Macroeconomic Theory 102
Winter 2015 - François Geerolf
Final Exam
Wednesday, March 18, 2015
Time Limit: 180 Minutes

Student ID Number: ____________

Last Name: ____________________

First Name: ____________________

Teaching Assistant: ____________________

Signature ____________________

Test A

The Final Exam contains 27 pages (including this cover page). You can earn 100 points, and 3 bonus points for the very last question.

Instructions:

1. Print your Last name, First Name, Teaching Assistant Name (as a reminder, teaching assistants are: Flavien Moreau, Keyyong Park, Matias Vieyra, and Gabriel Zaourak), Student ID Number and Signature at the top of this page.
2. The only items which should be on your desk are pencils and/or pens. NO other items are allowed. Place any other item UNDER your desk. Calculators are NOT allowed.
3. Once the exam begins, you are not allowed to leave the room until you hand in your exam.

Good luck ! Budget your time wisely ! (skip the question or even the exercice if you get stuck)

Grade Table (FOR TEACHER USE ONLY)

Question	Points	Score
1	20	
2	12	
3	18	
4	7	
5	8	
6	15	
7	23	
Total:	103	

1 Multiple Choice Questions (20 points)

1. Mark box if true - each multiple choice question has only one correct answer.

   (a) (1 point) In the Solow model, with population growth:

   - ◯ there is no steady state in output per person
   - ◯ the economy never settles down to a steady state and exhibits growth in output per person
   - ◯ the economy eventually settles down to a steady state in output per person
   - ◯ the economy eventually settles down to a steady state with no growth in aggregate output
   - ◯ None of these answers are correct.

   (b) (1 point) In the combined Solow-Romer model, the total output growth rate:

   - ◯ equals the growth rate of ideas.
   - ◯ is greater than the growth rate of ideas.
   - ◯ is lower than the growth rate of ideas
   - ◯ equals the rate of capital depreciation
   - ◯ equals the growth in labor productivity.

   (c) (1 point) The Great Depression stimulated __________ to write __________, which is considered to be the birth of modern macroeconomics.

   - ◯ John Hicks; *Value and Capital*
   - ◯ Karl Marx; *Das Kapital*
   - ◯ David Ricardo; *Principles of Political Economy and Taxation*
   - ◯ Thomas Piketty; *Capital in the 21st century*
   - ◯ John Maynard Keynes; *The General Theory of Employment, Interest, and Money*

   (d) (1 point) Suppose you put \$100 dollars in the bank on January 1, 2015. If the annual nominal interest rate is 2 percent, and the inflation rate is 5 percent, you will be able to buy ________ worth of goods on January 1, 2016.

   - ◯ \$95
   - ◯ \$102
   - ◯ \$97
   - ◯ \$103
   - ◯ -\$3.

   (e) (1 point) Compared to the nominal interest rate, the real interest rate is:

   - ◯ negative
   - ◯ always smaller
   - ◯ always greater than zero
   - ◯ relatively stable

- ◯ relatively volatile.

(f) (1 point) The costs associated with changing prices in times of inflation are called:
- ◯ inflation risks
- ◯ price staggering
- ◯ transaction costs
- ◯ shoe-leather costs
- ◯ menu costs.

(g) (1 point) When we add the financial friction to the AD curve it:
- ◯ is represented by a downward movement along the AD curve
- ◯ is represented by a downward movement along the AS curve
- ◯ shifts the AD curve down
- ◯ shifts the AS curve up
- ◯ has no impact on the AD curve.

(h) (1 point) "DSGE" stands for:
- ◯ dynamic stochastic general equilibrium
- ◯ deterministic simulated generalized estimation
- ◯ demand supply generated effects
- ◯ discrete stationary generated equilibrium
- ◯ demand and supply government expenditures.

(i) (1 point) Suppose a worker gets a new computer; this would:
- ◯ decrease the nominal wage
- ◯ increase total factor productivity
- ◯ increase the amount of leisure demanded
- ◯ increase the marginal product of labor
- ◯ None of these answers are correct

(j) (1 point) Using the labor market in the stylized DSGE model, from workers' perspectives, which of the following would be beneficial to them? (i) a reduction in the sales tax rate, (ii) a reduction in the income tax rate, (iii) a reduction in TFP.
- ◯ (ii)
- ◯ (ii) and (iii)
- ◯ (iii)
- ◯ (i), (ii) and (iii)
- ◯ (i) and (ii)

(k) (1 point) If the growth rate of the capital stock is 9 percent, the user cost of capital is 13 percent, the capital depreciation rate is 3 percent, and capital gain is 1 percent, then the investment rate is about:

- ○ 6.6 percent
- ○ 30.8 percent
- ○ 11.7 percent
- ○ 1.4 percent
- ○ 50 percent

(l) (1 point) The investment-GDP ratio will rise if:

- ○ depreciation falls.
- ○ user cost falls.
- ○ capital growth is negative.
- ○ the marginal product of labor is positive.
- ○ stock prices fall.

(m) (1 point) When capital depreciation is included in the arbitrage equation for capital, the user cost of capital is given by:

- ○ $MPK + R + \bar{d}$
- ○ $\frac{R\bar{d}}{MPK}$
- ○ $R + \bar{d} - \frac{\Delta p_k}{p_k}$
- ○ $R + \bar{d} - MPK$
- ○ $\bar{d} + \Delta p_s$

(n) (1 point) Let P denote the price of goods in the United States, P^w denote the price of goods in the foreign country, and E the exchange rate, measured as the number of units of foreign currency that can be purchased with one dollar. According to the law of one price:

- ○ $P = EP^w$
- ○ $P^w = EP$
- ○ $E = PP^w$
- ○ $P^w = E + P$
- ○ $P = E + P^w$

(o) (1 point) An explanation for a potential depreciation of the dollar vis-à-vis the euro is that:

- ○ incomes are higher in the euro area than in the United States
- ○ interest rates are higher in the United States than in the euro area
- ○ inflation in the United States is higher than in the euro area
- ○ there is less risk in the United States than in the euro area
- ○ None of these answers are correct.

(p) (1 point) Which of the following can be used to explain the failure of the law of one price with respect to Big Macs?

- ◯ transportation costs
- ◯ wage equalization across borders
- ◯ differences in the money supply
- ◯ over- or undervalued currency
- ◯ real interest rate differences.

(q) (1 point) Free flow of international assets is desirable because it allows countries to:

- ◯ borrow when times are bad in order to smooth consumption
- ◯ borrow when times are good in order to increase consumption
- ◯ buy in higher value asset markets
- ◯ hold gold
- ◯ maintain price stability.

(r) (1 point) From the national income identity, we have:

- ◯ $NX < 0 \quad \Rightarrow \quad (C - T) + I + G < Y.$
- ◯ $NX > 0 \quad \Rightarrow \quad C + I + G > Y.$
- ◯ $NX < 0 \quad \Rightarrow \quad C + I + G > Y.$
- ◯ $NX < 0 \quad \Rightarrow \quad (C - T) + I + G > Y$
- ◯ None of these answers is correct.

(s) (1 point) Suppose, in the North, one unit of labor produces f_n units of fish or c_n units of chips. In the South, one unit of labor produces f_s units of fish or c_s units of chips. For the South to specialize in fish, which of the following must be true?

- ◯ $f_s > c_s$
- ◯ $\frac{f_s}{c_s} > \frac{f_n}{c_n}$
- ◯ $\frac{c_s}{f_s} < \frac{c_n}{f_n}$
- ◯ $\frac{f_s}{c_s} < \frac{f_n}{c_n}$
- ◯ $f_s c_s < f_n c_n$.

(t) (1 point) Free labor migration is more effective at improving welfare in low-income countries than moving capital because:

- ◯ it moves labor to the place where productivity is high.
- ◯ low-income countries do not have much capital per person.
- ◯ it moves labor to the place where there is a lot of capital per person.
- ◯ it moves labor to the place where there is not much capital per person.
- ◯ it is cheaper to move labor than capital.

2 Short-Run: an Oil Price Shock (12 points)

2. Take the usual AS/AD model, ruling out Aggregate Demand shocks, so with $\bar{a} = 0$, but assuming a one-time, unexpected oil price shock $\bar{o}_0 > 0$. One time means that the oil price shock lasts only for one period, in period $t = 0$, and that $\bar{o}_t = 0$ for all subsequent $t \in \{1, 2, ...\}$. Unexpected means that the economy was originally in steady-state, and in particular that $\pi_0 = \bar{\pi}$. Unless otherwise noted, agents have adaptive expectations about inflation. The economy is described by an AS/AD model. In particular, the AS curve is given by (be careful about the convention on the timing of the oil shock $\bar{o}_{t-1}$!):

$$\pi_t = \pi_t^e + \bar{\nu}\tilde{Y}_t + \bar{o}_{t-1}.$$

The AD curve is the standard one used throughout the course.

(a) (1 point) What are the values of π_1 and $\tilde{Y}_1$ in terms of the parameters of the model? (in particular the size of the oil price shock, $\bar{o}_0$)

(b) (1 point) Show analytically[1] the effect of a more agressive monetary policy on inflation and short-run output in period 1: do inflation and short-run output increase or decrease with a more agressive monetary policy?

[1] That is, in mathematical terms.

(c) (1 point) Illustrate this on two graphs with the AS/AD curves: show one AS/AD diagram with a soft monetary policy, and next to it another AS/AD diagram with an agressive monetary policy. Show π_1 and $\tilde{Y}_1$ as well as the long run values of inflation and short-run output on these graphs.

(d) (1 point) Eliminating $\tilde{Y}_t$ from the AS/AD model, find a difference equation[2] for π_t, for $t \in \{2, 3, ...\}$.

[2] A difference equation is an expression of an economic quantity as a function of its previous (lagged) values, generally the value in the previous period. For example π_t expressed as a function of π_{t-1} is a difference equation for π_t

(e) (1 point) Substracting $\bar{\pi}$ on both sides in the difference equation for π_t, show that $\pi_t - \bar{\pi}$ satisfies a simpler difference equation than π_t.[3] Solve for this difference equation. This should give you an expression for π_t as a function of π_1. Then replace π_1 with the value found in question (a), to get π_t as a function of time and the parameters of the model.

(f) (1 point) Use the (AD) curve to then calculate $\tilde{Y}_t$ as a function of time and the parameters of the model.

[3] Simpler in the sense that you can solve for it. For example, a simple difference equation is one of the form $u_t = \rho u_{t-1}$, whose solution is $u_t = \rho^{t-1} u_1$.

(g) (1 point) Numerical Application: Suppose the parameters of the AS and AD curves take the following values: $\bar{o}_0 = 2\%$, $\bar{a} = 0$, $\bar{b} = 1/2$, $\bar{m} = 1/2$, $\bar{\nu} = 1/2$, and $\bar{\pi} = 2\%$. Solve for the value of short-run output and the inflation rate for the first 2 years after the shock. (express your result as a single fraction, since you do not have a calculator !)

(h) (1 point) Calculate how much realized inflation differs from expected inflation, or $\pi_t - \pi_t^e$, for any $t \geq 1$, in this model. Simplify the expression so that its sign appears clearly.[4]

[4] That is, it should be clear from your expression of $\pi_t - \pi_t^e$ whether it is positive or negative

(i) (1 point) Represent on a graph the behavior of inflation π_t as a function of time. In particular, represent $t = 0$, $t = 1$, $t = +\infty$, $\bar{\pi}$ and π_1 on this graph.

(j) (1 point) Represent on a graph the behavior of short-run output $\tilde{Y}_t$ as a function of time. In particular, represent $t = 0$, $t = 1$, $t = +\infty$, $\tilde{Y}_\infty$ and $\tilde{Y}_1$ on this graph.

(k) (1 point) Why are these adaptive expectations where $\pi_t^e = \pi_{t-1}$ considered as "non rational" ? What is irrational about them?

(l) (1 point) What would $\pi_t - \pi_t^e$ be equal to under rational expectations?

3 Short-Run: an Aggregate Demand Shock (18 points)

3. Take the usual AS/AD model, ruling out Oil Price shocks, so with $\bar{o} = 0$, but assuming a one-time, unexpected aggregate demand shock $\bar{a}_1 > 0$, materializing in the period 1 (AD) demand equation. One time means that the aggregate demand shock lasts only for one period, in period $t = 1$, and that $\bar{a}_t = 0$ for all subsequent $t \in \{2, 3, ...\}$, and also that $\bar{a}_0 = 0$. Unexpected means that the economy was originally in steady-state, so that $\tilde{Y}_0 = 0$ and $\pi_0 = \bar{\pi}$. Unless otherwise noted, agents have adaptive expectations about inflation. The economy is described by an AS/AD model. In particular, the AD curve is given by (be careful about the convention on the timing of the aggregate demand shock $\bar{a}_t$!):

$$\tilde{Y}_t = \bar{a}_t - \bar{b}\bar{m}(\pi_t - \bar{\pi}).$$

From the above assumptions on the aggregate demand shock, the AD curve is given as follows for $t \in \{0, 2, 3, ...\}$:

$$\tilde{Y}_t = -\bar{b}\bar{m}(\pi_t - \bar{\pi}).$$

For $t = 1$, the AD curve is in contrast given by:

$$\tilde{Y}_1 = \bar{a}_1 - \bar{b}\bar{m}(\pi_1 - \bar{\pi}).$$

The AS curve is the standard one used throughout the course, and has no oil price shock.

(a) (2 points) What are the values of π_1 and $\tilde{Y}_1$ in terms of the parameters of the model? (in particular the size of the aggregate demand shock, $\bar{a}_1$)

(b) (2 points) Show analytically[5] the effect of a more agressive monetary policy on inflation and short-run output in period 1: do inflation and short-run output increase or decrease with a more agressive monetary policy?

(c) (1 point) Illustrate this on two graphs with the AS/AD curves: show one AS/AD diagram with a soft monetary policy, and next to it another AS/AD diagram with an agressive monetary policy. Show π_1 and $\tilde{Y}_1$ as well as the long run values of inflation and short-run output on these graphs.

[5] That is, in mathematical terms.

(d) (1 point) Eliminating $\tilde{Y}_t$ from the AS/AD model, find a difference equation[6] for π_t, for $t \in \{2, 3, ...\}$.

(e) (2 points) Substracting $\bar{\pi}$ on both sides in the difference equation for π_t, show that $\pi_t - \bar{\pi}$ satisfies a simpler difference equation than π_t.[7] Solve for this difference equation. This should give you an expression for π_t as a function of π_1. Then replace π_1 with the value found in question (a), to get π_t as a function of time and the parameters of the model.

[6] A difference equation is an expression of an economic quantity as a function of its previous (lagged) values, generally the value in the previous period. For example π_t expressed as a function of π_{t-1} is a difference equation for π_t

[7] Simpler in the sense that you can solve for it. For example, a simple difference equation is one of the form $u_t = \rho u_{t-1}$, whose solution is $u_t = \rho^{t-1} u_1$.

(f) (2 points) Use the (AD) curve to then calculate $\tilde{Y}_t$ as a function of time and the parameters of the model, for $t \in \{2, 3, ...\}$.

(g) (1 point) What is the sign of $\tilde{Y}_t$ (positive or negative?) for $t \in \{2, 3, ...\}$?

(h) (3 points) What is the economic intuition for that ? Explain precisely.

(i) (1 point) Represent on a graph the behavior of inflation π_t as a function of time. In particular, represent $t = 0$, $t = 1$, $t = +\infty$, $\bar{\pi}$ and π_1 on this graph.

(j) (2 points) Represent on a graph the behavior of short-run output $\tilde{Y}_t$ as a function of time. In particular, represent $t = 0$, $t = 1$, $t = 2$, $t = +\infty$, and $\tilde{Y}_0$, $\tilde{Y}_1$, $\tilde{Y}_2$, $\tilde{Y}_\infty$ on this graph.

(k) (1 point) Why is this exercice illustrating the fundamental reason why central banks want to stabilize short-run output?

4 Long-Run: The Solow Growth model (7 points)

4. Consider an Solow economy with the following Cobb-Douglas, constant returns to scale, production function:

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}.$$

Assume that of this production, a fraction $\bar{s}$ is saved (and invested), and a fraction $1 - \bar{s}$ is consumed. Assume that the labor force is fixed: $L_t = \bar{L}$. Assume that capital depreciates at rate $\bar{d}$.

(a) (1 point) Write the law of motion for capital.[8] Solve for the steady state value of capital K^*, and the steady state value of Y^*, in this economy.

(b) (1 point) Comment the dependence of Y^* with $\bar{A}$. Compare with the production model. Is there a difference? Why?

[8] A law of motion for u_t is an expression of the form: u_{t+1} as a function of u_t, or Δu_{t+1} as a function of u_t.

(c) (1 point) Draw the Solow Diagram in this case: represent capital K on the x axis, and investment, depreciation, and output, on the y axis. Represent Y^* and K^* on this diagram.

(d) (1 point) Draw the same diagram again, but only with depreciation and investment on the y axis. Show on the graph how the economy converges to the steady state if it starts with a higher than steady-state level of capital.

(e) (1 point) Imagine that the government can enact policies aimed at targeting people's savings rate $\bar{s}$. Which savings rate would maximize steady-state production ? What would steady-state consumption then be equal to?

(f) (1 point) Using that the steady state value for consumption is equal to $C^* = (1 - \bar{s})Y^*$, show that the savings rate $\bar{s}$ has two opposing effects on consumption. Give an intuition.

(g) (1 point) Adam Smith once wrote: "Consumption is the sole end and purpose of all production". Calculate the steady-state consumption maximizing level of the savings rate $\bar{s}$ in the Solow model.

5 Long-Run: the AK model (8 points)

5. Consider a Solow model where the production function no longer exhibits diminishing returns to capital accumulation. In other words, now $Y_t = \bar{A}K_t$. This is not particularly realistic, for reasons discussed during the course. But it is interesting to consider this case nonetheless because of what it tells us about the workings of the Solow model. The rest of the Solow model is unchanged: the savings rate is still $\bar{s}$, and the depreciation rate is still $\bar{d}$. The initial capital stock is given, equal to $\bar{K}_0$.

   (a) (1 point) Write the law of motion for capital.

   (b) (2 points) Solve for K_t analytically[9] as a function of K_0, time t, and the parameters of the model. Solve for Y_t analytically, also as a function of K_0, time t, and the parameters of the model.

   (c) (1 point) Assume that the savings rate is relatively high, and in particular that $\bar{s} > \frac{\bar{d}}{\bar{A}}$. What happens to the capital stock and to output over time? Give an economic intuition for this result.

[9] Analytically means in mathematical terms.

(d) (1 point) Illustrate this with the help of a Solow Diagram. Show K_0 on this graph, K_1, K_2, etc.

(e) (1 point) Assume that the savings rate is relatively low, in the sense that $\bar{s} < \frac{\bar{d}}{\bar{A}}$. What happens to the capital stock and to output over time? What is the economic intuition for this result?

(f) (1 point) Illustrate this with the help of a Solow Diagram. Show K_0 on this graph, K_1, K_2, etc.

(g) (1 point) Assume that the savings rate is exactly such that $\bar{s} = \frac{\bar{d}}{\bar{A}}$. What happens then? Why?

6 Microfoundations: The Neoclassical Consumption Model with Log Utility and $\beta \neq 1$ (15 points)

6. Consider a consumer consuming goods in period 0 ("today") in quantity c_0, and goods in period 1 ("tomorrow") in quantity c_1. Assume that in period 0, the consumer is born with a financial wealth equal to f_0, and that he chooses his level of **savings** f_1 (or, equivalently, of **saving** $f_1 - f_0$) to maximize a lifetime utility function given by:

$$U = \log(c_0) + \beta \log(c_1).$$

The consumer also earns a labor income equal to y_0 in period 0, and equal to y_1 in period 1. The interest rate he earns on his savings is given by R.

(a) (1 point) Write the two budget constraints that the consumer faces in period 0, and period 1.

(b) (2 points) Eliminate f_1 from these equations so as to write the consumer's intertemporal budget constraint. Show the steps you use to get to this expression. (you won't get any credit if you don't)

(c) (2 points) Maximize the lifetime utility function of the consumer under the previous intertemporal budget constraint to derive the Euler equation. (again, please give details to get credit)

(d) (1 point) What is the intuition for this equation?

(e) (1 point) Using the Euler equation and the intertemporal budget constraint, solve for c_0 and c_1 as a function of the parameters of the model.

(f) (1 point) What is the marginal propensity to consume out of current earnings?

(g) (1 point) Why does it not depend on the interest rate?

(h) (1 point) How does it vary with β ? What is the intuition for that?

(i) (1 point) What is the marginal propensity to consume out of future earnings?

(j) (1 point) Why does it depend on the interest rate?

(k) (1 point) What is the marginal propensity to consume out of current wealth f_0?

(l) (2 points) Assume that $\beta = 1$, and that a consumer were to live and consume 70 periods instead of just 2. How would you then write his lifetime utility function? What would marginal propensity to consume out of wealth then be?

7 Long-Run / Microfoundations: a Growth model with an endogenous saving behavior (20 points + 3 bonus points)

7. Consider a closed economy with the following Cobb-Douglas, constant returns to scale, production function:

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}.$$

Assume that the labor force is fixed to unity: $L_t = \bar{L} = 1$. Assume that labor is taxed at rate τ (that is, if the employer pays w_t in wages, the worker receives only $(1-\tau)w_t$). Assume that capital depreciates at rate $\bar{d} = 1 = 100\%$. (that is, capital fully depreciates each period)[10]

(a) (1 point) Assuming firms are competitive and maximize profits, what is the wage w_t paid by employers in period t as a function of $\bar{A}$ and the level of the capital stock in period t, K_t ?

(b) (1 point) What is the wage received by workers, as a function of τ, $\bar{A}$ and the level of the capital stock in period t, K_t ?

(c) (2 points) Represent the labor market (labor supply and labor demand), with employment on the x axis, and the wage w_t on the y axis. Show the effect of the imposition of a tax. Show the deadweight loss.

[10] We will later interpret the length of time between t and $t+1$ as a generation, or about $30 - 40$ years.

(d) (1 point) Who effectively pays the tax? What is the economic intuition for that?

(e) (1 point) Taking K_t as given, what is the effect of the tax on output Y_t?

From now on, and until the end of this exercice, we assume that $\tau = 0$. Moreover, instead of assuming that savings are a fixed fraction $\bar{s}$ of output, we will instead derive the saving behavior from microfoundations.

We assume that people in this economy live only for two periods. However, instead of calling them "0" and "1", or "today" and "tomorrow", we now call them "t" and "$t+1$. People are called "young" in the first period of their life, and "old" in the second. People from generation t are young in period t, and old in period $t+1$. We denote their consumption when young by c_t^y and their consumption when old by c_{t+1}^o. In terms of the lectures, you should really think of c_t^y as c_0, and of c_{t+1}^o as c_1. People work when young, and then receive a wage given by w_t (remember, we assumed $\tau = 0$). They retire when old, and then do not work. Their lifetime utility is logarithmic with $\beta = 1$:

$$U = \log(c_t^y) + \log(c_{t+1}^o).$$

Their intertemporal budget constraint is given by:

$$c_t^y + \frac{c_{t+1}^o}{1+R} = w_t.$$

There is always two generations living in period t: the previous period's young, born in period $t-1$, now old, consuming the return from their savings; and this period's young, newly born (in period t).

(f) (2 points) Calculate c_t^y and c_{t+1}^o as a function of w_t. (you can give the solution without an explanation) Calculate saving by the young, given by $w_t - c_t^y$.

(g) (1 point) What is the relationship between savings by the young and investment in this economy? (Hint: remember that the economy is closed !)

(h) (3 points) Write the law of motion for capital as a function of the parameters of the model. Note that it takes a very similar form as that in the Solow model. What is the savings rate in this model corresponding to $\bar{s}$ in the Solow model?

(i) (1 point) Write the arbitrage condition for the firm between the marginal product of capital, depreciation, and the interest rate R, assuming that the price of capital is constant and equal to one.

(j) (2 points) What is an expression for R as a function of K_t and the parameters of the model?

Assume now that people work half time when young, and half time when old, earning wage $w_t/2$ in each period (total labor supply is still 1 in each period), so that the new intertemporal budget constraint is given by:

$$c_t^y + \frac{c_{t+1}^o}{1+R} = \frac{w_t}{2} + \frac{w_t}{2(1+R)}.$$

(k) (3 points) Write the law of motion for capital ONLY as a function of the parameters of the model. (in particular, substitute out R)

(l) (2 points) Why does it differ strikingly from the Solow model?

(m) (3 points) **Bonus:** With an analogy to the Solow model, explain how you could can nevertheless solve for this model. Show that the steady-state level of capital accumulation is quite low. What is the intuition for that? (use the back of the sheet if needed)