frac{1}{4}\bar{A}K_t^{1/4} - \frac{1}{2}K_t - \bar{d}K_t.$$

Therefore:

$$\Delta K_{t+1} = \frac{1}{4}\bar{A}K_t^{1/4} - \frac{3}{2}K_t.$$

(10) (2 pts) Why does it differ strikingly from the Solow model?

Solution: Note that in order to have the previously derived law of motion square with the Solow model, one would need the following parameters: $\bar{s} = 1/4$, and $\bar{d} = 150\%$. The implied depreciation rate, of course, does not make any economic sense: would be impossible that more than the previously accumulated capital stock depreciate each period.

The reason why this model differs strikingly from the Solow model is that because of the wealth effect, R impacts also how much is saved today. When the capital stock is higher, R is lower, and savings are also lower as a consequence. Therefore there are two things that enter negatively in the law of motion for capital:

- Depreciation: the usual $\bar{d}K_t$ term.
- The effect of interest rates (wealth effects): the (unusual) $\frac{1}{2}K_t$ term.

Long-Run / Solow-Romer Model (20 points)

D. Consider the following Solow model. Output Y_t is produced using capital K_t and labor L_t according to the production function: $Y_t = \bar{A}K_t^{1/4}L_t^{3/4}$. (note the exponents !) The amount of labor is fixed $L_t = \bar{L}$, and a fraction of $\bar{s}$ of output is saved by agents in this economy. Capital depreciates every period at a rate $\bar{d}$.

(1) (2 pts) Write the law of motion for capital. Solve for the steady state value of capital K^*, and the steady state value of Y^*, in this economy.

Solution:

$$\Delta K_{t+1} = K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t$$
$$= \bar{s}\bar{A}K_t^{1/4}\bar{L}^{3/4} - \bar{d}K_t$$

Steady-state obtains with $\Delta K_{t+1} = 0$, so that:

$$\bar{s}Y^* = \bar{d}K^*.$$

The production function gives a second equation:

$$\bar{s}Y^* = \bar{A}\left(K^*\right)^{1/4}\bar{L}^{3/4}.$$

Again, this is a two equations/two variables system whose solution is:

$$K^* = \left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{4/3}\bar{L} \qquad Y^* = \left(\frac{\bar{s}}{\bar{d}}\right)^{1/3}\left(\bar{A}\right)^{4/3}.$$

(2) (2 pts) Show graphically that the Solow growth model converges to a steady-state. Use words to explain your graph.

Solution: See the course.

(3) (2 pts) Comment the dependence of Y^* with $\bar{A}$. Compare with the production model. Why is there a difference?

Solution: There is an exponent 4/3. The production function has linearity in total factor productivity. The reason is that capital accumulation amplifies productivity differences, because more productive countries accumulate more capital, which increases their production further.

(4) (2 pts) Imagine that the government can enact policies aimed at targeting people's savings rate $\bar{s}$. Which savings rate would maximize steady-state production ? What would steady-state consumption then be equal to?

Solution: Y^* is maximized when savings are maximum, so the savings rate maximizing steady state production would be $\bar{s} = 1$. However people would be starving in this economy. All its output would be used towards increased capital accumulation.

(5) (2 pts) Using that the steady state value for consumption is equal to $C^* = (1-\bar{s})Y^*$, show that the savings rate $\bar{s}$ has two opposing effects on consumption. Give an intuition.

Solution: $(1-\bar{s})$ decreases in $\bar{s}$ while Y^* increases in $\bar{s}$. On the one hand, steady state consumption increases when savings increase because there is then more production. But on the other, consumption decreases because a higher fraction of production is then saved rather than consumed.

(6) (2 pts) Adam Smith once wrote: "Consumption is the sole end and purpose of all production". Calculate the steady-state consumption maximizing level of the savings rate $\bar{s}$ in the Solow model.

Solution: You need to maximize the following:

$$C^* = (1-\bar{s})\left(\frac{\bar{s}}{\bar{d}}\right)^{1/3}\left(\bar{A}\right)^{4/3}.$$

The first order condition gives (note you actually just need to maximize $(1-\bar{s})(\bar{s})^{1/3}$):

$$\frac{\partial C^*}{\partial \bar{s}} = 0 \quad \Rightarrow \quad -\bar{s}^{1/3} + \frac{1}{3}\bar{s}^{-2/3}(1-\bar{s}) = 0 \quad \Rightarrow \quad \boxed{\bar{s} = \frac{1}{4} = 25\%}.$$

(7) (2 pts) Assume that $K_0 = 2$, $\bar{A} = 2$, $\bar{d} = 9\%$, $\bar{s}$ is the steady state consumption maximizing level of the savings rate, and $\bar{L} = 1$. Calculate K_t for $t = 1, 2, 3, 4$.

Solution:

$$K_1 = K_0 + \frac{1}{2}K_0^{1/4} - 0.09K_0 = K_0 + \frac{1}{2}\sqrt{\sqrt{K_0}} - 0.09K_0 \approx 2.41,$$
$$K_2 = K_1 + \frac{1}{2}K_1^{1/4} - 0.09K_1 = K_1 + \frac{1}{2}\sqrt{\sqrt{K_1}} - 0.09K_1 \approx 2.82,$$
$$K_3 = K_2 + \frac{1}{2}K_2^{1/4} - 0.09K_2 = K_2 + \frac{1}{2}\sqrt{\sqrt{K_2}} - 0.09K_2 \approx 3.21,$$
$$K_4 = K_3 + \frac{1}{2}K_3^{1/4} - 0.09K_3 = K_3 + \frac{1}{2}\sqrt{\sqrt{K_3}} - 0.09K_3 \approx 3.59.$$

(8) (2 pts) Consider now a Solow-Romer model where productivity grows because researchers engage in production. Assume that the efficiency of research is $\bar{z} = 2$, that the fraction of researchers in R&D is $\bar{l} = 3\%$, and that population is still given by $\bar{L} = 1$, and that $A_0 = 2$. What is A_t as a function only of t?

Solution: From the Romer model we know that the evolution of the number of ideas is given by:

$$\Delta A_{t+1} = \bar{z}L_{at}A_t \quad \Rightarrow \quad A_t = \left(1 + \bar{z}\bar{l}\bar{L}\right)^t A_0 = 2 * 1.06^t.$$

(9) (2 pts) As in question (6), calculate K_t for $t = 1, 2, 3, 4$ when productivity is growing as above.

Solution:

$$K_1 = K_0 + \frac{1}{2}K_0^{1/4} - 0.09K_0 = K_0 + \frac{1}{2}\sqrt{\sqrt{K_0}} - 0.09K_0 \approx 2.41,$$
$$K_2 = K_1 + \frac{1.06}{2}K_1^{1/4} - 0.09K_1 = K_1 + \frac{1.06}{2}\sqrt{\sqrt{K_1}} - 0.09K_1 \approx 2.86,$$
$$K_3 = K_2 + \frac{1.06^2}{2}K_2^{1/4} - 0.09K_2 = K_2 + \frac{1.06^2}{2}\sqrt{\sqrt{K_2}} - 0.09K_2 \approx 3.33,$$
$$K_4 = K_3 + \frac{1.06^3}{2}K_3^{1/4} - 0.09K_3 = K_3 + \frac{1.06^3}{2}\sqrt{\sqrt{K_3}} - 0.09K_3 \approx 3.84.$$

(10) (2 pts) What is the rate of growth of output in the long run, as a function of $\bar{z}$, $\bar{l}$ and $\bar{L}$, on a balanced growth path? What is a numerical value for the growth rate of output in the long run, with the given values for $\bar{z}$, $\bar{l}$ and $\bar{L}$ above? Give an intuition for why the growth rate of output and the growth rate of productivity are different.

Solution:

$$Y_t = A_t K_t^{1/4} L_{yt}^{3/4} \quad \Rightarrow \quad g_Y = g_A + \frac{1}{4}g_K.$$

Then, from balanced growth (that is, we know that in the long run, all variables K_t Y_t end up growing at the same rate, so that in particular the capital over output ratio K_t/Y_t goes to a constant that we denote from now on as $(K/Y)^*$), we have that $g_K = g_Y$ in the long run so that from the last equation:

$$g_Y = g_K = \frac{4}{3}g_A = \frac{4}{3}\bar{z}\bar{l}\bar{L}.$$

Since $g_A = 6\%$, we find that $g_Y = 8\%$ The growth rates of output and productivity are different because output also grows because of capital accumulation. In fact, in the balanced growth equilibrium, capital grows at the same rate as output, so that output overall grows faster than productivity.

Scratch Paper

Scratch Paper

Scratch Paper

Scratch Paper